Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2018

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

JTMATHE2

This question paper consists of 15 pages, including 1 information sheet, and a special answer book.

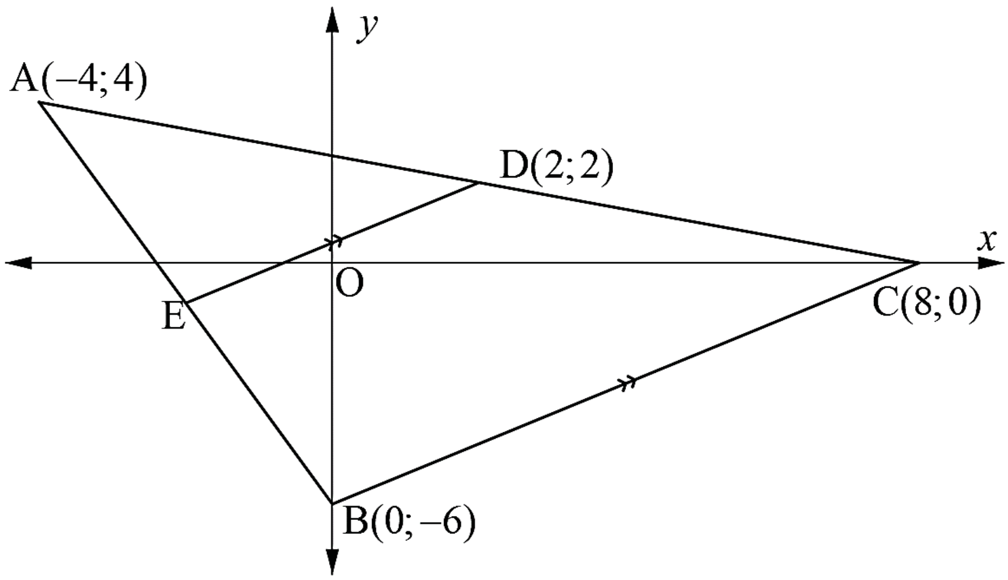
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Diagrams are not necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

In the diagram below $A(-4;4)$, $B(0;-6)$ and $C(8; 0)$ are the vertices of ΔABC with $D(2; 2)$ and E on AC and AB respectively such that $BC \parallel DE$.



Determine:

1.1 The length of BC (3)

1.2 The coordinates of E, the midpoint of AB (2)

1.3 The gradient of BC (2)

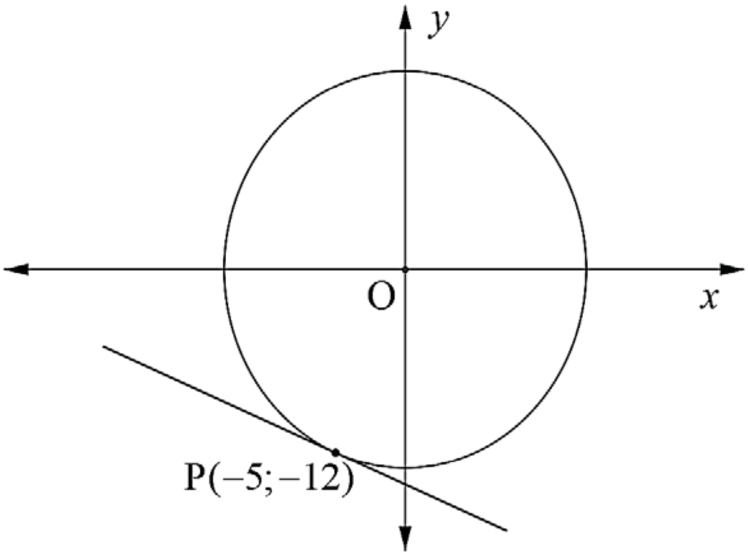
1.4 The equation of the line passing through points D and E (4)

1.5 The size of $A\hat{B}C$ (6)

[17]

QUESTION 2

2.1 The diagram below shows a circle with centre at the origin O with the tangent passing through point $P(-5;-12)$.



Determine:

