Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2019

TECHNICAL MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

JTMATHE1

This question paper consists of 15 pages, 1 answer sheet and a formula sheet consisting of 2 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions.

3. Answer QUESTION 3.3 and QUESTION 5.3 on the ANSWER SHEET provided. Write your name in the spaces provided and then hand in the ANSWER SHEET with your ANSWER BOOK.

4. Number the answers correctly according to the numbering system used in this question paper.

5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

6. Answers only will not necessarily be awarded full marks.

7 You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. If necessary, round off to TWO decimal places, unless stated otherwise.

9. Diagrams are NOT necessarily drawn to scale.

10. Write neatly and legibly.

QUESTION 1

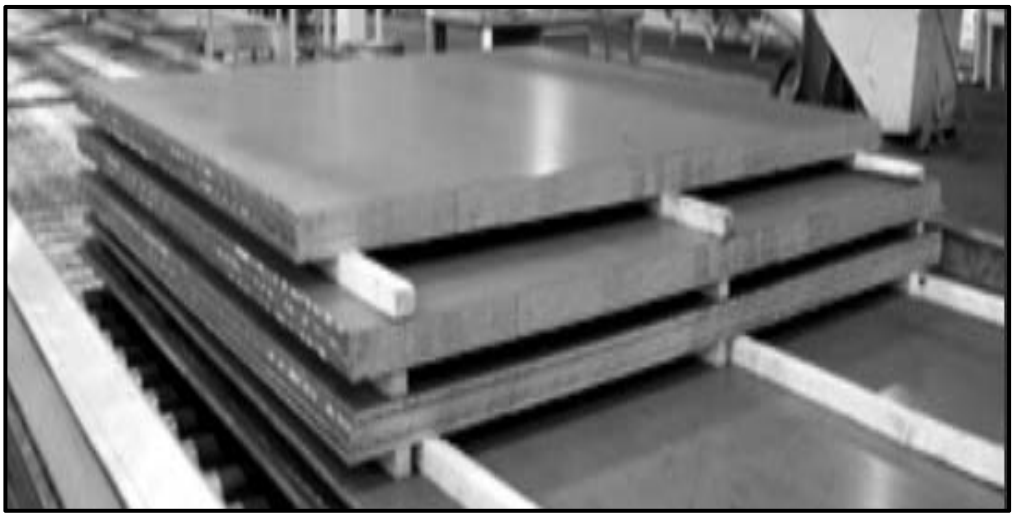
1.1 Solve for x:

1.1.1 $2(7x-1)(x+2)=0$ (2)

1.1.2 $(x-2)(3x-1)=1$ (correct to TWO decimal places) (4)

1.1.3 $-x^2-4x+5\geq 0$ (3)

1.2 The picture below shows packaged metal sheets by a metal sheet manufacturing company.



Each pack contains 300 metal sheets and has a thickness of 151 mm.

Determine the thickness (in metres) of one metal sheet in **Scientific Notation**. (3)

1.3 Given: $2y+6x=4$ and $y^2-25x^2=4$

1.3.1 Factorise y^2-25x^2 (1)

1.3.2 Hence or otherwise, solve for x and y simultaneously (6)

1.4 Given: $\text{M}=\sqrt{\dfrac{5-p}{2}}$; $p\in\{1;2;3;4;5;6\}$

Determine the:

1.4.1 Value(s) of p for which M will be real (2)

1.4.2 Value(s) of p for which M will be non-real (1)

1.5 Describe the nature of the roots of $ax^2+bx+c=0$ if $a<0,\ b>0$ and $c=0$. (2)

1.6 Write 86 as a binary number. (2)

[26]

QUESTION 2

2.1 Simplify the following without using a calculator:

2.1.1 $3^n.3^4$, to a single base term (1)

2.1.2 $$\frac{7.3^{n+2}}{3^{n+4}-6.3^{n+1}}$$ (2)

2.1.3 $\sqrt{32}-\sqrt{72}+\sqrt{18}$ (2)

2.1.4 $-\log_3 243+\log_3 1$ (2)

2.2 Solve for x:

2.2.1 $$\frac{\left(4^x\right)^{2x}.\sqrt{16^{-3}}}{4^x}=\left(4^x\right)^0$$ (6)

2.2.2 $$x=\frac{\log 6-\log 2}{\log 9\left(2\log 5+\log 4\right)}$$ (4)

2.3 Solve for x and y if $x+2yi=\left(-2+6i\right)\left(4-7i\right)$. (4)

2.4 Write $z=\sqrt{5}-3i$ in the form $r\,cis\,\theta$ (5)

[26]

QUESTION 3

Given: $g(x) = \left(\frac{1}{3}\right)^x$ and $h(x) = -\frac{3}{x}$

3.1 Write down the equations of the asymptotes of *h*. (2)

3.2 Determine the *y*-intercept of *g*. (1)

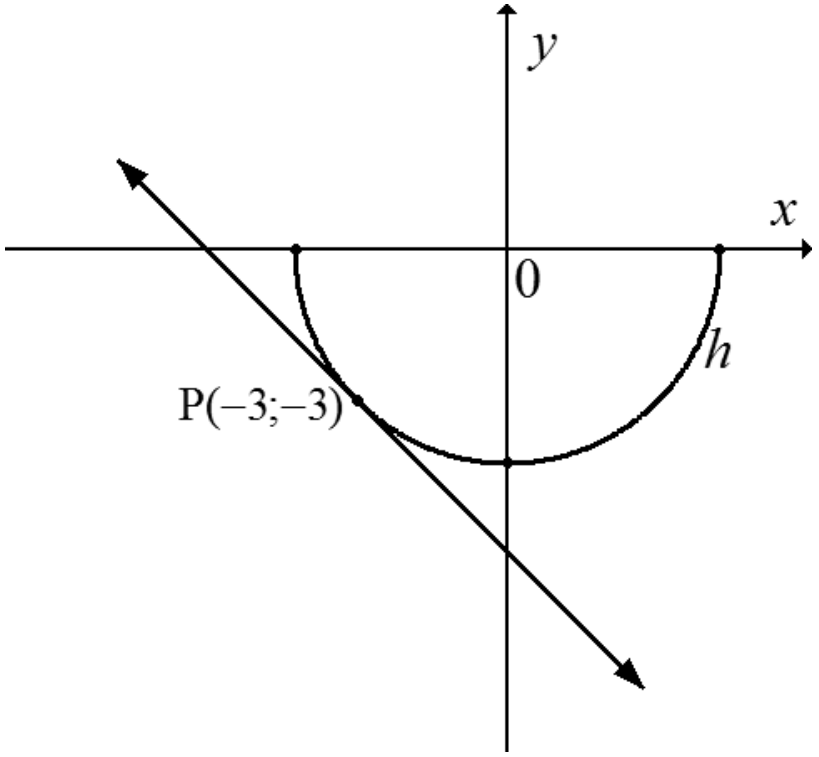
3.3 Sketch the graphs of *g* and *h* on the same set of axes on the ANSWER SHEET provided. Clearly show the asymptotes and the intercepts with the axis. (3)

3.4 Write down the domain of *h*. (1)

3.5 Determine the value(s) of *x* for which $g(x) > h(x)$. (2)
[9]

QUESTION 4

In the diagram drawn below, is a semi-circle defined by $h(x) = -\sqrt{r^2 - x^2}$ and a tangent to a semi-circle at point $P(-3;-3)$.



Determine the:

4.1 Equation of the semi-circle, *h* (2)

4.2 Equation of the tangent to the semi-circle at P in the form $y = \ldots$ (4)

4.3 Range of *h* (2)
[8]

QUESTION 5

The graph of the function f defined by $f(x) = ax^2 + bx - 12$ intersects the x-axis at $x = -2$ and $x = 6$.

5.1 Show by means of calculations that $a = 1$ and $b = -4$. (4)

5.2 Determine the coordinates of the turning point of f. (3)

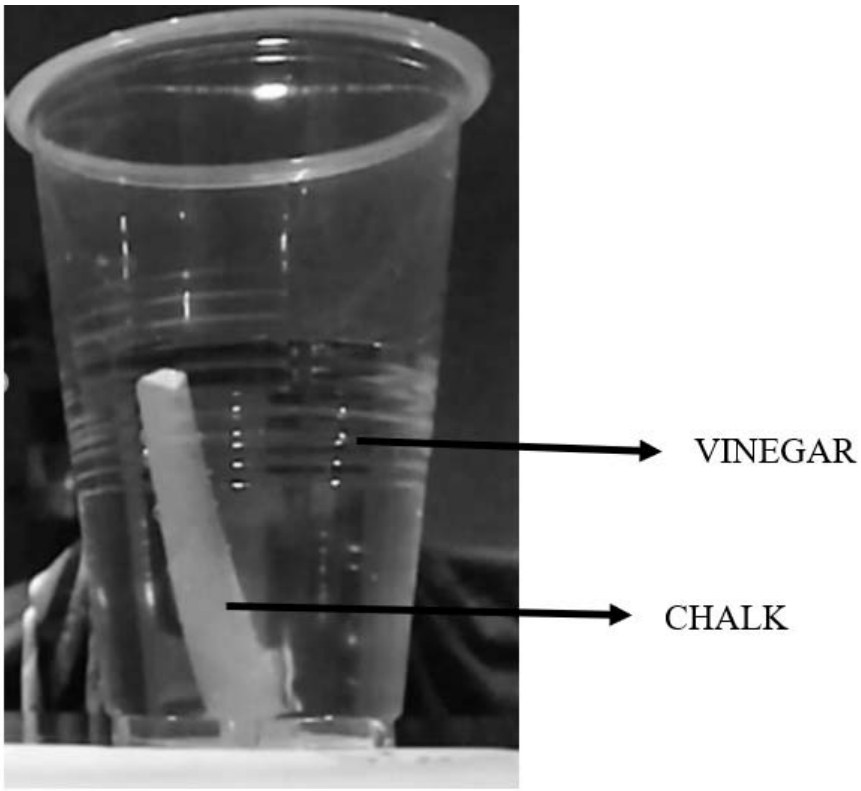
5.3 Sketch the graph of f on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axis and the turning points of the graph. (4)

[11]

QUESTION 6

6.1 Determine the nominal interest rate compounded quarterly if the effective interest rate is 8% per annum. (3)

6.2 In an experiment conducted by learners in a science class, as shown in the picture below, a 90 mm piece of chalk is immersed in a cup containing vinegar to test the effects of an acid on a piece of chalk. The chalk dissolves at a rate of 5% per minute, compound decrease.



Determine, how long (in minutes) it will take for the chalk to dissolve to half its original length. (5)

6.3 R300 000 is invested at 12% per annum, compound interest for 3 years.
After 3 years a deposit of R65 000 is made at 9% per annum compounded quarterly for the remaining 4 years.

Calculate how much will be in the account at the end of 7 years. (6)
[14]

QUESTION 7

7.1 Determine the derivative of $f(x) = 5x - 12$ by using FIRST PRINCIPLES. (5)

7.2 Determine:

7.2.1 $\mathrm{D}_x\left(3\pi + \frac{2}{x^2} - 5x^3\right)$ (4)

7.2.2 $\frac{dy}{dx}$ if $y = 11x^{-3} + \frac{2}{5}x^{\frac{1}{5}} + \sqrt{x^2}$ (4)

7.3 7.3.1 Determine the gradient of a tangent to the graph defined by: $h(x) = 3x^2 - 7x + 2$ at $x = 0,5$ (3)

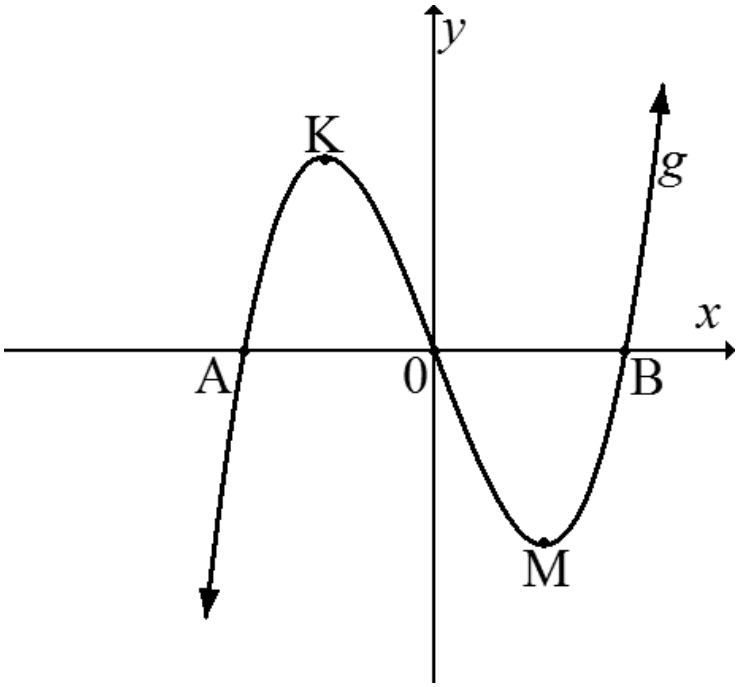
7.3.2 Determine the average gradient of $h(x) = 3x^2 - 7x + 2$ between $x = 1$ and $x = 3$. (4)

[20]

QUESTION 8

The graph of the function *g* defined by $g(x) = x^3 - x$ is drawn below.

- Points A, (0;0) and B are the intercepts of the graph with the *x*-axis.
- The *y*-axis is at the origin.
- K and M are the turning points of *g*.



Determine the:

8.1 Coordinates of points A and B (4)

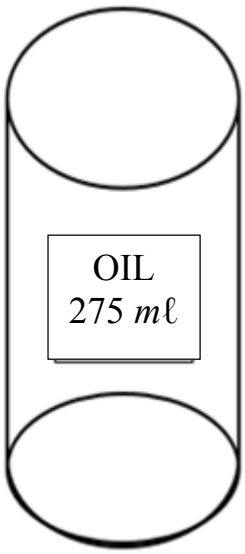
8.2 Coordinates of K and M, the turning points of *g* (5)

8.3 Value(s) of *x* for which $g'(x) \leq 0$ (3)

[12]

QUESTION 9

An oil manufacturing company manufactures cylindrically shaped plastic containers.
Each container must hold 275 $m\ell$ of oil.



FORMULAE

$$V = \pi r^2 h$$
$$SA = 2\pi rh + 2\pi r^2$$

9.1 Write h, the height of the container in terms of r. (3)

9.2 Show that $SA = \frac{550}{r} + 2\pi r^2$ is the surface area of the container. (3)

9.3 Determine r, the radius of the container for which a minimum amount of plastic will be used. (5)

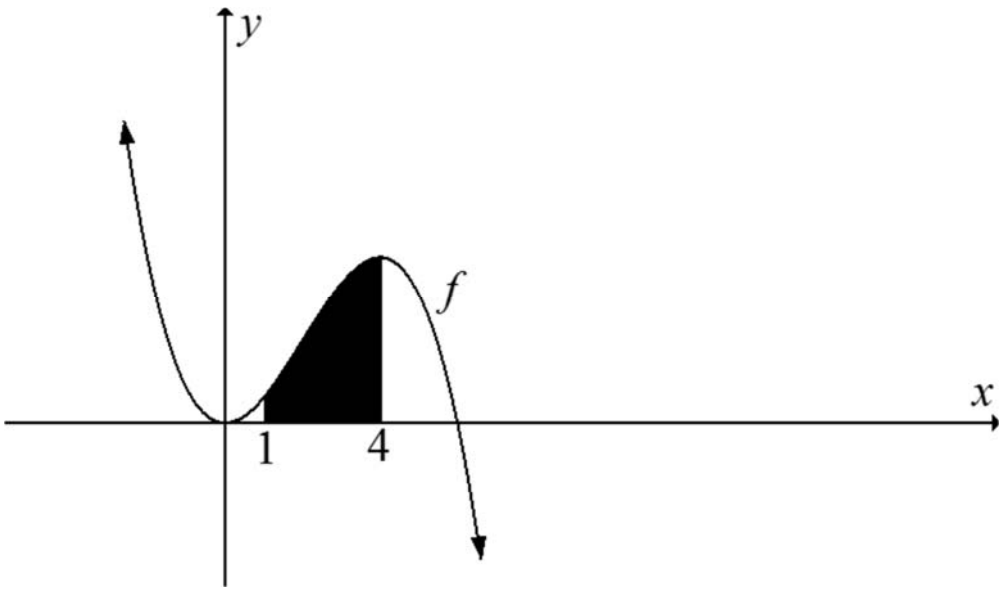
[11]

QUESTION 10

10.1 Determine the integral:

$$\int \frac{2x^8 + x^5 - 13x^2}{x^3}\, dx \qquad (5)$$

10.2 The graph of f defined by $f(x) = -x^3 + 6x^2$ is drawn below.



Determine:

10.2.1 $\int \left(-x^3 + 6x^2\right) dx$ (3)

10.2.2 The area of the shaded region of the graph of f bounded by the graph and the x-axis, between $x = 1$ and $x = 4$. (5)

[13]

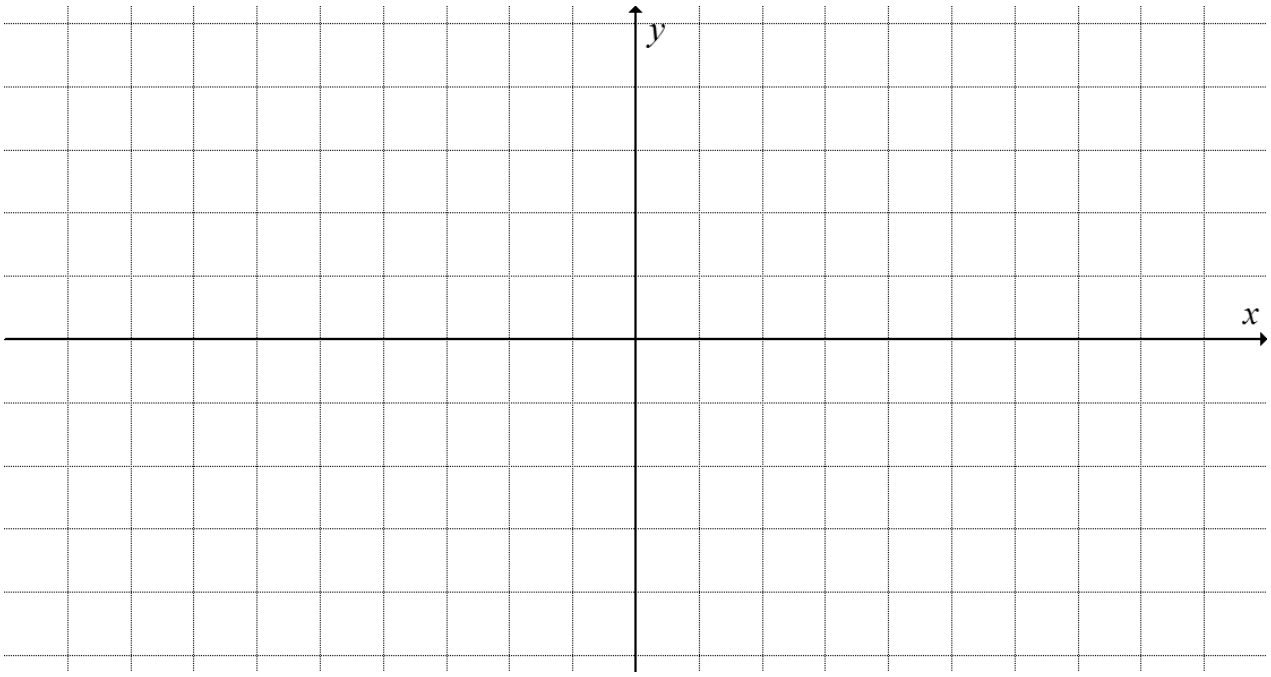
TOTAL: 150

ANSWER SHEET

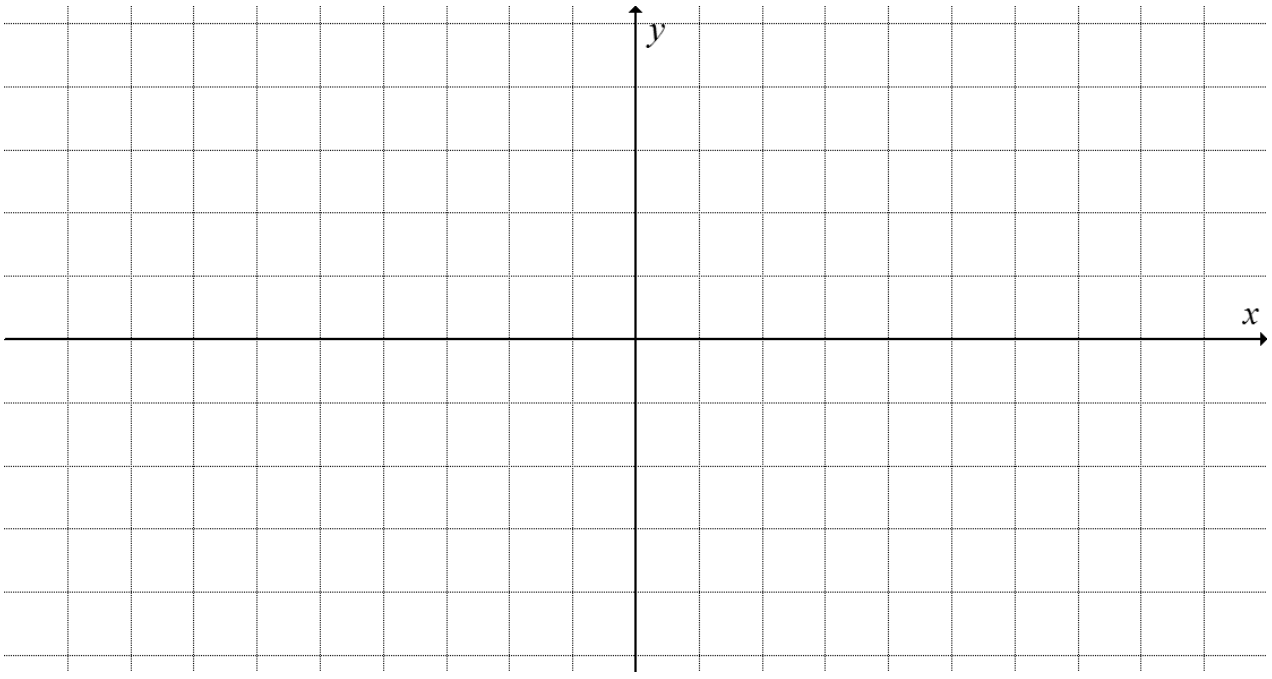
Name: ……

School: ……

QUESTION 3.3



QUESTION 5.3



INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b \ , \qquad a > 0 \ , \ a \neq 1 \ \text{and} \ b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \qquad x > 0 \qquad\qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta ABC: \ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad\qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ of\ \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $\cot^2\theta + 1 = \text{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates