Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2019

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

JTMATHE2

This question paper consists of 17 pages, including an information sheet consisting of 2 pages.

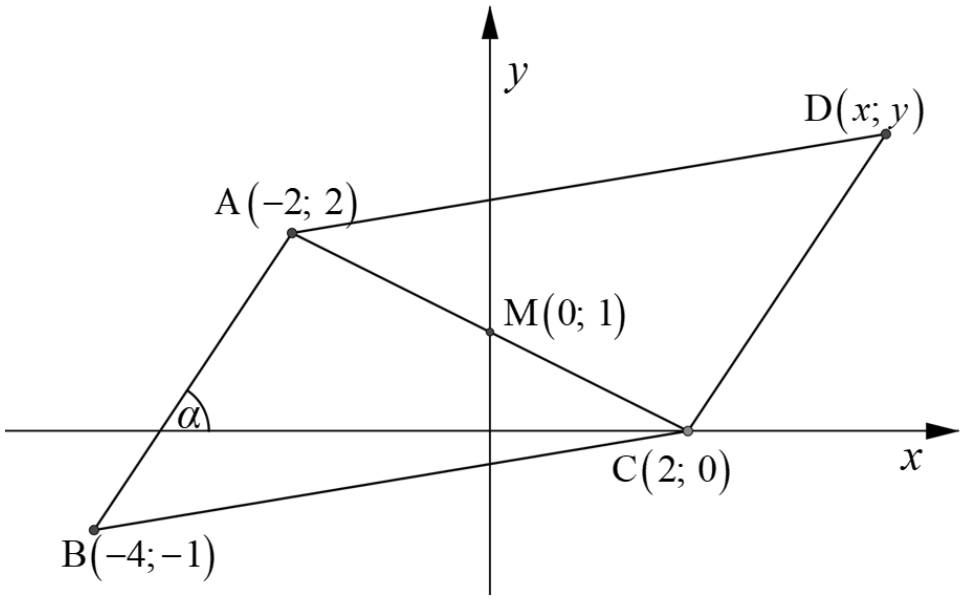
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The diagram below is a parallelogram with vertices
$A(-2; 2)$; $B(-4; -1)$; $C(2; 0)$ and $D(x; y)$
α is the angle which AB forms with the x-axis.
M(0; 1) is the midpoint of AC.



Determine:

1.1 The length of AB (2)

1.2 The gradient of AB (2)

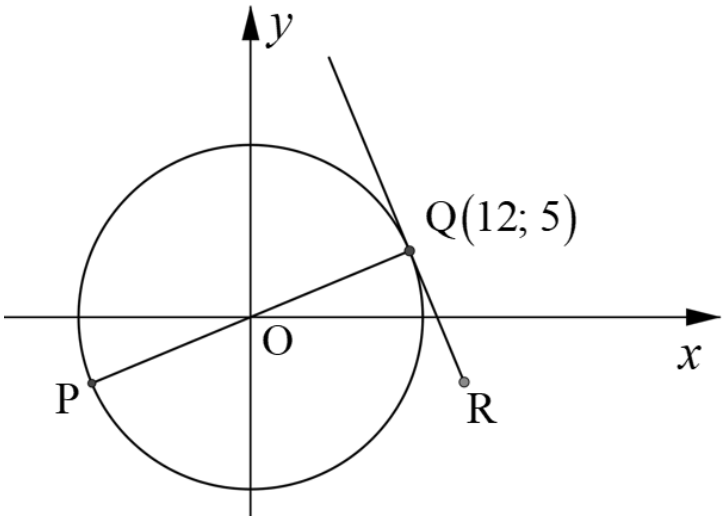
1.3 The equation of line DC in the form $y = mx + c$ (3)

1.4 The size of α (rounded off to TWO decimal places) (2)

1.5 The y-coordinate of D (2)
[11]

QUESTION 2

2.1 In the figure below, O is the centre of the circle. P and $Q(12; 5)$ are two points on the circumference of the circle. POQ is a straight line. The point R lies on the tangent to the circle at Q.



Determine the equation of:

2.1.1 The circle (2)

2.1.2 The tangent RQ in the form $y = mx + c$ (4)

2.2 The equation of an ellipse, with centre at the origin is given by $9x^2 + 16y^2 = 144$:

2.2.1 Write down this ellipse equation in standard form (2)

2.2.2 Determine the length of the major axes and the minor axes respectively (2)

2.2.3 Draw on the grid provided, a neat sketch graph of the ellipse. Show ALL intercepts with axes and clearly indicate the major and minor axes (3)

[13]

QUESTION 3

3.1 Use a calculator to determine the value of (rounded to 3 decimal places):

$$\frac{\cos ec^2 100^\circ}{-\sec 80^\circ} \qquad (2)$$

3.2 Simplify to a single trigonometric function:

3.2.1 $\tan(180^\circ - \theta).\cos^2(180^\circ + \theta) + \cos(180^\circ - \theta).\sin\theta.\sec\frac{\pi}{3}$ (7)

3.2.2 $$\frac{1 - sin^2\, x \cdot cot^2\, x}{2\, sin^2\, x + 2\, cos^2\, x} \qquad (4)$$

3.3 Prove the identity:

$$\frac{2}{\sec\theta \sec(360^\circ - \theta) - \tan 45^\circ} = 2\cot^2\theta \qquad (4)$$

[17]

QUESTION 4

4.1 If $6\sin\theta + 4 = 1$ and $\cos\theta > 0$, draw a diagram on a Cartesian plane, showing the position of θ and evaluate without using a calculator:

4.1.1 $\tan\theta$ (4)

4.1.2 $\sin\theta + \sec\theta$ (3)

4.2 Solve for $\theta \in [0^o; 360^o]$, without using a calculator, if:

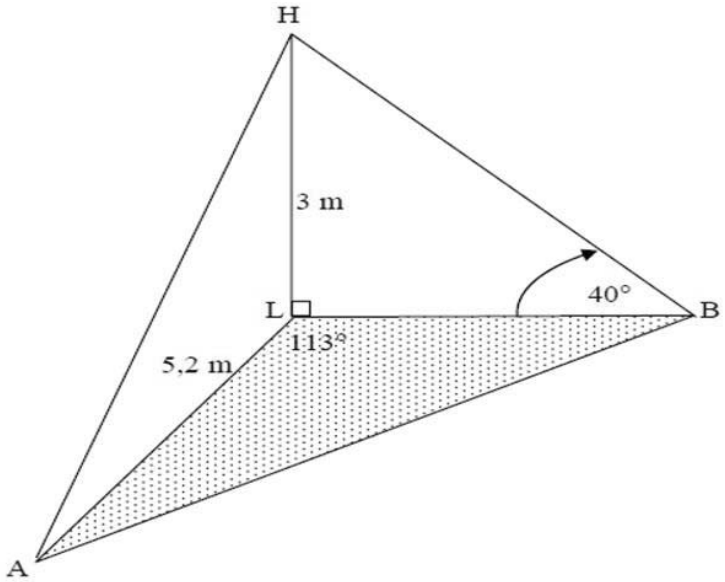
$2\sin\theta - \cos\theta = 0$ (5)

[12]

QUESTION 5

In the diagram below, A, B and L are points on the same horizontal plane.

- HL is a vertical pole of 3 meters high.
- AL = 5,2 m.
- $A\hat{L}B = 113°$ and angle of elevation of H from B is 40°.



5.1 Determine the size of $B\hat{H}L$. (1)

5.2 Calculate the length of LB. (2)

5.3 Hence or otherwise, determine the length of AB. (3)

5.4 Calculate the area of ΔALB. (3)
[9]

QUESTION 6

Given $f(x) = 2\cos x$ and $g(x) = \sin(x - 30°)$ for $x \in [0°; 360°]$

6.1 Draw the graphs of f and g on the same set of axes. Clearly show the intercepts with the axes as well as the turning points of the graphs. (6)

6.2 Write down the amplitude of f. (1)

6.3 Determine the period of $g(x)$. (1)

6.4 What value(s) of x is $g(x) \leq 0$? (4)

[12]

Give reasons for ALL your statements in QUESTIONS 7, 8 AND 9.

QUESTION 7

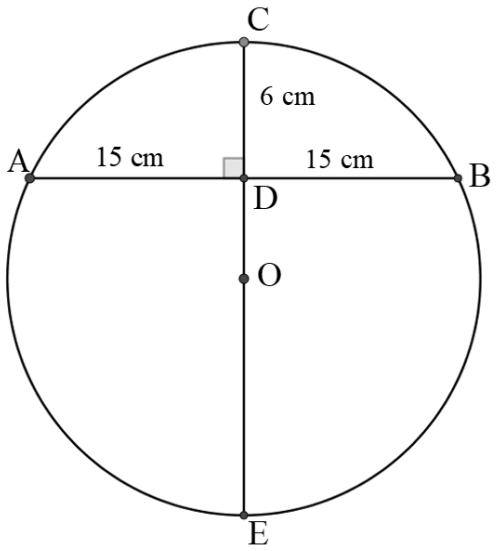
7.1 Complete the following theorems by writing down the missing word(s) in each case:

7.1.1 A line drawn from the centre of a circle to the midpoint of a chord, is … to the chord. (1)

7.1.2 Angles opposite equal sides in an isosceles triangle are … (1)

7.1.3 Opposite angles of a cyclic quadrilateral are … (1)

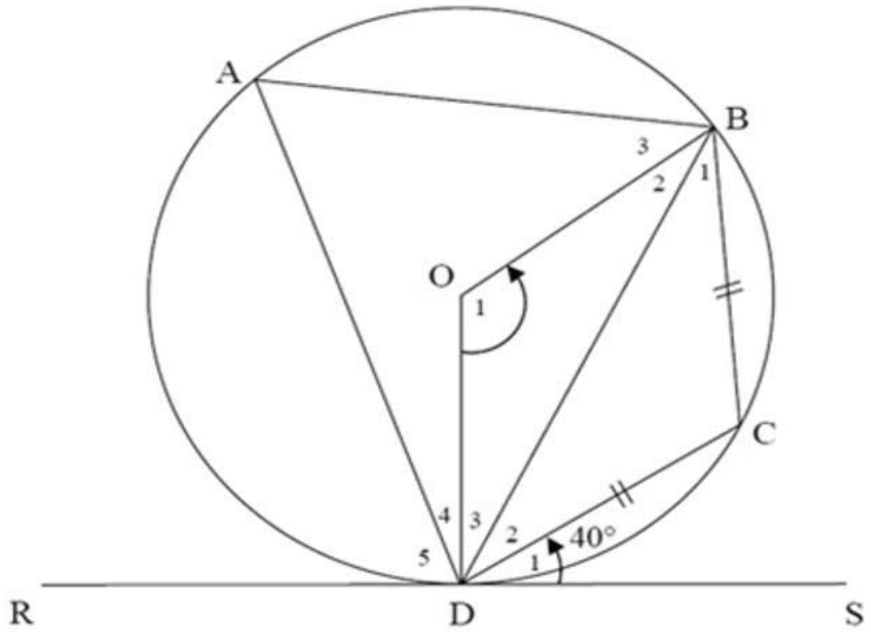
7.2 In the given diagram, O is the centre of the circle. Points A, C, B and E are on the circle. Diameter COE and chord AB intersect at right angles at D.
AD = DB = 15 cm and CD = 6 cm.



7.2.1 Calculate the length of DO. (4)

7.2.2 Calculate the size of $A\hat{O}B$. (3)

7.3 The figure below models the wheel of a sport car with O the central point through which the axle of the car passes.
The wheel touches the straight tared surface at point D only.
BC = DC and $C\hat{D}S = 40^{\circ}$



7.3.1 What geometric name is given to RS? Explain your answer. (2)

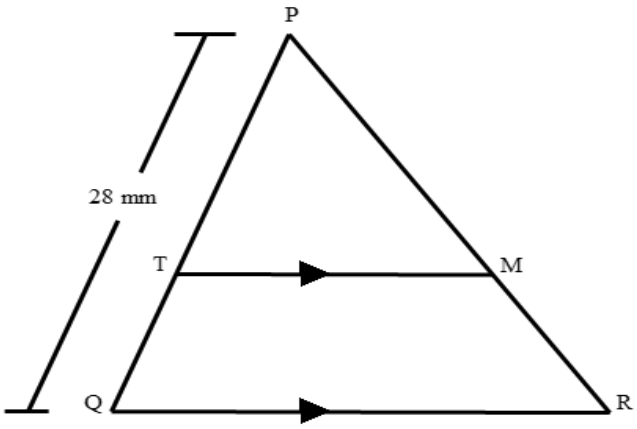
7.3.2 Determine the size of $C\hat{D}B$. (3)

7.3.3 Calculate the size of $\hat{B}_2$. (3)

7.3.4 Determine the size of Â. (3)
[21]

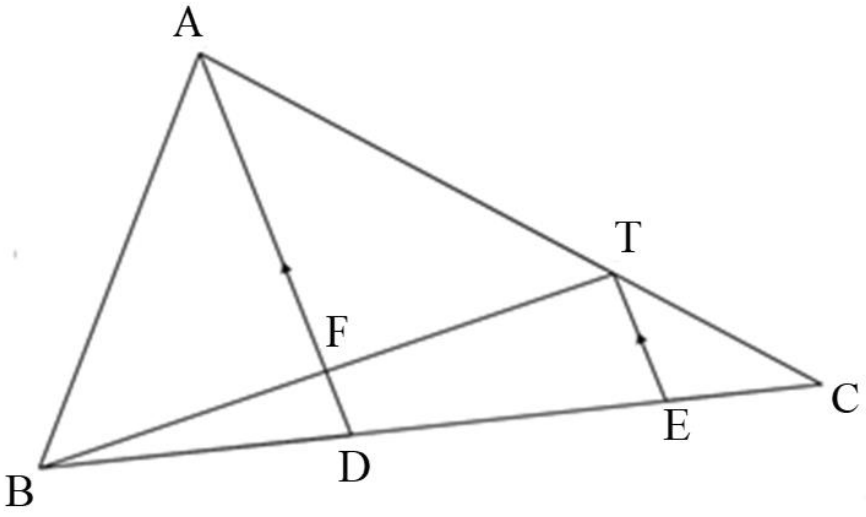
QUESTION 8

8.1 In the figure below, ΔPQR has TM|| QR, PQ $=$ 28 mm and PM : MR $=$ 4 : 3



Determine the length of TQ. (4)

8.2 Below is the geometric model of a portion of a roof truss.
In the model, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD // TE.



8.2.1 Write down the numerical value of $\frac{CE}{ED}$. (2)

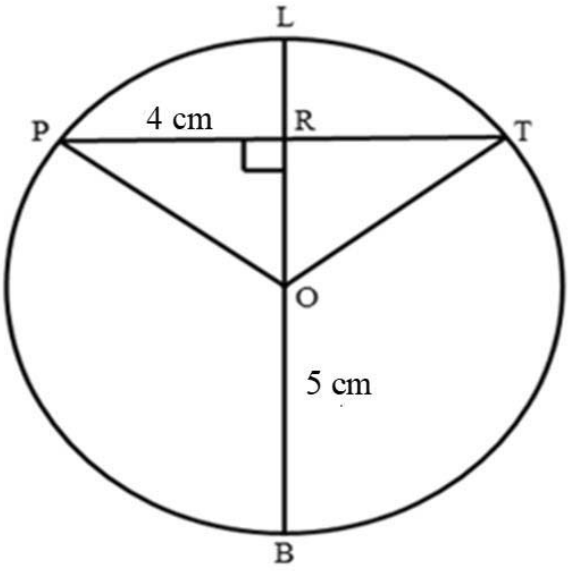
8.2.2 Show that D is the midpoint of BE. (4)

8.2.3 If FD = 2 cm, calculate the length of TE. (4)

8.2.4 Calculate the numerical value of
$\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (3)
[17]

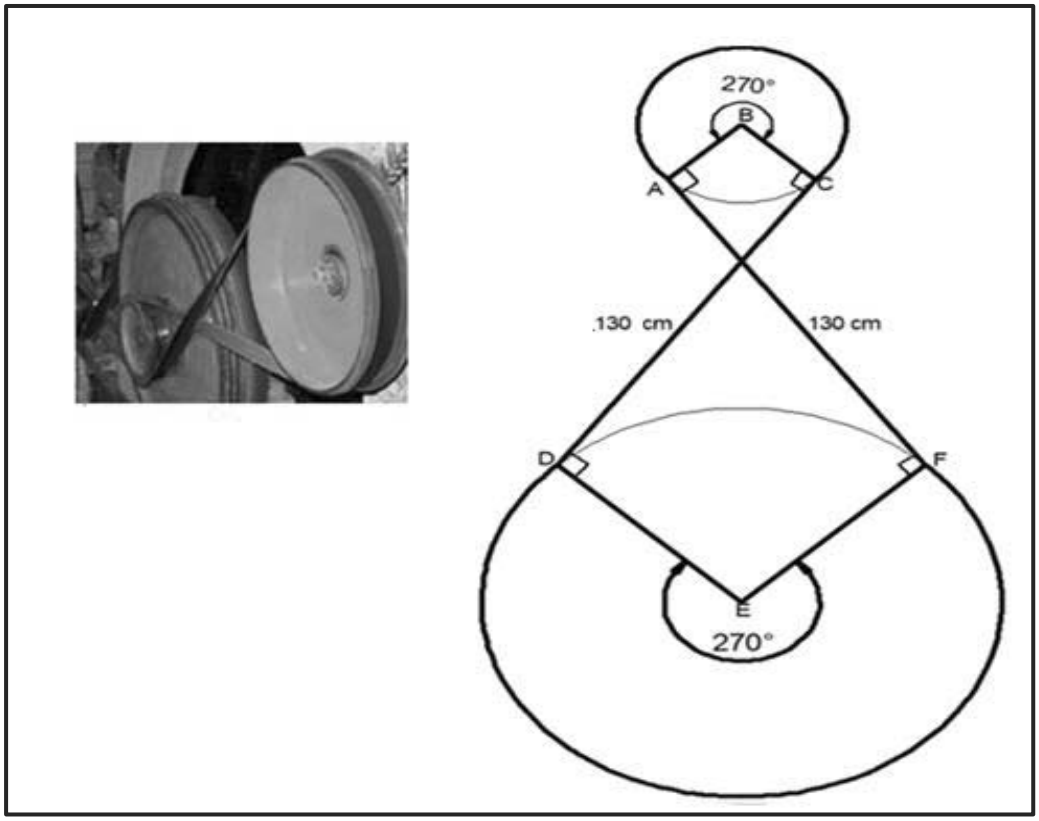
QUESTION 9

9.1 In the figure below, a circle with centre O and OR ⊥ PT is given.
Radius OB = 5 cm and PR = 4 cm.



Calculate the heights of the segments of the circle. (5)

9.2 A mechanical technician needs to replace a cross belt on a lath machine as shown in the picture below. The two pulleys have radii of 35 and 85 respectively. The length of the driving belt A to F and C to D, which are points of contacts is 130 cm.



9.2.1 Calculate the length of the major arc DF, to the nearest integer. (4)

9.2.2 If the length of the major arc AC is 165 cm, determine the total length of the driving belt to be replaced. (2)

9.3 A pulley rotates at 420 *r/min*.

Calculate:

9.3.1 The angular velocity of the pulley in radians per second (4)

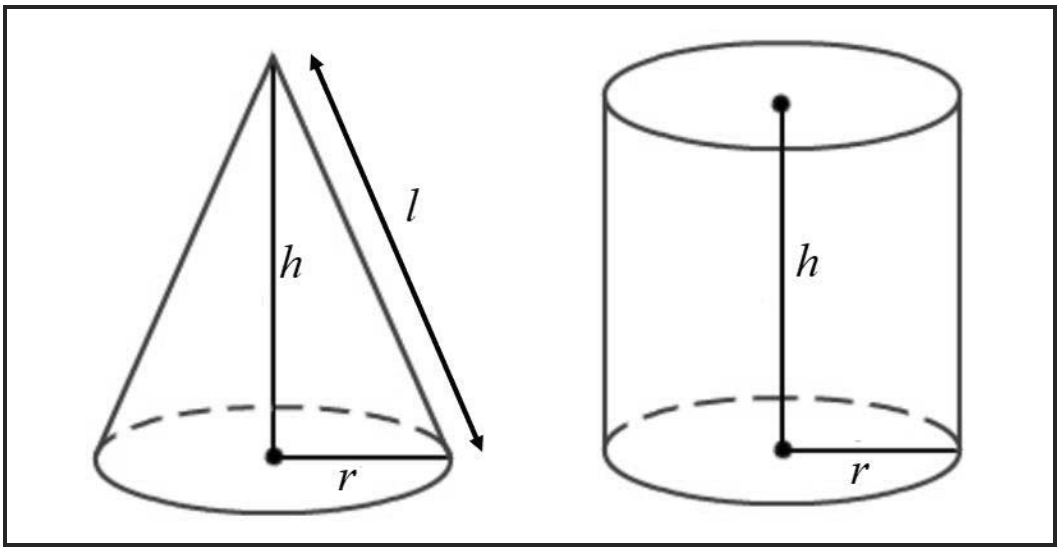
9.3.2 The circumferential velocity of the pulley in metres per second, if the diameter of the pulley is 240 *mm* (5)

[20]

QUESTION 10

10.1 Consider the following cylinder and cone. Both objects have the same radius, r cm and height, h cm.

$$V = \pi r^2 h \; ; \; V = \frac{1}{3}\pi r^2 h \; ; \; A = 2\pi r^2 + 2\pi rh \; ; \; A = \pi r^2 + \pi rl$$



10.1.1 Express the slant height, l of the cone in terms of r and h. (1)

10.1.2 What is the relationship between the volumes of the objects? (1)

10.1.3 If the curved surface area of the cylinder and the cone are the same, $\pi rl = 2\pi rh$, show that $r^2 = 3h^2$. (2)

10.1.4 Hence, express the volume of the cone in terms of h only. (2)

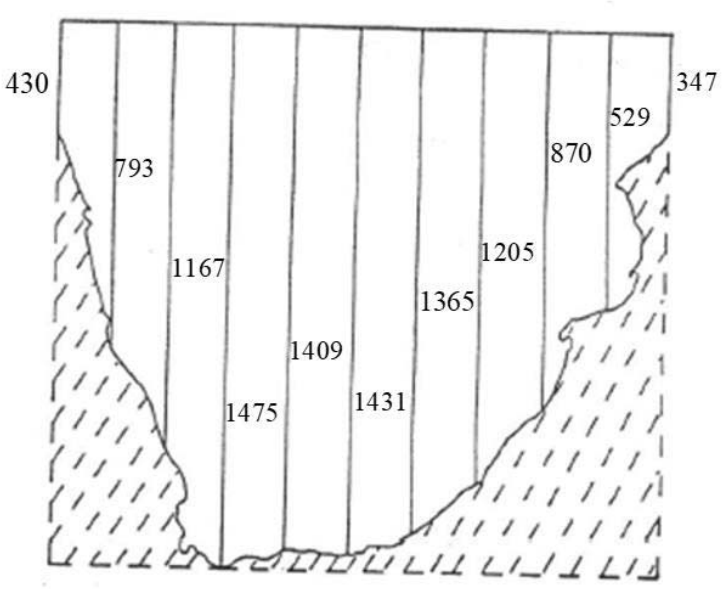
10.1.5 If the two volumes differ by 54π:

(a) Show that $h = 3$ (4)

(b) Find the value of r (2)

10.2 The map below represents a part of Southern Africa. Ordinates are drawn on the map such that the distance between the ordinates is 110 km.
ALL measurements are in kilometres.

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
430	793	1167	1475	1409	1431	1365	1205	870	529	347



Part of Southern Africa

Determine:

10.2.1 The area of the region represented by the map, using the mid-ordinate rule (3)

10.2.2 The area of the shaded region (3)
[18]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = log_a\, b \; a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C\,, n \neq -1$$

$$\int \frac{1}{x} dx = ln(x) + C, x > 0 \qquad \int a^x dx = \frac{a^x}{ln\, a} + C\,, a > 0$$

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m\left(x - x_1\right) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{Area} = \frac{1}{2} ab.\sin C$$

$$sin^2\,\theta + cos^2\,\theta = 1 \qquad 1 + tan^2\,\theta = sec^2\,\theta$$

$$cot^2\,\theta + 1 = cos\,ec^2\theta$$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360^0 n$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$A_T = a\left(\frac{o_1+o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates

OR

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