2.1.1 The equation of the circle. (3)

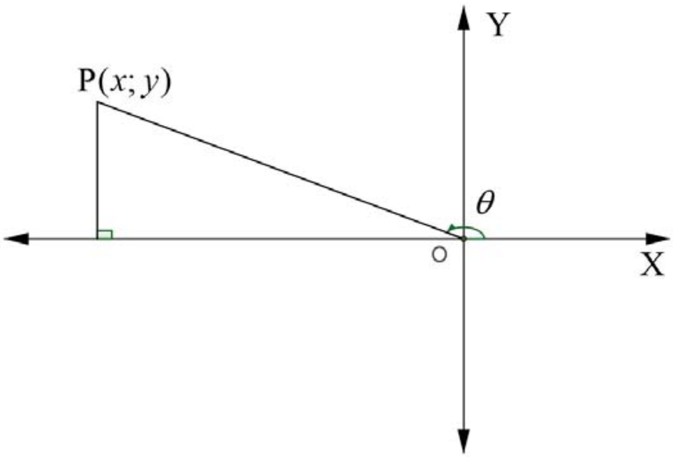
2.1.2 The equation of the tangent to the circle at point P, in the form $y = mx + c$ (4)

2.2 Sketch the graph defined by $4x^2 + 9y^2 = 36$. Clearly show all intercepts with axes. (5)

[12]

QUESTION 3

3.1 In the diagram below, it is given that $13\sin\theta = 12$ and $\theta \in [90°;180°]$.
Use the diagram as provided in the SPECIAL ANSWER BOOK and answer the questions that follows.



3.1.1 Determine the coordinates of P. (4)

3.1.2 Determine the numerical value of $\tan\theta + \sec\theta$. (3)

3.1.3 Determine the size of θ, rounded off to ONE decimal place. (2)

3.2 Simplify the following:

$$\frac{\tan(180° - \theta).\sqrt{1 - \sin^2\theta}}{\cos^2(180° + \theta) + \sin^2(360° - \theta)}$$ (7)

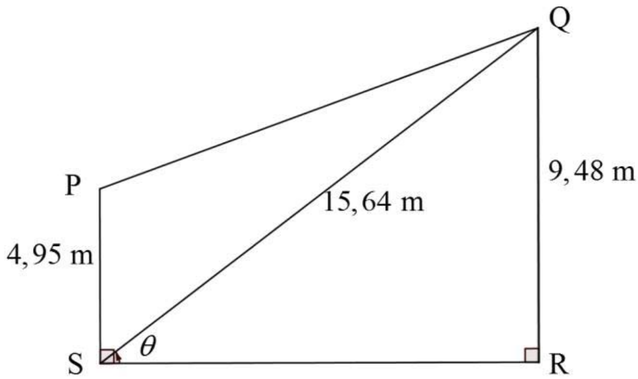
3.3 Prove, using basic trigonometry identities, that:

$$\frac{1 + \cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{1 + \cos\alpha} = 2\operatorname{cosec}\alpha$$ (6)

[22]

QUESTION 4

In the figure below, PS and QR represent two poles perpendicular to SR ground level. Furthermore, the angle of elevation, from S to Q is θ. The height of the one pole, PS = 4,95 m and the other pole QR = 9,48 m. The poles are connected to each other with two cables, namely PQ and SQ, with SQ = 15,64 m.



4.1 Determine the size of the angle of elevation, θ. (2)

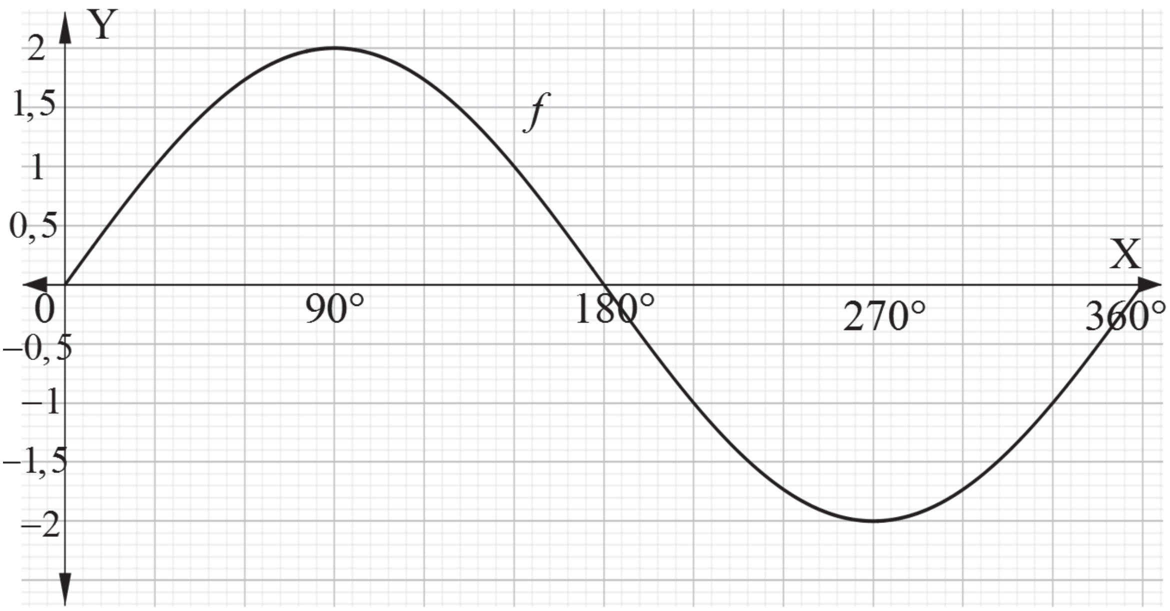
4.2 Determine the area of ΔPSQ. (4)

4.3 Calculate the distance between P and Q, rounded off to TWO decimal digits. (5)

[11]

QUESTION 5

Given $f(x) = 2\sin x$ and $x \in [0°; 360°]$



5.1 Use the figure in the SPECIAL ANSWER BOOK and draw on the same set of axes, the graph of $g(x) = \cos(x - 30°)$. (4)

5.2 Use your graphs to answer the following questions.

5.2.1 Write down the amplitude of f. (1)

5.2.2 Write down the range of g. (2)

5.2.3 Which values of x are $f(x) - g(x) = 1,5$? (1)

5.2.4 Which values of x are $f(x) \leq g(x)$? (3)

[11]

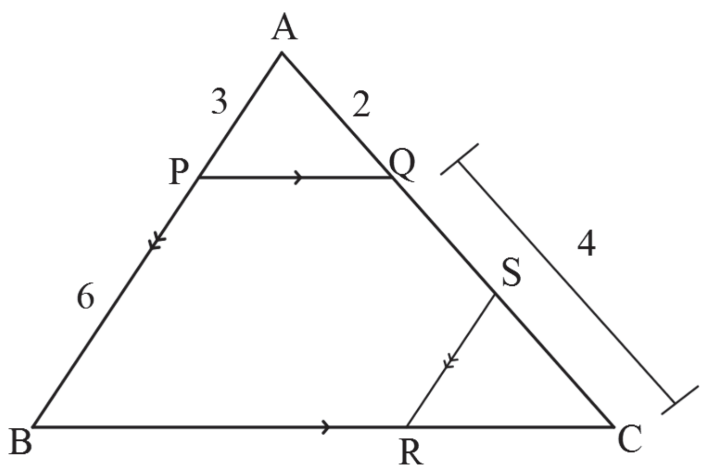
Give reasons for ALL your statements in QUESTION 6, 7 AND 8.

QUESTION 6

6.1 Complete the following statement:

A line parallel to one side of a triangle, divides the other two sides ... (1)

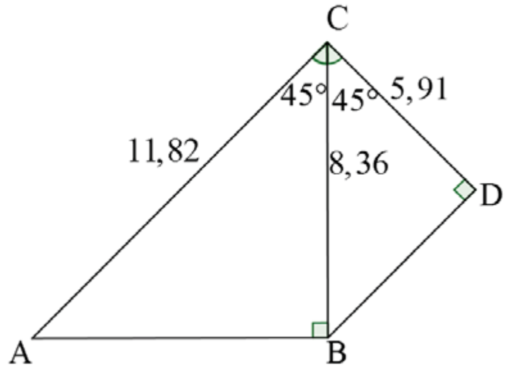
6.2 In ΔABC, AP = 3, PB = 6, AQ = 2 and QC = 4 units. PQ || BC and RS || BA.



If CR : RB = 1 : 3, calculate QS. (7)
[8]

QUESTION 7

7.1 In the right-angled triangle ABC, AC = 11,82 cm, BC = 8,36 cm and $A\hat{C}B = 45°$.
In ΔDCB, CD = 5,91 cm and $D\hat{C}B = 45°$.



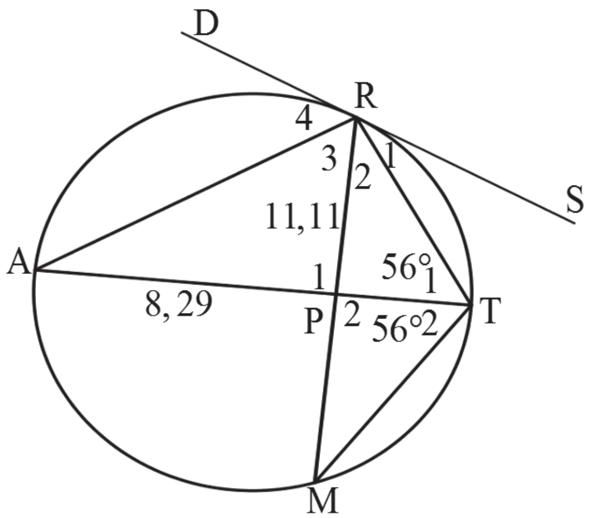
7.1.1 Write down with reasons the lengths of AB and BD. (3)

7.1.2 Determine the following ratios: $\frac{AB}{BD}, \frac{BC}{DC}$ and $\frac{AC}{BC}$ (3)

7.1.3 Use your answers obtained in QUESTION 7.1.2 to make a conclusion about ΔABC and ΔBDC. (1)

7.2 In the diagram below, DRS is a tangent to the circle, ARTM. Chords AT and RM intersect at P.

$\hat{T}_1 = 56° = \hat{T}_2$



7.2.1 Determine the size of the following angles:

(a) $\hat{R}_3$ (2)

(b) $\hat{R}_4$ (2)

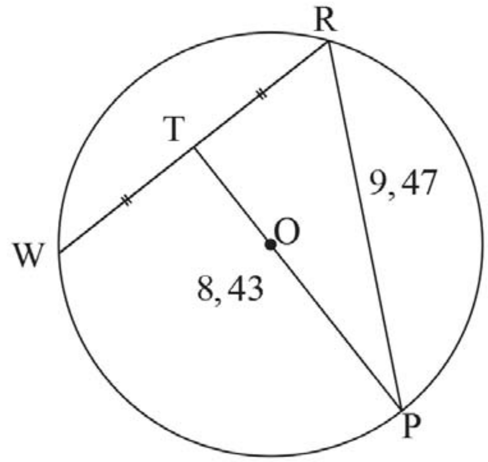
7.2.2 Write down two angles that are equal to $\hat{A}$. (2)

7.2.3 Given $\Delta APR \parallel\!| \Delta MPT$ with PR = 11,11 cm, AP = 8,29 cm and AT = 12,01 cm, calculate the length of MP. (5)

[18]

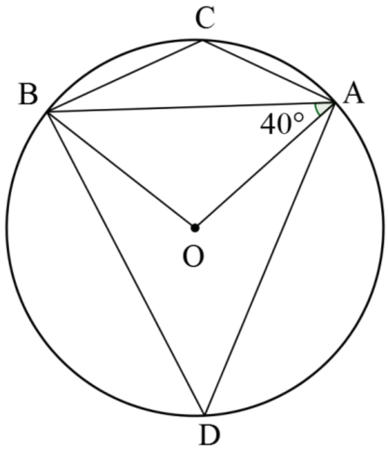
QUESTION 8

8.1 In the circle below, O is the centre, PO is a radius extended to bisect WR at T.
PR = 9,47 units and TP = 8,43 units.



Calculate the length of the chord WR. (6)

8.2 In the diagram below, O is the centre of the circle and $B\hat{A}O = 40°$.

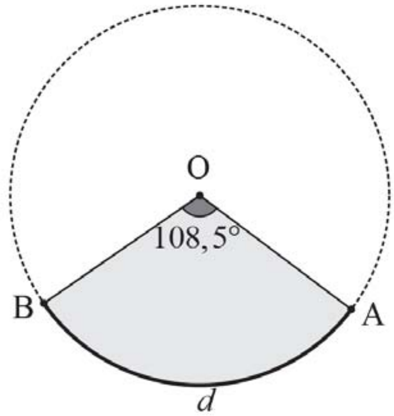


Calculate the size of the angle $B\hat{C}A$. (6)

[12]

QUESTION 9

9.1 In the diagram below, O is the centre of the circle $A\hat{O}B = 108,5°$. The diameter of the circle is 10,84 cm in length.



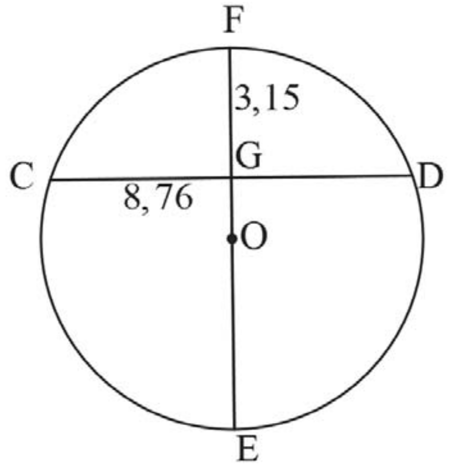
9.1.1 Convert 108,5° to radians. (2)

9.1.2 Determine the radius. (1)

9.1.3 Determine, ***d***, the length of the arc of the shaded sector. (3)

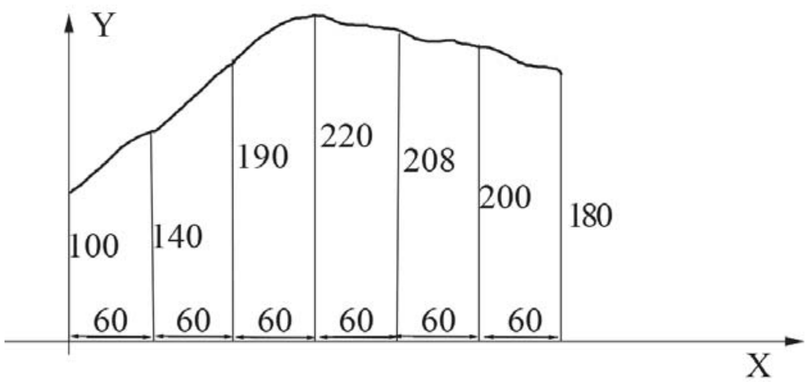
9.1.4 Determine the area of the shaded sector. (3)

9.2 In the diagram below the length of chord, CD = 8,76 units. The height of the smaller segment, FG = 3,15 units.



Calculate EF, the length of the diameter of the circle. (4)

9.3 The picture below represents a piece of off-cut metal. To determine the area of the metal, it is divided into equal sections on the horizontal with different heights as illustrated in the diagram.



Calculate the approximated area of the piece of metal by using the mid-ordinate rule. Measurements are in mm. Give your answer in cm^2. (5)

[18]

QUESTION 10

10.1 A wheel of diameter 570 mm rotates at 15 revolutions per second.

Calculate:

10.1.1 The angular velocity (3)

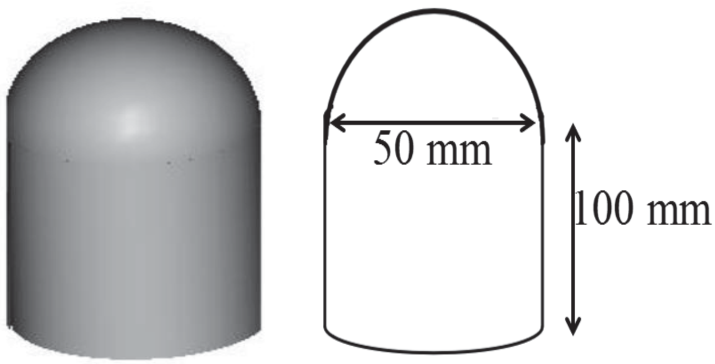
10.1.2 The circumferential velocity in m/s (4)

10.1.3 The distance that a point on the circumference of the wheel will cover in 2 min, to the nearest meter (4)

10.1.4 The angular displacement during 0,3 s (3)

10.2 A shape as shown below is made from a cylinder, 50 mm in diameter and 100 mm long with a hemisphere on one end.

$$\text{Volume} = \pi r^2 h\ ; \quad \text{Volume} = \frac{1}{3}\pi r^2 h \quad \text{Volume} = \frac{4}{3}\pi r^3$$



Determine the volume of the shape. (7)
[21]

TOTAL: 150

INFORMATION SHEET

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\dfrac{b}{2a}$ $\qquad y = \dfrac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b \;\; a > 0 \;,\; a \neq 1 \text{ and } b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$

$A = P(1 + i)^n$ $\qquad A = P(1 - i)^n$ $\qquad i_{eff} = \left(1 + \dfrac{i^m}{m}\right)^m - 1$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad \mathrm{M}\left(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$ $\qquad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ $\qquad \int x^n dx = \dfrac{x^{n+1}}{n+1} + C \quad , \; n \neq -1$

$\int \dfrac{1}{x} dx = \ln(x) + C, \quad x > 0$ $\qquad \int a^x dx = \dfrac{a^x}{\ln a} + C \quad , \; a > 0$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360^0 n$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

Area of a sector $= \dfrac{rs}{2} = \dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

In ΔABC:

$\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos \mathrm{A}$ $\qquad \text{Area} = \dfrac{1}{2} ab.\sin \mathrm{C}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad \cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\mathrm{A}_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, o_i = i^{th} ordinate and n = number of ordinates

OR

$\mathrm{A_T} = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates