Province of the
EASTERN CAPE
EDUCATION

EXAMINATIONS AND ASSESSMENT CHIEF DIRECTORATE
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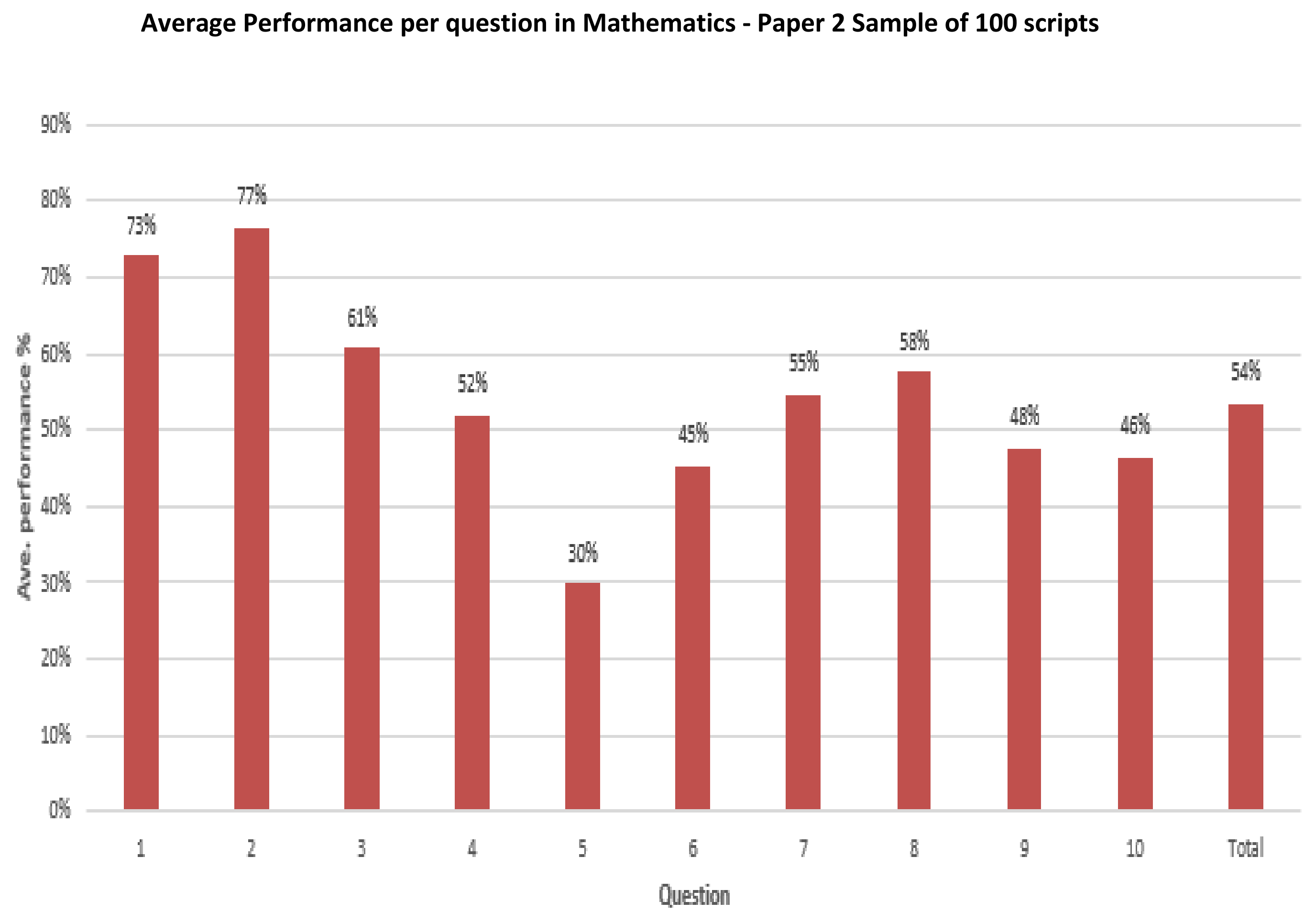
REPUBLIC OF SOUTH AFRICA, Website: www.ecdoe.gov.za

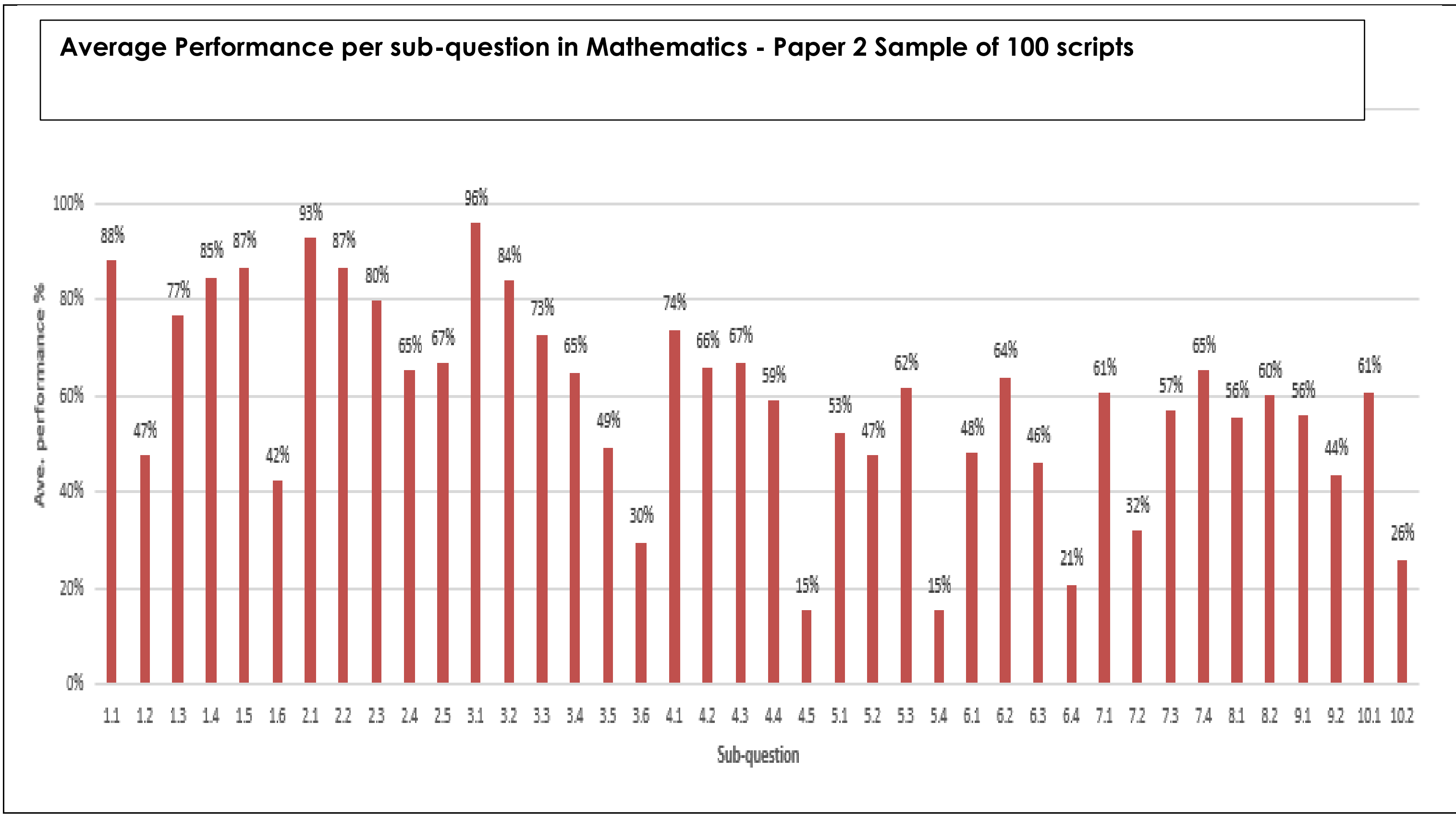
2020 NSC CHIEF MARKER'S REPORT

SUBJECT	**MATHEMATICS**
PAPER	**2**
DURATION OF PAPER	**3 hours**

SECTION 1: (General overview of Learner Performance in the question paper as a whole)

Candidates results covered the full spectrum from no marks to almost full marks. The graph below shows an analysis of the marks for 100 scripts drawn from good, average and weak candidates with an even distribution of marks from 0 to almost full marks. The graph indicates that these candidates performed best in routine questions (1 and 2) and worst in questions requiring applications and higher order thinking (3, 4, 5 and 6).
This paper tested whether maths is being taught in our classrooms and whether learners are not just coached to answer exam papers. It is time for learners to realise that one does not absorb Maths through being in the "presence" of Maths. It requires hard work, dedication and perseverance to achieve goals.





SECTION 2: Comment on candidates' performance in individual questions

The bar graphs generated from the Rasch analysis are included for each question. Please note that this is drawn from 100 scripts ranging from 0 to almost full marks and does not give a true reflection of the overall achievement of all candidates but gives a good indication of how the results for the sub questions vary. The overall achievement of candidates was very poor as too many learners lack the basic knowledge and understanding of Mathematics.

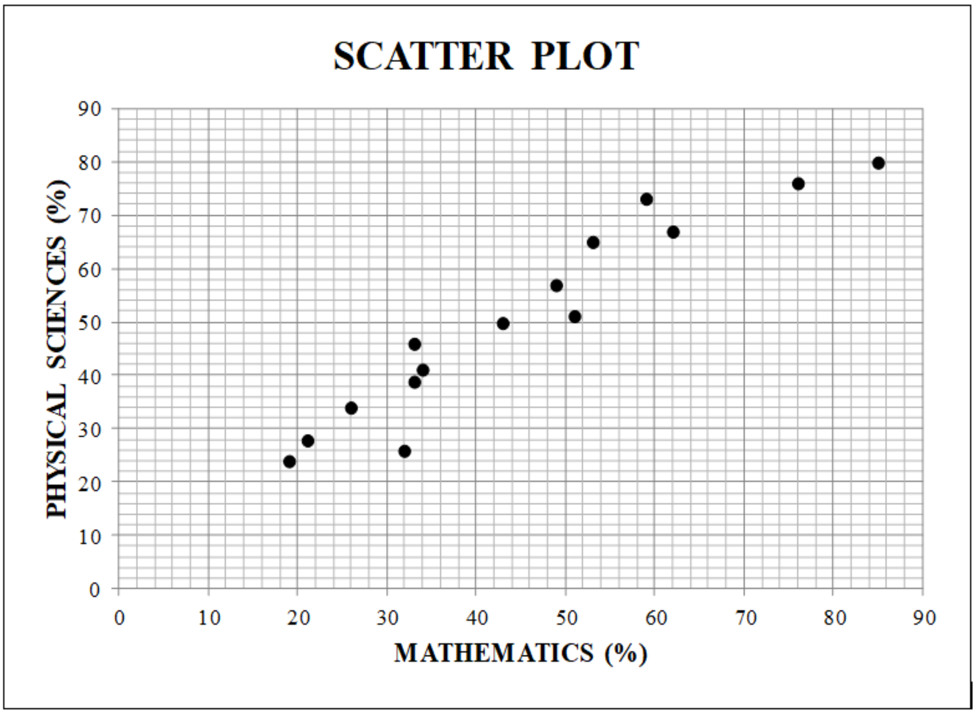
Brief comments are made on common mistakes made and advice is given to educators to implement so that future candidates can achieve optimal results. Comments are also included to assist educators with internal marking as well as comments on the setting of internal papers. It is advised that educators read this report in conjunction with the official marking guideline, CAPS document and Examination guidelines.

Educators must remember that additional notes implemented at the marking venues only apply for the paper of 2020 and it cannot be perceived as policy or a way of teaching maths forward.

QUESTION 1

A Mathematics teacher was curious to establish if her learners' Mathematics marks influenced their Physical Sciences marks. In the table below, the Mathematics and Physical Sciences marks of 15 learners in her class are given as percentages (%).

MATHEMATICS (AS %)	26	62	21	33	53	76	32	59	43	33	49	51	19	34	85
PHYSICAL SCIENCES (AS %)	34	67	28	46	65	76	26	73	50	39	57	51	24	41	80

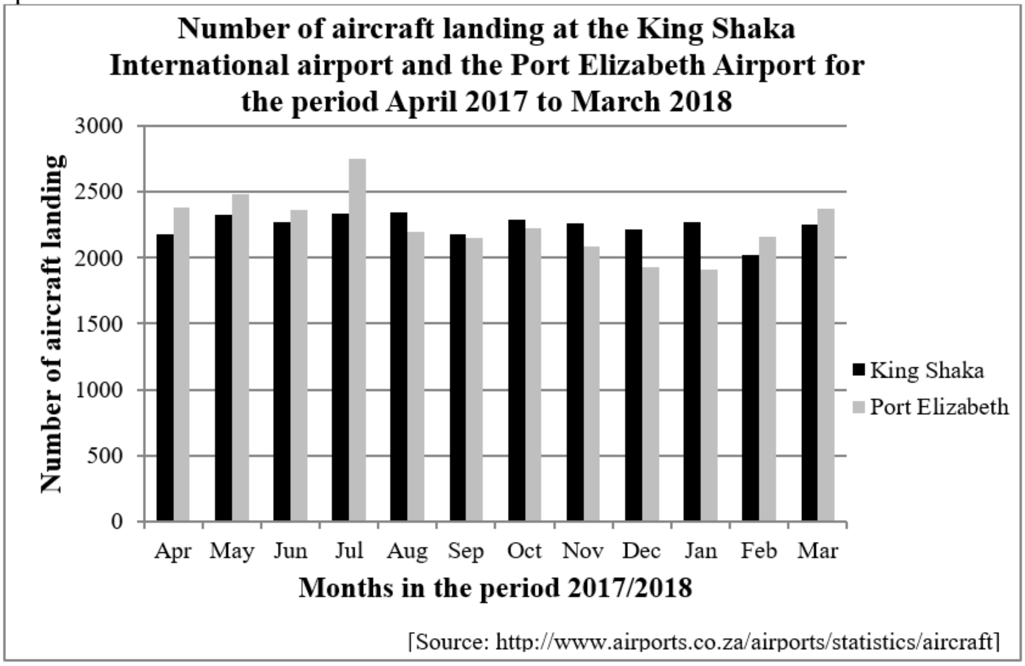


1.1	Determine the equation of the least squares regression line for the data.	(3)
1.2	Draw the least squares regression line on the scatter plot provided in the ANSWER BOOK.	(2)
1.3	Predict the Physical Sciences mark of a learner who achieved 69% for Mathematics.	(2)
1.4	Write down the correlation coefficient between the Mathematics and Physical Sciences marks for the data.	(1)
1.5	Comment on the strength of the correlation between the Mathematics and Physical Sciences marks for the data.	(1)
1.6	What trend did the teacher observe between the results of the two subjects?	(1)
		[10]

It is expected that question 1 should be one of the best answered question as it tests the use of a calculator. From the sub-question graph it is clear which questions were well or poorly answered
1.1 Consistent accuracy was applied for wrong equation.
1.2 No marks are allocated the regression line is having a negative slope.
1.3 No marks were awarded for incorrect substitution. Consistency accuracy applied from 1.1
1.4 Accuracy mark for correlation coefficient.
1.5 Consistent mark for the strength on the correlation coefficient from 1.4
1.6 This was a challenging question for candidates and it was one of the most poorly answered questions. Candidates could not see the trend that had been observed by the teacher. .
General comments • Scatter plot was the last topic taught in grade 12. • Learners struggled to get marks for the equation of the least squares regression line and also the correlation coefficient. Teachers needs to encourage learners to have their own calculators (Casio fx82 preferable) when they are teaching this topic. Without the knowledge of how to use their calculators (Casio fx82) they cannot get any marks in this topic. • Predictions on the Physical Sciences mark was fairly answered • Strength on the correlation also seemed to be a challenge. Learners were confusing strength with the trend. • Learners struggled to explain the trend between Mathematics and Physical Sciences Marks • Learners struggled to plot the regression line. To draw the regression line one needs to have 2 points, by substituting independent values in the equation to get dependent values. Or else it can be y-intercept and the mean point. Some learners just draw the line of best fit of which they are not the same.

QUESTION 2

The number of aircraft landing at the King Shaka International Airport and the Port Elizabeth Airport for the period starting in April 2017 and ending in March 2018, is shown in the double bar graph below.



2.1	The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During which month is this difference the greatest?	(1)
2.2	The number of aircraft landing at the King Shaka International Airport during these months are:	

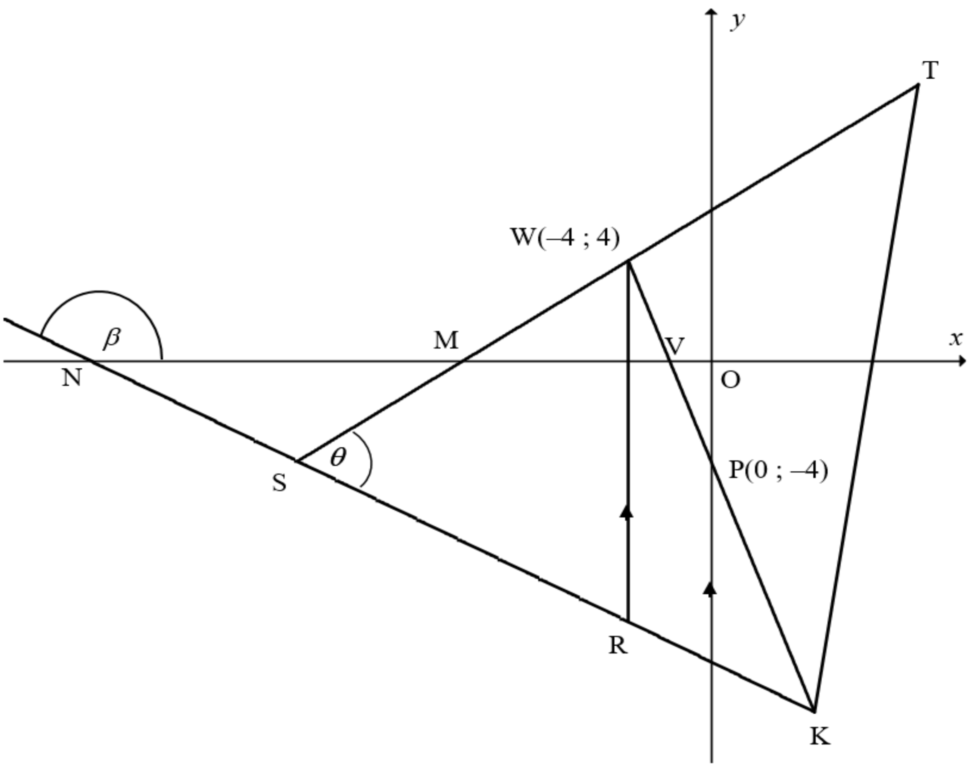
2 182	2 323	2 267	2 334	2 346	2 175
2 293	2 263	2 215	2 271	2 018	2 254

	Calculate the mean for the data.	(2)
2.3	Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period.	(2)
2.4	Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean.	(3)
2.5	Which ONE of the following statements is CORRECT?	
	A. During December and January, there were more landings at the Port Elizabeth Airport than at the King Shaka International Airport.	
	B. There was a greater variation in the number of aircraft landing at the King Shaka International Airport than at the Port Elizabeth Airport for the given period.	
	C. The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport.	(1) **[9]**

It was expected that question 2 should be the best answered question as it tested the use of a calculator.
2.1 This question was a knowledge question which required learners to read their answers from the double bar graph of which some failed to do so. 2.2 – 2.3 were very routine questions and many candidates from the 100 scripts scored good marks in this section. A few candidates lost marks because they were unable to add all the number of aircrafts landed. Some did not even write the standard deviation which is supposed to be found by use of calculator.
2.4 Was a follow up from 2.2 and 2.3. Many candidates performed well in this question as it was also a routine question.
2.5 Candidates were not able to interpret the scenario according to the double bar graph
General comments: • Educators should teach learners on how to use a calculator in this topic (Data Handling) as early as at the early stages. • To get mean or standard deviation learners should be taught to use a calculator (Casio fx82). • Correct symbol for standard deviation from the calculator is so much important. Learners must not use this (Sx) symbol for standard deviation from their calculators but they must use this ((σx) symbol • Learners did not seem to know how to apply the interval. Teachers need to teach them on how to apply the interval whether it is within or outside the data. • Calculator use in this topic is very important. • Teachers must not teach grade 12 work in isolation; they must always consolidate with grade 8 to grade 11 work and assess thereafter

QUESTION 3

ΔTSK is drawn. The equation of ST is $y = \frac{1}{2}x + 6$ and ST cuts the x-axis at M. W(–4 ; 4) lies on ST and R lies on SK such that WR is parallel to the y-axis. WK cuts the x-axis at V and the y-axis at P(0 ; –4). KS produced cuts the x-axis at N. $T\hat{S}K = \theta$.

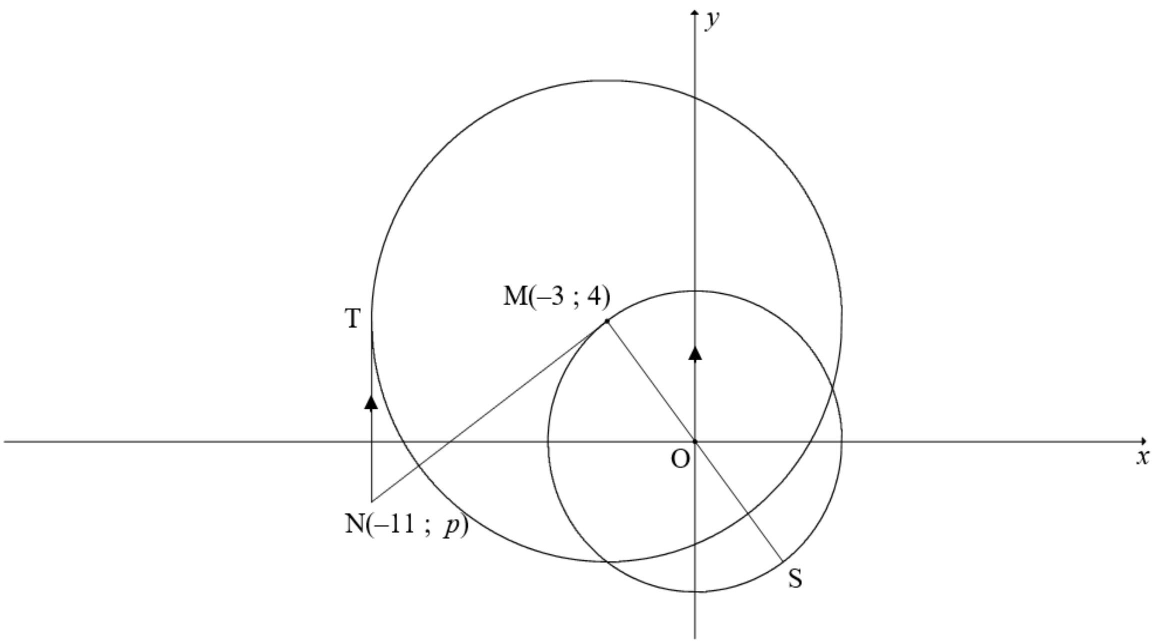


3.1	Calculate the gradient of WP.	(2)
3.2	Show that $WP \perp ST$.	(2)
3.3	If the equation of SK is given as $5y + 2x + 60 = 0$, calculate the coordinates of S.	(4)
3.4	Calculate the length of WR.	(4)
3.5	Calculate the size of θ.	(5)
3.6	Let L be a point in the third quadrant such that SWRL, in that order, forms a parallelogram. Calculate the area of SWRL.	(4)
		[21]

From the sampling of 100 scripts this question was fairly well answered by many candidates.
3.1 Routine question where candidates have to calculate the gradient of a straight line. Well answered question although there were a few learners who still wrote the formula as $m=\frac{x_2-x_1}{y_2-y_1}$ which is a **Breakdown**. Learners who wrote such a formula got no marks.
3.2 Well answered question from the 100 scripts, although some learners made a misconception by starting with $m_{WP}\times m_{ST}=-1$ while the question required candidates to show that the lines are perpendicular to each other.
3.3 Well answered question. S is the point of intersection for the 2 lines given on the statement ST and SK. What was needed was to solve both equations simultaneously to get the coordinates of S.
3.4 Fairly answered question. Some candidates could not connect WR which was parallel to y-axis and intersecting at the equation of SK. They struggled to get the coordinates of R although they showed an understanding of getting the length of the line.
3.5 Not well answered question. Many tempted to calculate the gradients whereas they were already given the equations of SK and ST. This question seems to be a problem to some learners.
3.6 Was more of a visualizing question differentiating between the poor performers and the top performers. Very few candidates scored this question
General comments • Expose learners to more problem-solving questions. • Many concepts in analytical geometry must be taught as early as in grade 10 integrating application of theorems with relevant polygons. • Teachers should train learners on how to write analytical formulae correctly • Teachers should emphasize that in any proof learners cannot start with the conclusion. • Teachers should teach learners on how to get information from the diagrams, many learners lost marks on higher order questions because they could not read from the diagram. • Continuity is very important in teaching grade 10 to grade 12, prior knowledge is essential in answering Analytical Geometry. • Stock up with equipment in models for topics like area and volume. • Teachers should discourage learners in assuming answers without showing any calculations.

QUESTION 4

M(–3 ; 4) is the centre of the large circle and a point on the small circle having centre O(0; 0). From N(–11 ; p), a tangent is drawn to touch the large circle at T with NT is parallel to the y-axis. NM is a tangent to the smaller circle at M with MOS a diameter.

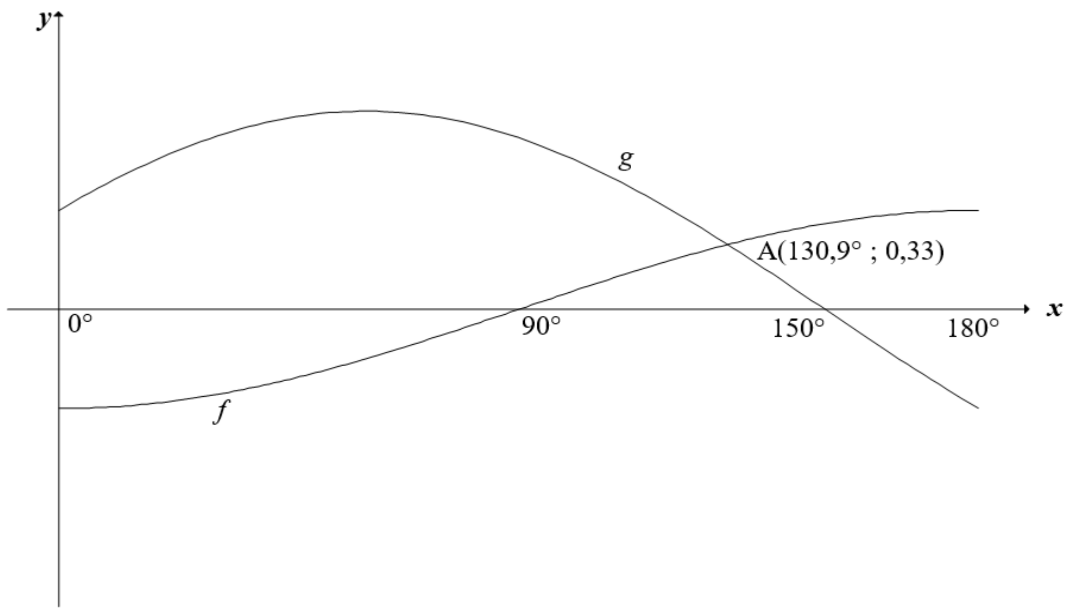


4.1	Determine the equation of the small circle.	(2)
4.2	Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$	(3)
4.3	Determine the equation of NM in the form $y=mx+c$	(4)
4.4	Calculate the length of SN.	(5)
4.5	If another circle with centre B(–2 ; 5) and radius k touches the circle centred at M, determine the value(s) of k, correct to ONE decimal place.	(5)
		[19]

This question was also relatively well answered. Candidates scored marks in the routine parts, but very few were able to score marks in parts at the complex procedures level.
4.1 Well answered question. Many learners scored marks on this question.
4.2 Some learners could not read the coordinates of T from the diagram relating with what has been stated.
4.3 Fairly answered question from 100 sampled scripts. Learners could not apply Euclidean Geometry to get the gradient of the tangent. Lack of knowledge in application of radius and a tangent theorem.
4.4 Fairly answered question although some assumed that SN=NM and their reason was radii which was incorrect.
4.5 Learners could not visualise the 3rd circle in 4.5
General comments • Teachers must integrate Euclidean Geometry in Analytical Geometry as early as in grade 10. • As learners are reading the given statement they must link that statement with the given diagram. • Teachers must teach learners on how to get information from the diagram. • Learners made unnecessary mistakes because of poor algebraic skills. Many learners were not prepared well for grade 12. • Teachers should not only drill basics but also emphasise interpretive questions.

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}\cos x$ and $g(x) = \sin(x + 30°)$, for the interval $x \in [0°; 180°]$, are drawn below. A(130,9° ; 0,33) is the approximate point of intersection of the two graphs.

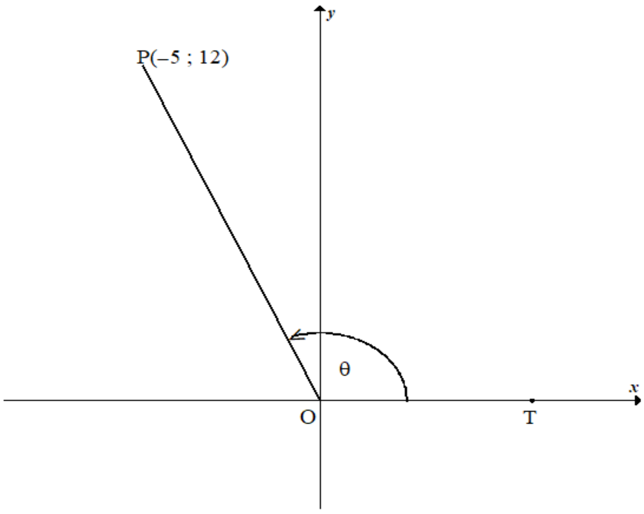


5.1	Write down the period of g.	(1)
5.2	Write down the amplitude of f	(1)
5.3	Determine the value of $f(180°) - g(180°)$	(1)
5.4	Use the graphs to determine the values of x, in the interval $x \in [0° ; 180°]$, for which:	
	5.4.1 $f(x - 10°) = g(x - 10°)$	(1)
	5.4.2 $\sqrt{3}\sin x + \cos x \geq 1$	(4)
		[8]

This was a fairly answered question from the 100 sampled scripts.
51-5.3. Not as well answered question as was expected. Some learners seemed not to know basics of trigonometric graphs. Amplitudes are taught as early as grade 10. Candidates made errors even in the knowledge-based early parts of the question.
5.4 Poorly answered question especially 5.4.2. Candidates did not see the relationship between the given inequality and the graph. Very few learners managed to score marks in this question.
General comments • Teachers needs to teach basic trigonometric graphs as early as in grade 10. Grade 10 and 11 work needs to be revised thoroughly in grade 12. • Graph interpretation needs to be emphasised in all grades. • Teachers needs to show learners that trigonometric graphs and algebraic graphs are very similar to interpret. • More time should be spent in grade 10 and 11 – like periods, amplitudes and translations.

QUESTION 6

6.1	In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $\hat{POT} = \theta$



	Answer the following **without using a calculator**:			
	6.1.1	Write down the value of $\tan\theta$		(1)
	6.1.2	Calculate the value of $\cos\theta$		(3)
	6.1.3	S(a ; b) is a point in the third quadrant such that $T\hat{O}S = \theta + 90°$ and OS = 6,5 units. Calculate the value of b.		(4)
6.2	Determine, **without using a calculator**, the value of the following trigonometric expression:			
	$\dfrac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$			(5)
6.3	Determine the general solution of the following equation:			
	$6\sin^2 x + 7\cos x - 3 = 0$			(6)
6.4	Given: $x + \dfrac{1}{x} = 3\cos A$ and $x^2 + \dfrac{1}{x^2} = 2$			
	Determine the value of $\cos 2A$ **without using a calculator**.			(5)
				[24]

This question was not as well answered as expected. It was disappointing to see learners who could not write basic trigonometric ratios.

6.1 Some candidates were able to score marks in the familiar first 2 parts of 6.1 . Very few displayed the deep understanding of basic definitions needed to do 6.1.3 without using a calculator.

6.2 Some were able to answer up to the first 3 marks and they struggled to simplify the expression. The use of brackets in reduction formula is very important. For example in the solution below, learners made a correct reduction then messed up by incorrect simplification in the third step of multiplying -1 by $-\sin x$ as the example below

$$\frac{\sin 2x.\cos\left(-x\right) + \cos 2x.\sin(360^0 - x)}{\sin(180^0 + x)}$$

$$= \frac{\sin 2x.\cos x + \cos 2x.-\sin x}{-\sin x}$$

$$= \frac{2\sin x\cos x + 2\cos^2 x - 1.-\sin x}{-\sin x}$$

$$= \frac{2\sin x\cos x + 2\cos^2 x + \sin x}{-\sin x}$$

Reduction is correct but simplification was a mess!

6.3 Not performed as expected although it was a routine question. Some learners they simply changed $\sin^2 x$ to be $\cos^2 x$ or $\cos x$ to be $\sin x$ of which that was mathematically incorrect. Some learners did miss $k \in Z$ which forms the gist of general solution.

6.4 Poorly answered question and it was intended to be one of higher order questions and very few candidates answered it correctly. Candidates did not relate algebra with trigonometry.

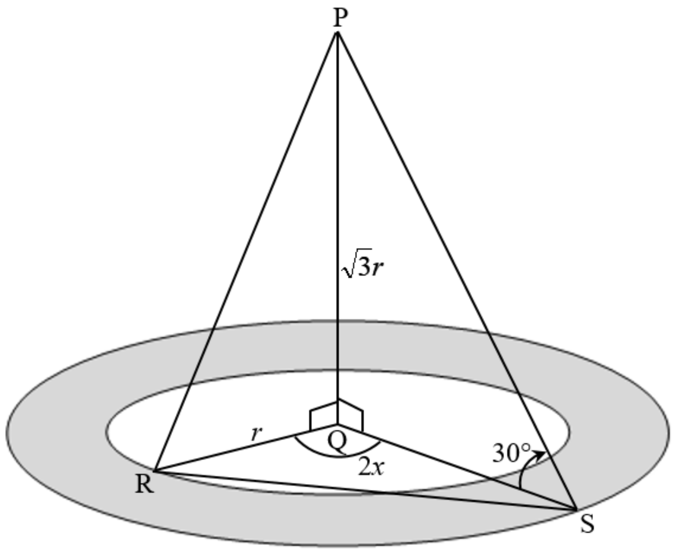
General comments

- Teachers need to teach basics of trigonometry as early as in grade 10 and 11. Grade 12 work only requires application on what they have been taught in grade 10 and 11. Basics are in lower grades.
- Reduction formula must be done thoroughly in grade 11. Simplification of trigonometric expressions seems to be a problem with our learners. The use of brackets when reducing trigonometric ratios is very useful not to miss the operations.
- Topic integration is very important in teaching when consolidating topics.
- Teachers must teach to make use of proper sketches which will lead to correct answers.
- Educators seem to underteach with little or no assessment on this topic of which it carries ± 40 of the paper.

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is $30°$.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



7.1	Show that $QS = 3r$	(3)
7.2	Determine, in terms of r, the area of the flower garden.	(2)
7.3	Show that $RS = r\sqrt{10 - 6\cos 2x}$	(3)
7.4	If $r = 10$ metres and $x = 56°$, calculate RS.	(2)
		[10]

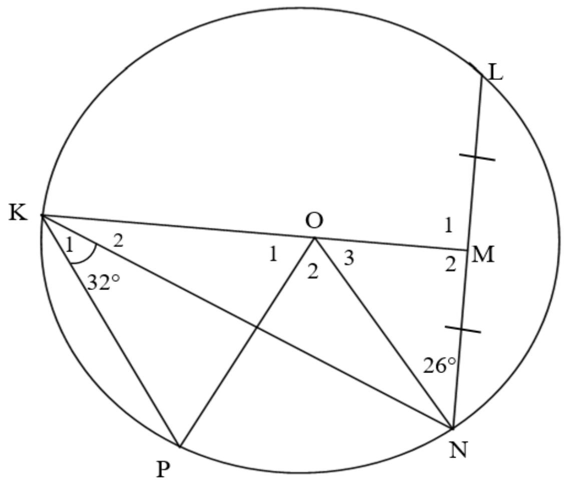
Question 7 was fairly answered from 100 scripts although the majority of learners poorly performed on this question some did not even touch the question.
7.1 Many learners did perform poorly in this question
7.2 was poorly answered. The question was not clear whether it needs the area of the flower garden or the area covered by the flowers.
7.3 The majority of learners seems to struggle with the application of triangle formula(sine rule, cosine rule, area rule and trigonometrical ratios).
7.4 Fairly answered from the graph. Some learners did not manage to substitute correctly on the proof that was done in 7.3

General comments:

- Teachers must drill learners on application of triangle formulae as early as in grade 10.
- Teachers must stop re-caring syllabus to the next grade as that is disadvantaging the learners. Syllabus must be covered in **ALL** grades as per ATPs and CAPS document.
- Continuous assessment must be done across all grades.
- Teachers needs to revise work of the previous grade before learners sit for examinations.
- Trigonometry tends to be the most poorly performed topic in paper 2 compared to other topics.
- Effective teaching and effective revision must be done across all grades.

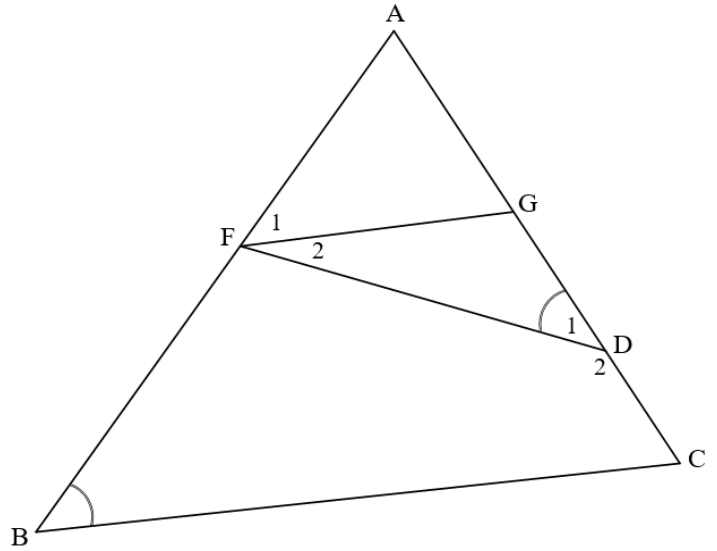
QUESTION 8

8.1	O is the centre of the circle.. KOM bisects chord LN and $M\hat{N}O = 26°$. K and P are points on the circle with $N\hat{K}P = 32°$. OP is drawn.		



	8.1.1	Determine, giving reasons, the size of:			
		(a)	$\hat{O}_2$		(2)
		(b)	$\hat{O}_1$		(4)
	8.1.2	Prove, giving reasons, that KN bisects $O\hat{K}P$.			(3)

8.2	In ΔABC, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.	



	8.2.1	If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that $FG \parallel BC$.	(4)

	8.2.1	If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that $FG \parallel BC$.	(4)

	8.2.2	If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x.	(4)
			[17]

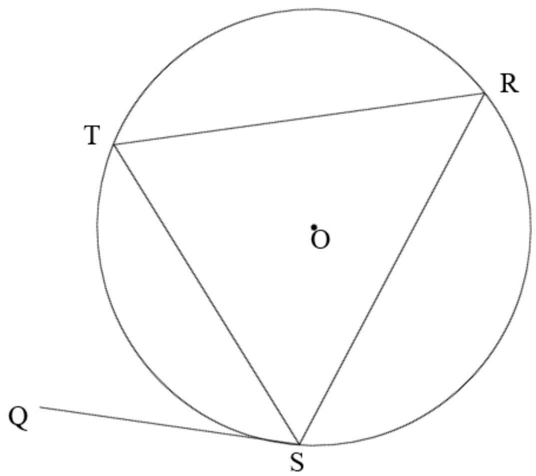
Fairly well answered and it was good to see the trend of improving perfomance in Euclidean Geometry. More learners who attempted it performed well.

8.1 Fairly answered question from 100 sample scripts, was less abstract and required the calculation of angles. Learners struggled with giving the correct reasons

8.2 Also proved to be accesssible to learners, fairly performed. More learners attempted the question and performed well. Learners were giving the ratio incorrectly ,not matching with the parallel lines. Struggled with substitution as they were expected to calculate one of the sides a triangle.
Educators must expose learners with various type of such question.

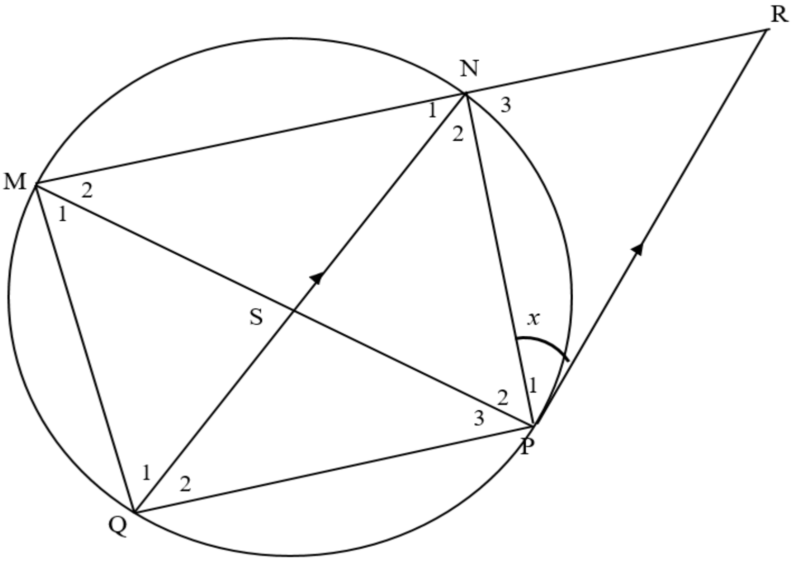
QUESTION 9

9.1	In the diagram, O is the centre of the circle. Points S, T and R lie on the circle. Chords ST, SR and TR are drawn in the circle. QS is a tangent to the circle at S.		



	Use the diagram to prove the theorem which states that $Q\hat{S}T = \hat{R}$.		(5)

9.2	Chord QN bisects $M\hat{N}P$ and intersects chord MP at S. The tangent at P meets MN produced at R such that QN ∥ PR. Let $\hat{P}_1 = x$.		



9.2.1	Determine the following angles in terms of x. Give reasons			
	(a)	$\hat{N}_2$		(2)
	(b)	$\hat{Q}_2$		(2)
9.2.2	Prove, giving reasons, that $\frac{MN}{NR} = \frac{MS}{SQ}$			(6)
				[15]

Fairly answered question.

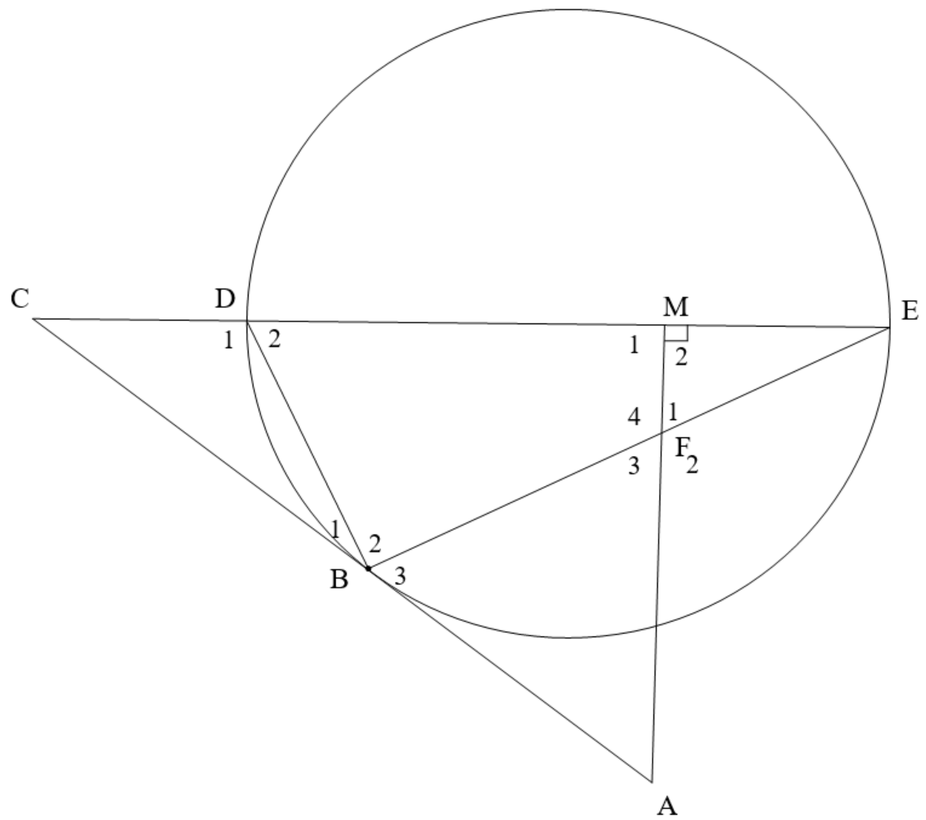
9.1 Required the proof of a theorem. Most candidates in the sample were able to answer this predictable lower order cognitive level question, which shows a growing realisation that a theory must be learnt.

9.2.1 Well performed question. Candidates were able to score marks in this routine question

9.2.2 Fairly perfomed according to the graph. Generally. very few candidates were able to complete the proof which required problem-solving skilsl. Candidates used wrong notation $\hat{B}_{1+2}$ instead of $\hat{B}_1 + \hat{B}_2$ as well as incorrect reasons (isosceles∆/ radii/ base angles) these reasons are not acceptable according examinations guidelines.

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that $AM \perp DE$. AM and chord BE intersect at F.



10.1	Prove, giving reasons, that:		
	10.1.1	FBDM is a cyclic quadrilateral	(3)
	10.1.2	$\hat{B}_3 = \hat{F}_1$	(4)
	10.1.3	$\Delta CDB \parallel\!| \Delta CBE$	(3)
10.2	If it is further given that CD = 2 units and DE = 6 units, calculate the length of:		
	10.2.1	BC	(3)
	10.2.2	DB	(4)
			[17]
		TOTAL:	**150**

Fairly perfomed question and continuous improvement has been displayed from the bar graph.

10.1 Fairy answered questiom from the sample of 100 scripts. Routine question where some candidates were able to score marks. Converse of theorems still seemed to be problematic.

10.2 This required problem solving skills. Poorly performed by many candidates from the 100 scripts. Very few candidates being able to answer Question 10.2.2 in particular. Building ratios from similar triangles and its application is also a problem.

General comments:

- There is a continuous improvement in the performance of learners in this topic.
- Geometry is a process that needs to be taught properly from grade 8.
- Logical reasoning in trying to prove something is a method to be taught by teachers.
- Teachers should provide correct reasons for learners. Each learner must have examination guideline when teaching Euclidean geometry starting from grade 8

The following general suggestions, observations and additional comments are given annually. A word of thanks must go out to all the dedicated educators who are really trying their best, often under difficult circumstances

GENERAL SUGGESTIONS FOR IMPROVEMENT IN RELATION TO TEACHING AND LEARNING
The foundation for basic mathematical skills must be laid in grade 8 and 9.
Educators should not assume that learners know how to use their calculators.
Don't simply coach learners for exams. Teach the syllabus. This approach applies even more for learners who intend to study further in Mathematics. We need to ensure the integrity of assessments.
Motivate learners to work through previous papers as to familiarize themselves with the various ways of asking the same topic but do not teach question paper.
Encourage learners to work independently during the year. Learners can benefit from study groups as well but the final 'test' depends on the individual's ability to think.
Educators should try to introduce more unseen questions to brighter learners. Integrate topics for higher level questions.
Teachers as well as learners must be committed in teaching and studying the subject.
Test learners on the selection of the correct formula from the information sheet. Make the information sheet available during all tests (formal and informal) and examinations in grade 12.
Learners must realize that they cannot expect great things to happen if they don't put in effort and some sacrifices to achieve their dreams.
Do not only focus on improving weaker learners but also focus on enriching stronger learners. Make an effort to look for higher order questions. Use the Independent Examination Board papers as reference as well.

OBSERVATIONS RELATING TO RESPONSES OF LEARNERS
There are too many learners taking Mathematics who lack the basic skills.
Candidates do not read the instructions/questions and do not motivate/explain an answer if asked for a motivation or explanation. They must give an equation if an equation is asked and not stop too soon. Give coordinates if coordinates are asked for.
The language barrier remains a problem for many candidates.
Motivate learners to write neatly and answer the questions in numerical order.
Point out the instruction that states that an answer only will not necessarily be awarded full marks.
Incorrect statement with correct reasoning is not awarded any mark as Euclidean Geometry requires logical reasoning.
If a sub-question is answered out of place from the rest of the question it is always good to write a note regarding the page on which it is redone.

ADDITIONAL COMMENTS USEFUL TO TEACHERS, SUBJECT ADVISORS,TEACHER DEVELOPMENT ETC.
Educators are encouraged to make use of this report throughout the year and not only read through once. They need to refer to this report every time they are going to introduce a new topic.
Educators must regard grades 10, 11 and 12 as one unit and not only focus on grade 12.
Focus should be placed on the training and development of grades 8 , 9 and 10 educators. The understanding of basic skills is promoted in these grades.
Educators need to constantly upgrade their own mathematical knowledge and skills, communicate with educators from surrounding schools and contact subject specialists.
When setting tests, teachers should also include unseen higher order questions.
If available, make use of technology in teaching certain topics. As mentioned, several times in the report, GeoGebra can be used to illustrate and teach various topics.
Be an enthusiastic maths teacher. You are involved in teaching a great subject.
Teachers should teach understanding and not only knowledge.
Subject advisors to continue visiting schools and assist educators in various ways.
Subject advisors could use a memo discussion session for non-markers to enrich them.
ECDOE must ensure that there is a Mathematics subject advisor appointed in each district.
All stakeholders must be congratulated for the various programs that have been implemented in our province to improve Mathematics.

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/ NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2020

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 24 pages.

* M A T H E 2 *

EASTERN CAPE

INSTRUCTIONS AND INFORMATION

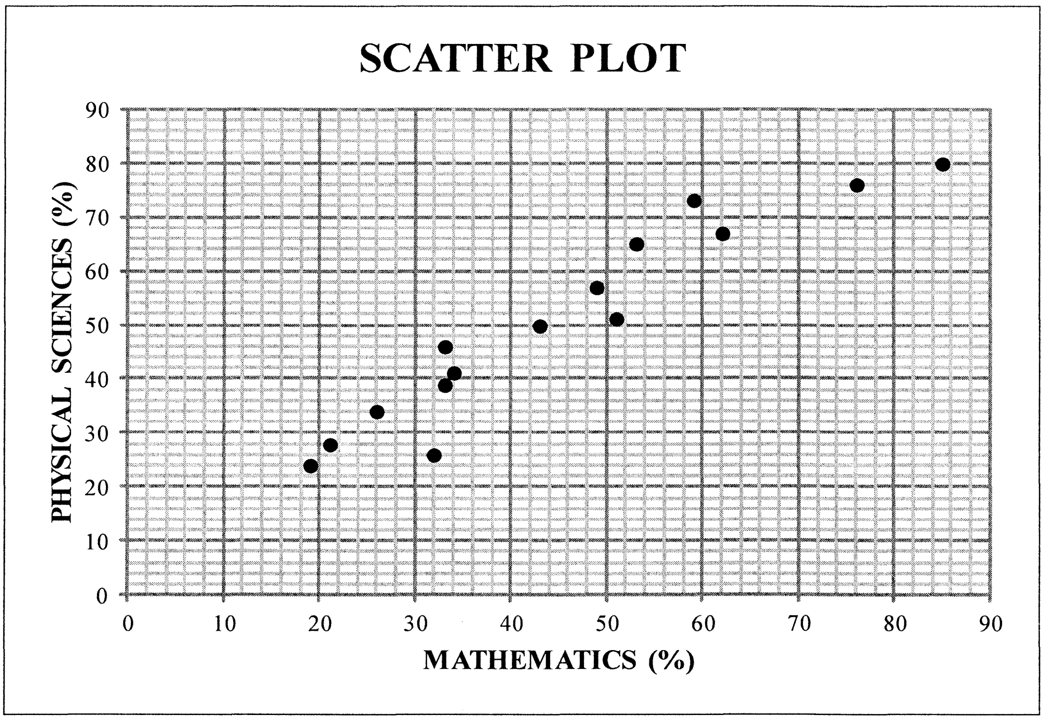
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A Mathematics teacher was curious to establish if her learners' Mathematics marks influenced their Physical Sciences marks. In the table below, the Mathematics and Physical Sciences marks of 15 learners in her class are given as percentages (%).

MATHEMATICS (AS %)	26	62	21	33	53	76	32	59	43	33	49	51	19	34	85
PHYSICAL SCIENCES (AS %)	34	67	28	46	65	76	26	73	50	39	57	51	24	41	80



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Draw the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)

1.3 Predict the Physical Sciences mark of a learner who achieved 69% for Mathematics. (2)

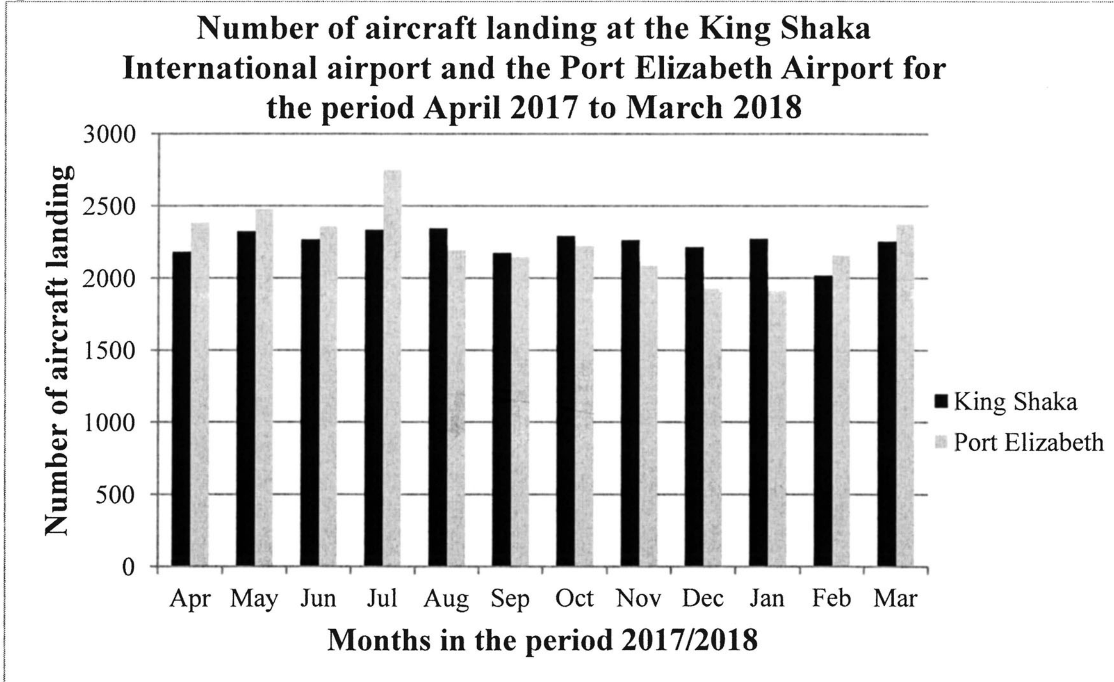
1.4 Write down the correlation coefficient between the Mathematics and Physical Sciences marks for the data. (1)

1.5 Comment on the strength of the correlation between the Mathematics and Physical Sciences marks for the data. (1)

1.6 What trend did the teacher observe between the results of the two subjects? (1)
[10]

QUESTION 2

The number of aircraft landing at the King Shaka International Airport and the Port Elizabeth Airport for the period starting in April 2017 and ending in March 2018, is shown in the double bar graph below.



[Source: http://www.airports.co.za/airports/statistics/aircraft]

2.1 The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During which month is this difference the greatest? (1)

2.2 The number of aircraft landing at the King Shaka International Airport during these months are:

2 182	2 323	2 267	2 334	2 346	2 175
2 293	2 263	2 215	2 271	2 018	2 254

Calculate the mean for the data. (2)

2.3 Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period. (2)

2.4 Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean. (3)

2.5 Which ONE of the following statements is CORRECT?

A. During December and January, there were more landings at the Port Elizabeth Airport than at the King Shaka International Airport.

B. There was a greater variation in the number of aircraft landing at the King Shaka International Airport than at the Port Elizabeth Airport for the given period.

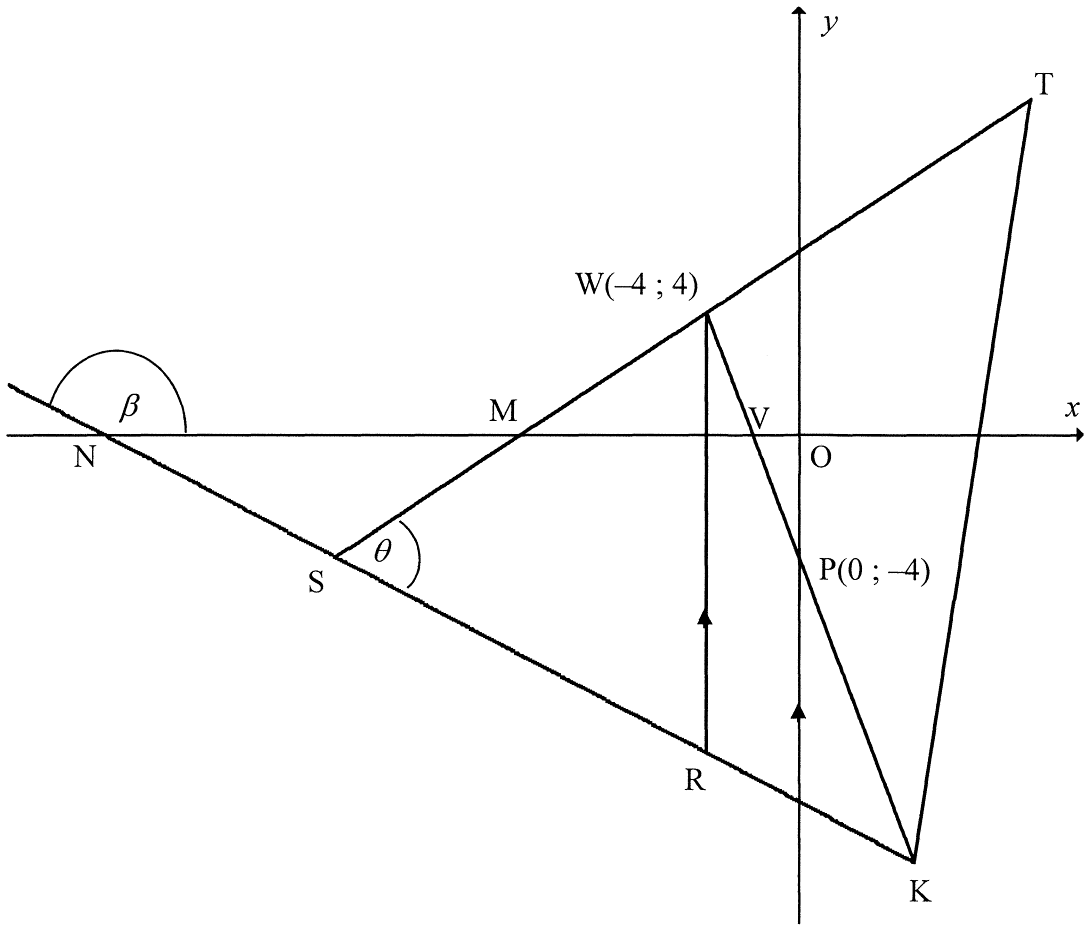
C. The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport. (1)

[9]

EASTERN CAPE

QUESTION 3

ΔTSK is drawn. The equation of ST is $y = \frac{1}{2}x + 6$ and ST cuts the x-axis at M. W(–4 ; 4) lies on ST and R lies on SK such that WR is parallel to the y-axis. WK cuts the x-axis at V and the y-axis at P(0 ; –4). KS produced cuts the x-axis at N. $\hat{TSK} = \theta$.



3.1 Calculate the gradient of WP. (2)

3.2 Show that WP $\perp$ ST. (2)

3.3 If the equation of SK is given as $5y + 2x + 60 = 0$, calculate the coordinates of S. (4)

3.4 Calculate the length of WR. (4)

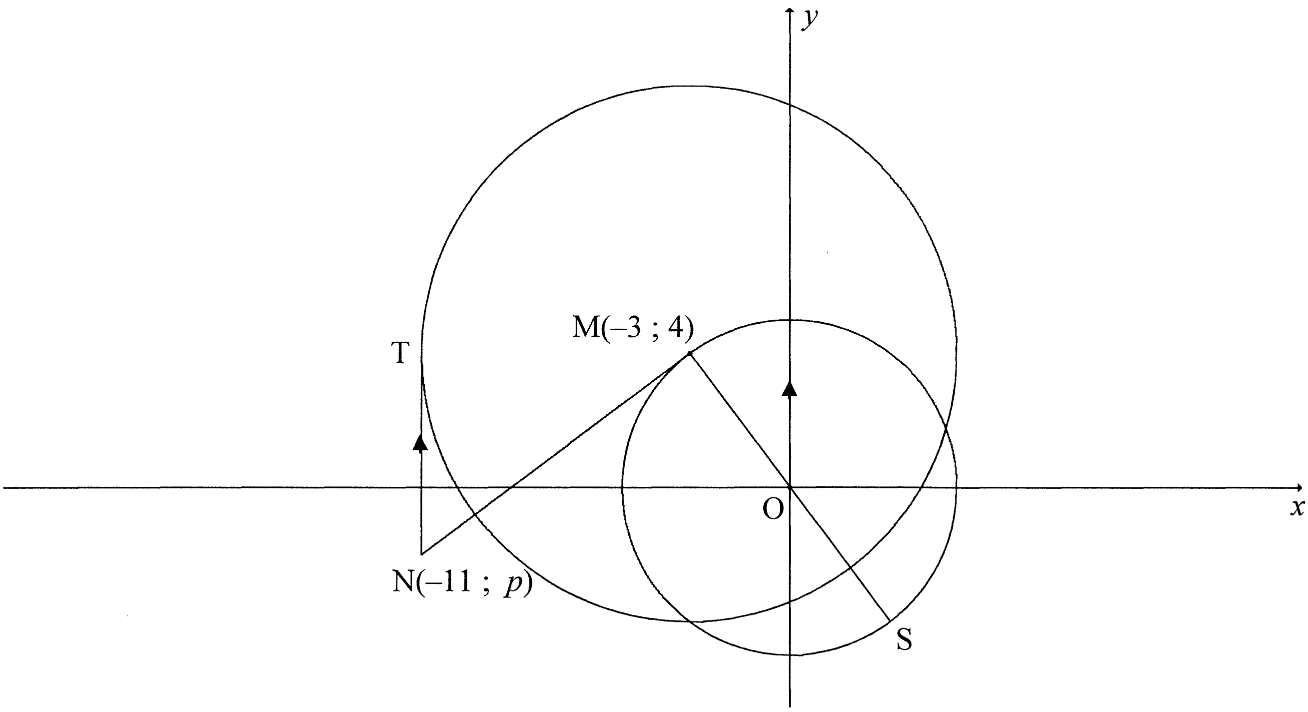
3.5 Calculate the size of θ. (5)

3.6 Let L be a point in the third quadrant such that SWRL, in that order, forms a parallelogram. Calculate the area of SWRL. (4)

[21]

QUESTION 4

M(–3 ; 4) is the centre of the large circle and a point on the small circle having centre O(0; 0). From N(–11 ; p), a tangent is drawn to touch the large circle at T with NT is parallel to the y-axis. NM is a tangent to the smaller circle at M with MOS a diameter.



4.1 Determine the equation of the small circle. (2)

4.2 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.3 Determine the equation of NM in the form $y=mx+c$ (4)

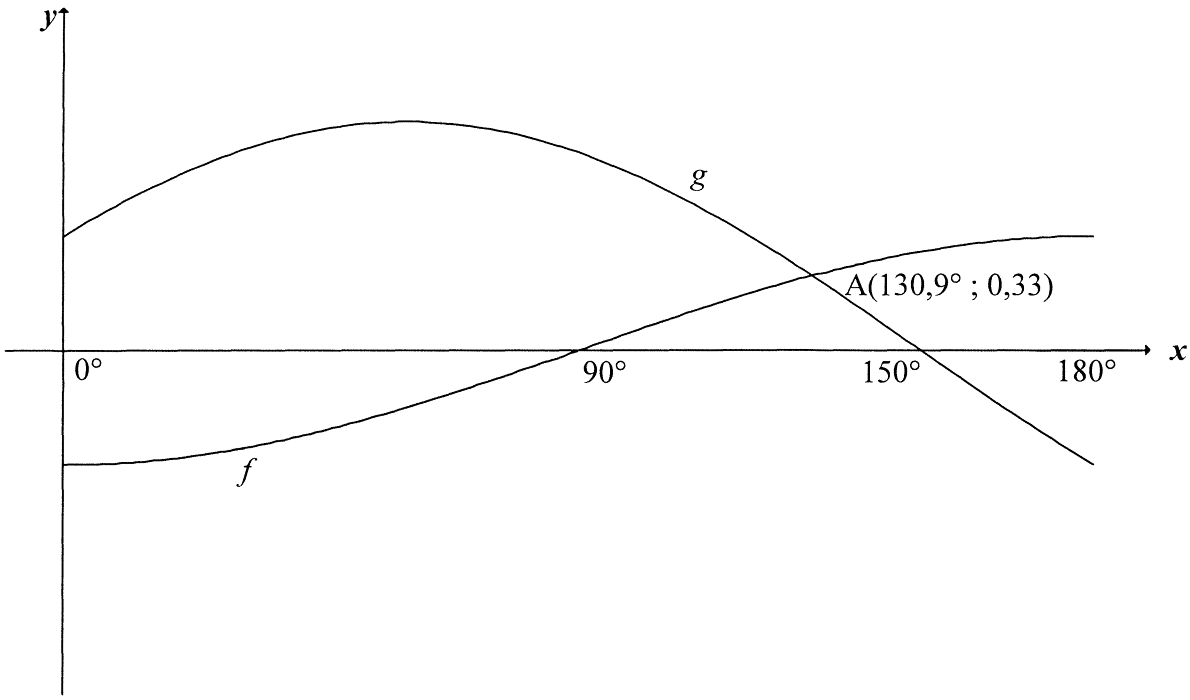
4.4 Calculate the length of SN. (5)

4.5 If another circle with centre B(–2 ; 5) and radius k touches the circle centred at M, determine the value(s) of k, correct to ONE decimal place. (5)

[19]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}\cos x$ and $g(x) = \sin(x + 30°)$, for the interval $x \in [0°; 180°]$, are drawn below. A(130,9° ; 0,33) is the approximate point of intersection of the two graphs.



5.1 Write down the period of g. (1)

5.2 Write down the amplitude of f (1)

5.3 Determine the value of $f(180°) - g(180°)$ (1)

5.4 Use the graphs to determine the values of x, in the interval $x \in [0° ; 180°]$, for which:

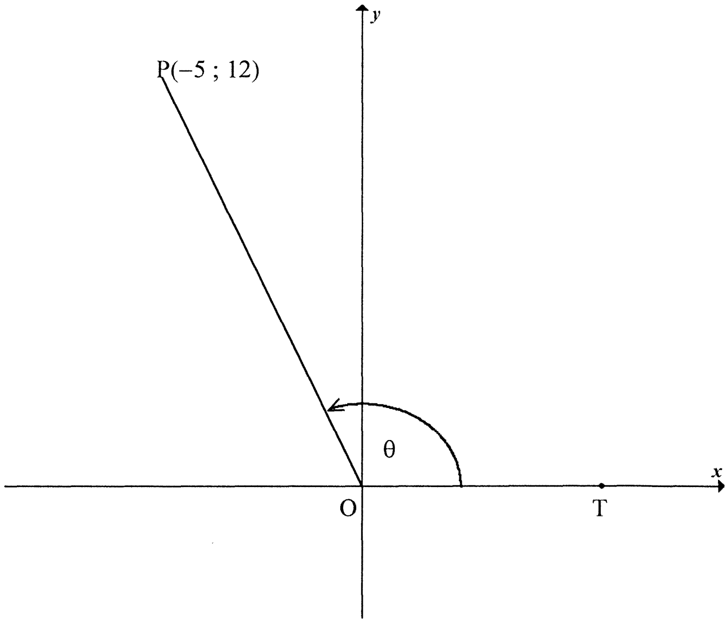
5.4.1 $f(x - 10°) = g(x - 10°)$ (1)

5.4.2 $\sqrt{3}\sin x + \cos x \geq 1$ (4)

[8]

QUESTION 6

6.1 In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $\hat{POT} = \theta$



Answer the following **without using a calculator**:

6.1.1 Write down the value of $\tan\theta$ (1)

6.1.2 Calculate the value of $\cos\theta$ (3)

6.1.3 S(a ; b) is a point in the third quadrant such that $\hat{TOS} = \theta + 90°$ and OS = 6,5 units. Calculate the value of b. (4)

6.2 Determine, **without using a calculator**, the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$$ (5)

6.3 Determine the general solution of the following equation:

$$6\sin^2 x + 7\cos x - 3 = 0$$ (6)

6.4 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

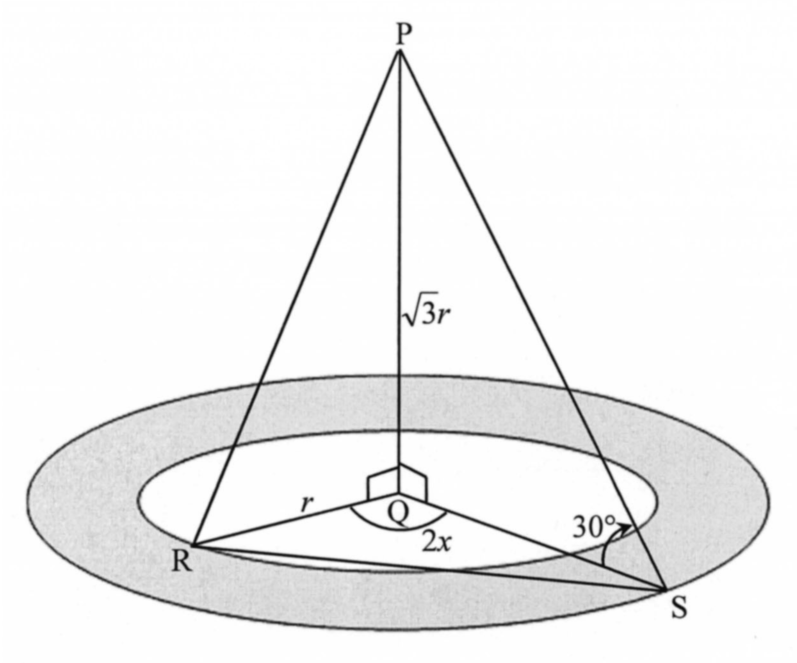
Determine the value of $\cos 2A$ **without using a calculator**. (5)

[24]

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is 30°.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



7.1 Show that $QS = 3r$ (3)

7.2 Determine, in terms of r, the area of the flower garden. (2)

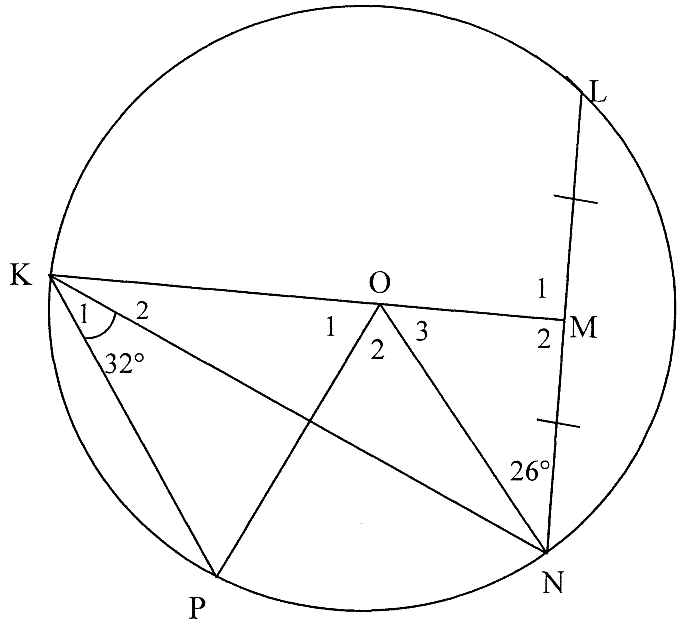
7.3 Show that $RS = r\sqrt{10 - 6\cos 2x}$ (3)

7.4 If $r = 10$ metres and $x = 56°$, calculate RS. (2)

[10]

QUESTION 8

8.1 O is the centre of the circle.. KOM bisects chord LN and $M\hat{N}O = 26°$. K and P are points on the circle with $N\hat{K}P = 32°$. OP is drawn.



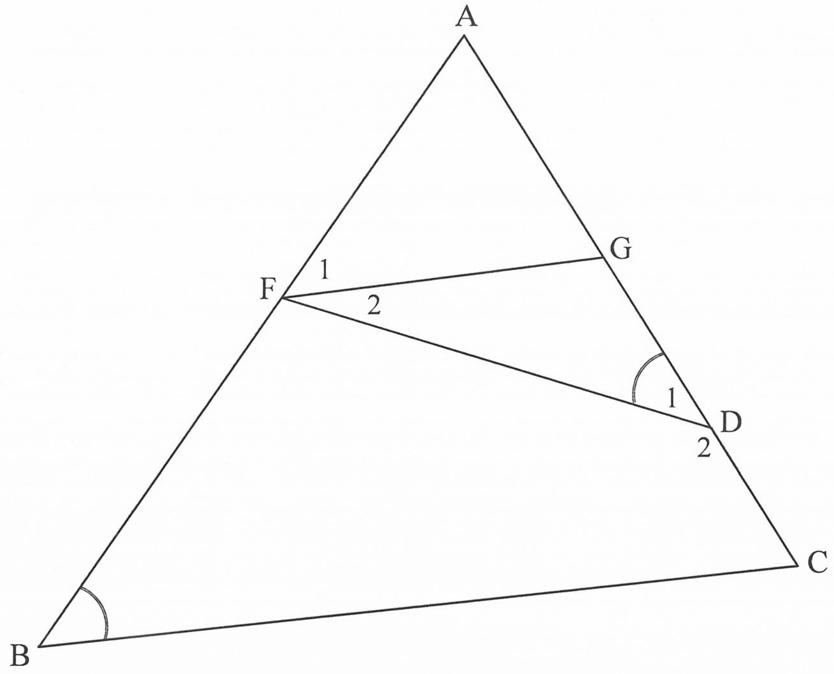
8.1.1 Determine, giving reasons, the size of:

(a) $\hat{O}_2$ (2)

(b) $\hat{O}_1$ (4)

8.1.2 Prove, giving reasons, that KN bisects $O\hat{K}P$. (3)

8.2 In ΔABC, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.



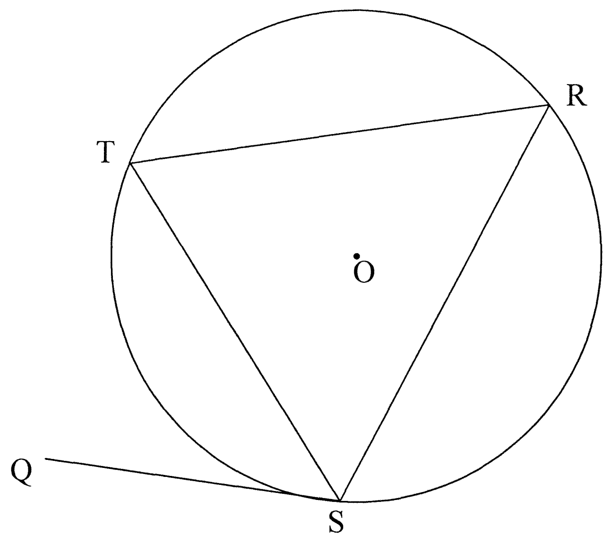
8.2.1 If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that FG || BC. (4)

8.2.2 If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x. (4)

[17]

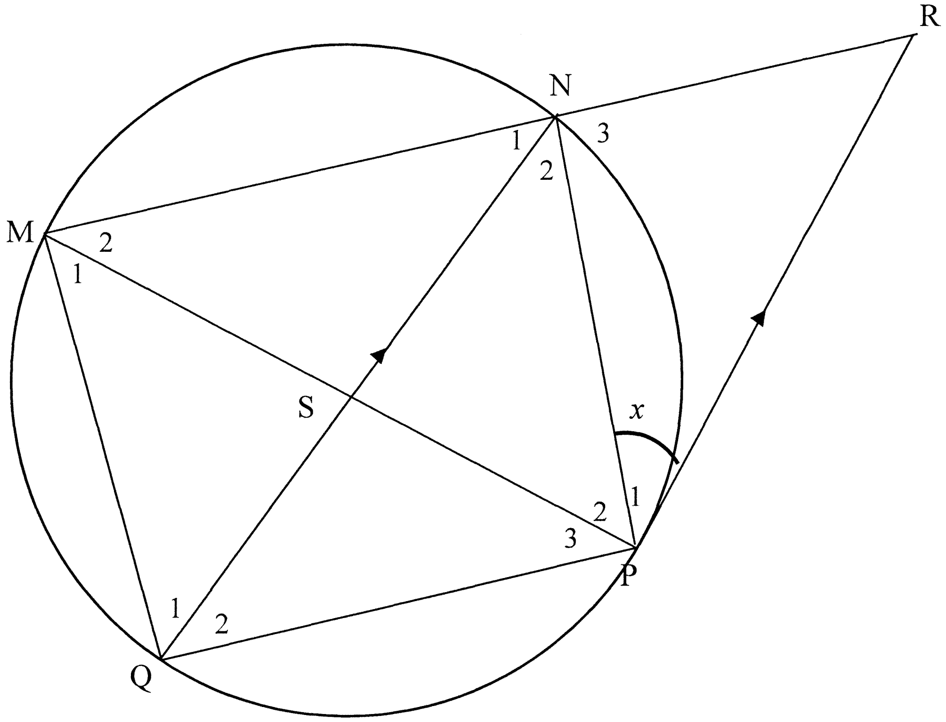
QUESTION 9

9.1 In the diagram, O is the centre of the circle. Points S, T and R lie on the circle. Chords ST, SR and TR are drawn in the circle. QS is a tangent to the circle at S.



Use the diagram to prove the theorem which states that $Q\hat{S}T = \hat{R}$. (5)

9.2 Chord QN bisects $M\hat{N}P$ and intersects chord MP at S. The tangent at P meets MN produced at R such that QN || PR. Let $\hat{P}_1 = x$.



9.2.1 Determine the following angles in terms of x. Give reasons

(a) $\hat{N}_2$ (2)

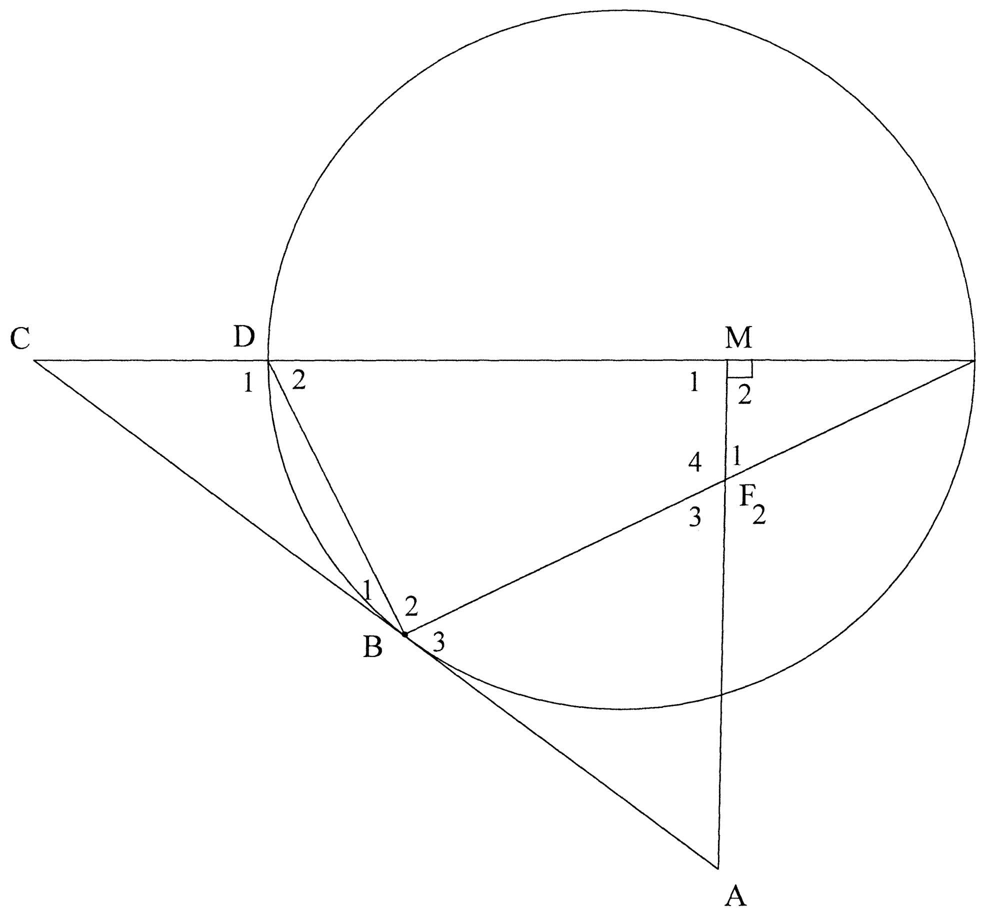
(b) $\hat{Q}_2$ (2)

9.2.2 Prove, giving reasons, that $\dfrac{MN}{NR} = \dfrac{MS}{SQ}$ (6)

[15]

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that AM ⊥ DE. AM and chord BE intersect at F.



10.1 Prove, giving reasons, that:

10.1.1 FBDM is a cyclic quadrilateral (3)

10.1.2 $\hat{B}_3 = \hat{F}_1$ (4)

10.1.3 $\Delta CDB \parallel\!| \Delta CBE$ (3)

10.2 If it is further given that CD = 2 units and DE = 6 units, calculate the length of:

10.2.1 BC (3)

10.2.2 DB (4)
[17]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta\text{ABC} = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/*SENIOR SERTIFIKAAT*
NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2020

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

DEPARTMENT OF BASIC EDUCATION
PRIVATE BAG X895, PRETORIA 0001
2020 -11- 23
APPROVED MARKING GUIDELINE
PUBLIC EXAMINATION

23/11/2020
approved

These marking guidelines consist of 27 pages.
Hierdie nasienriglyne bestaan uit 27 bladsye.

Approved
2020-11-23

NOTE:

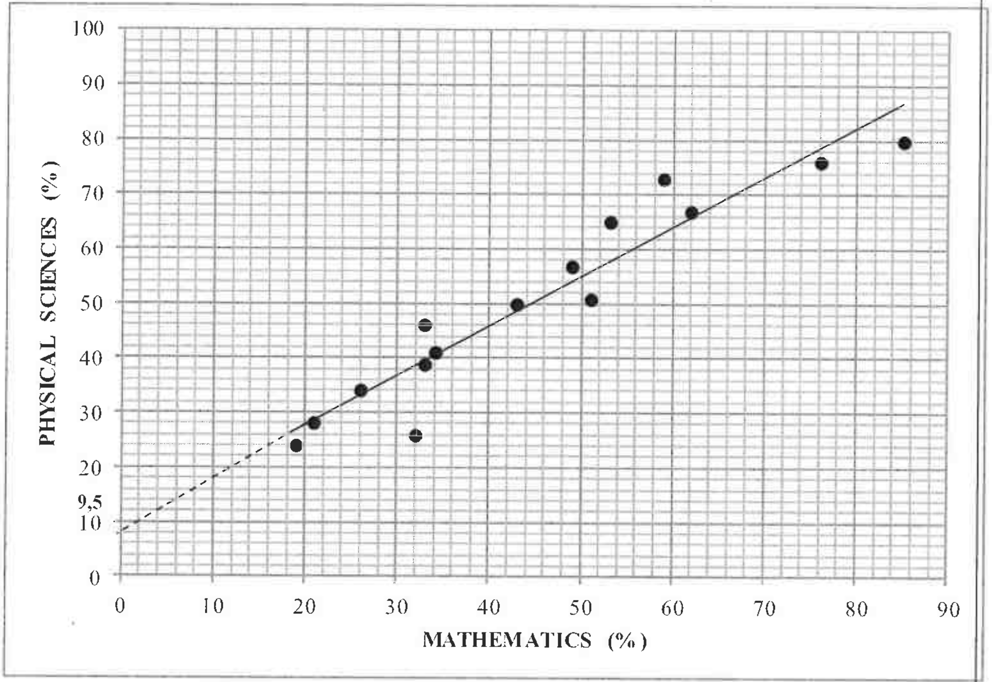
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

<table>
<tr><td>1.1</td><td>$a=9{,}5$
$b=0{,}909..=0{,}91$
$\hat{y}=9{,}5+0{,}91x$</td><td>✓ $a=9{,}5$
✓ $b=0{,}91$
✓ equation
(3)</td></tr>
<tr><td>1.2</td><td>
</td><td>✓✓ correct slope going through 2 points:
(50 ; 55) or
(40 ; 46) or
(60 ; 64) or
(0 ; 9,5) or
(45 ; 50)
(2)</td></tr>
<tr><td>1.3</td><td>Final exam mark ≈ 72,22% (calculator)

OR

$\hat{y}=9{,}5+0{,}91(69)$
$\approx 72{,}29\%$</td><td>✓✓ answer
(2)

✓ substitution
✓ answer
(2)</td></tr>
<tr><td>1.4</td><td>$r=0{,}95$</td><td>✓ answer(A)
(1)</td></tr>
<tr><td>1.5</td><td>There is a very strong positive correlation between the Mathematics and Physical Sciences mark.
Daar is 'n baie sterk positiewe korrelasie tussen die Wiskunde en Fisiese Wetenskappunte.</td><td>✓ strong/ sterk
(1)</td></tr>
<tr><td>1.6</td><td>The teacher concludes that the higher the learners' Mathematics marks, the higher the learners' Physical Sciences marks.
Die onderwyser het waargeneem dat hoe hoër die wiskunde punte is, hoe hoër is die Fisiese Wetenskappunte.</td><td>✓ answer
(1)</td></tr>
<tr><td></td><td></td><td>[10]</td></tr>
</table>

QUESTION/*VRAAG* 2

2 018	**2 175**	**2 182**	**2 215**	**2 254**	**2 263**	**2 267**	**2 271**	**2 293**	**2 323**	**2 334**	**2 346**

2.1	July / *Julie*	✓ answer (1)
2.2	$\bar{x} = \dfrac{26941}{12}$ $= 2\,245{,}083.. \approx 2\,245{,}08$ aircraft landings Answer only: Full marks	✓ 26 941 ✓ answer (2)
2.3	Standard deviation for landings at the King Shaka International airport: $\sigma = 86{,}30$	✓✓ answer (2)
2.4	$(\bar{x}-\sigma\,;\bar{x}+\sigma) = (2\,245{,}08 - 86{,}30\ ;\ 2\,245{,}08 + 86{,}30)$ limit = (2 158,78 ; 2 331,38) There were **9** months when the aircraft arrivals at the King Shaka International airport were within one standard deviation of the mean.	✓ $\bar{x}-\sigma$ ✓ $\bar{x}+\sigma$ ✓ answer (3)
2.5	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of arrivals at the King Shaka International Airport **OR C.**	✓ answer (1)
		[9]

DEPARTMENT OF BASIC EDUCATION
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2020 -11- 23
APPROVED MARKING GUIDELINE
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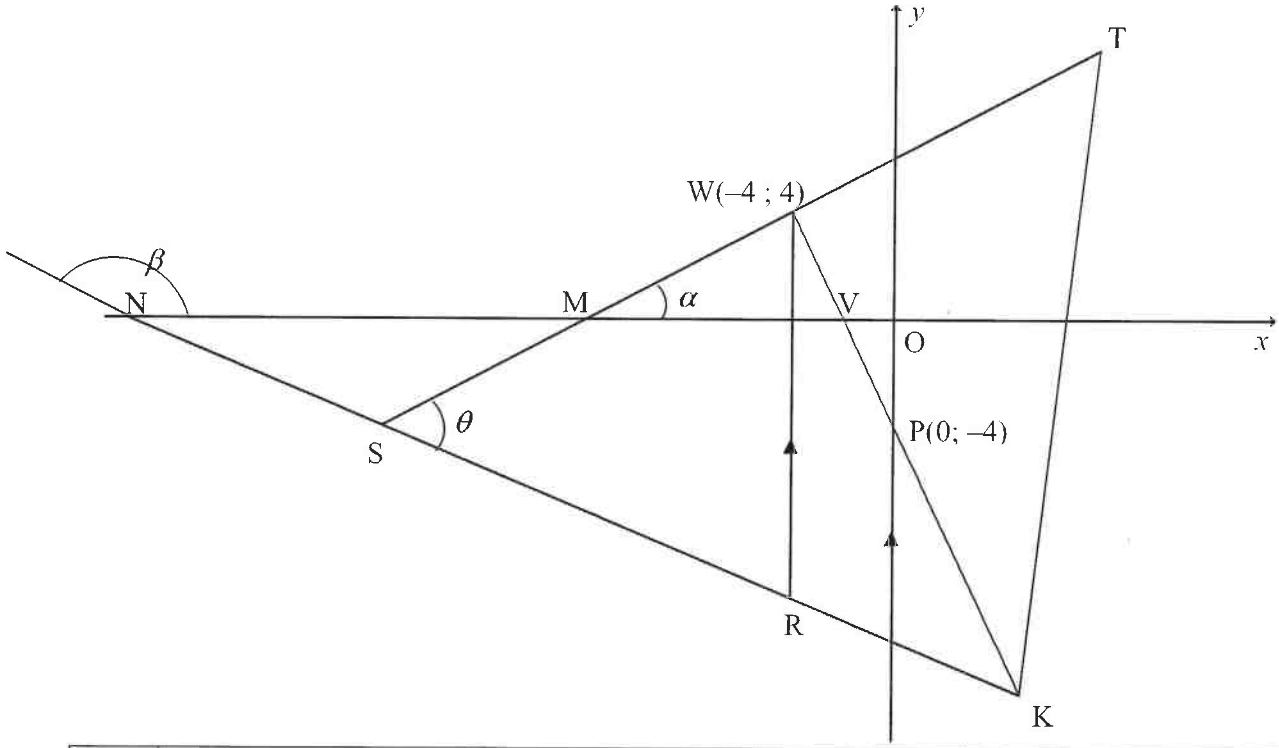
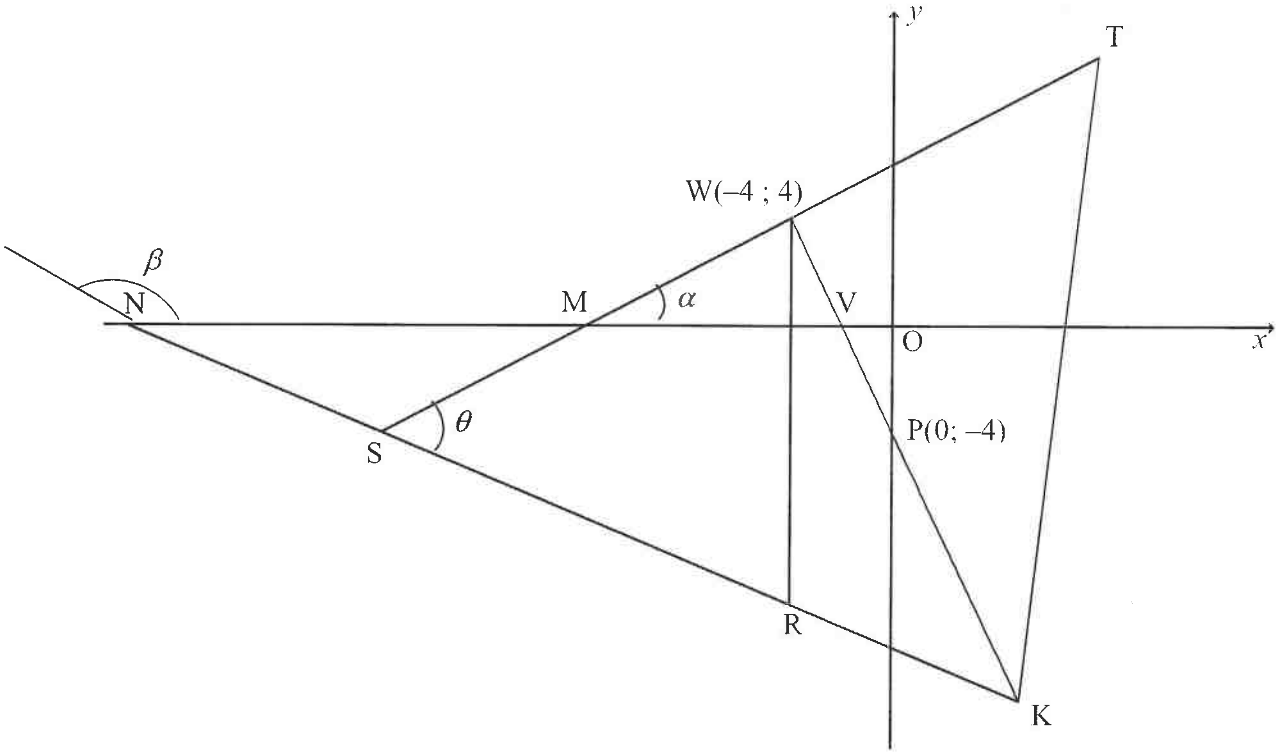
QUESTION/*VRAAG* 3



3.1	$m_{WP}=\dfrac{4-(-4)}{-4-0}=\dfrac{8}{-4}$ $m_{WP}=-2$	✓ substitution of W and P ✓ m_{WP} (2)
3.2	$m_{ST}=\dfrac{1}{2}$ (given) $(m_{WP})(m_{ST})=(-2)\left(\dfrac{1}{2}\right)$ $=-1$ $\therefore ST\perp WP$	✓ $(m_{WP})(m_{ST})$ ✓ $(m_{WP})(m_{ST})=-1$ (2)
3.3	$5y+2x+60=0$ $\therefore y=-\dfrac{2}{5}x-12$ $-\dfrac{2}{5}x-12=\dfrac{1}{2}x+6$ $-4x-120=5x+60$ $9x=-180$ $x=-20$ $\therefore y=-\dfrac{2}{5}(-20)-12$ $\therefore y=-4$ $\therefore$ S(–20; –4) **OR**	✓ equating ✓ x value ✓ substitution ✓ y value (4)

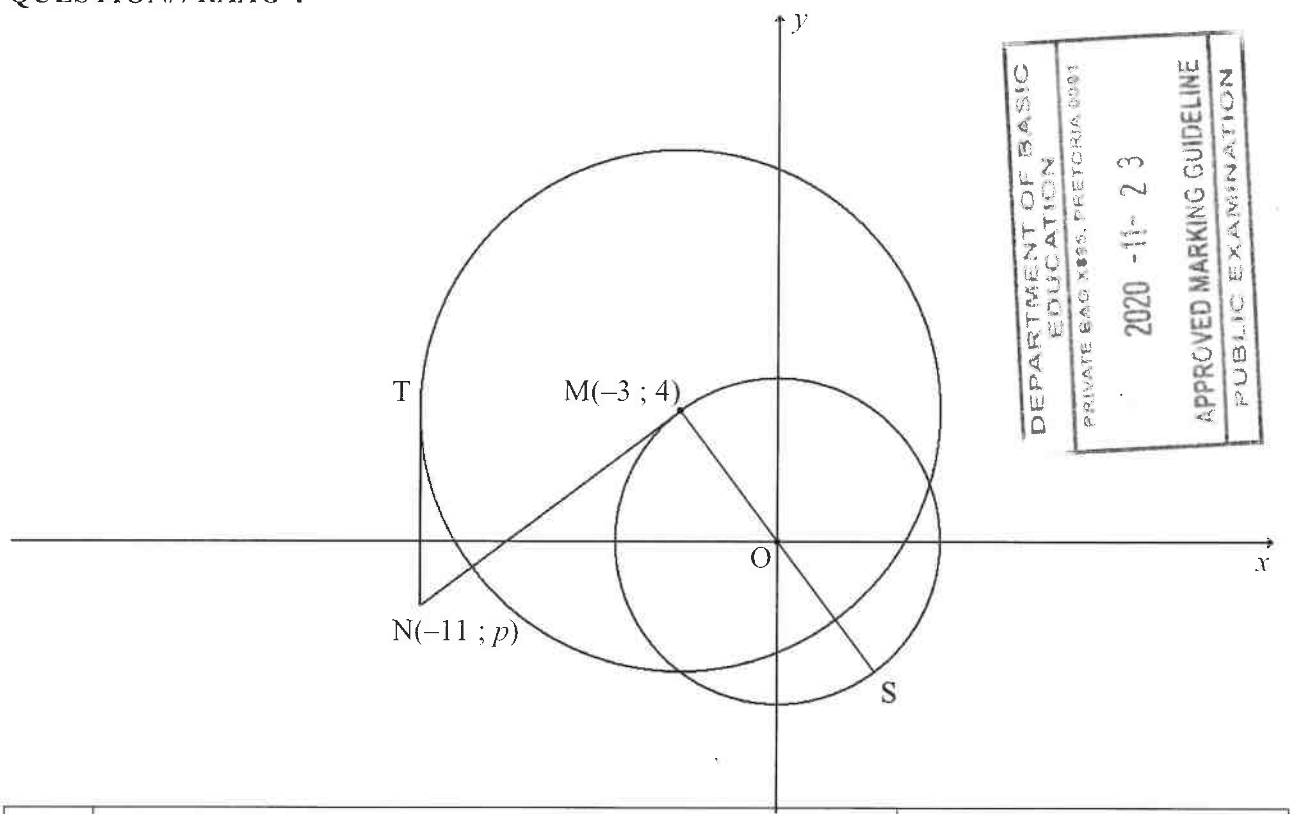


	$5y+2x+60=0$ $5\left(\frac{1}{2}x+6\right)+2x+60=0$ $\frac{5}{2}x+30+2x+60=0$ $\frac{9}{2}x=-90 \qquad \therefore x=-20$ $\therefore y=-\frac{2}{5}(-20)-12$ $\therefore y=-4$ $\therefore$ S(–20; –4) **OR** $5y+2x=-60$ ....(1) $2y-x=12$ ....(2) $(1)+2(2): 9y=-36$ $y=-4$ $2(-4)-x=12$ $x=-20$	✓ substitution ✓ x value ✓ substitution ✓ y value (4) ✓ adding ✓ y value ✓ substitution ✓ x value (4)

3.4	$y=-\frac{2}{5}(-4)-12$ OR $5y+2(-4)+60=0$ $y=-\frac{52}{5}$ $\therefore R\left(-4;-\frac{52}{5}\right)$ OR R(–4 ; –10,4) $\therefore WR=4-\left(-\frac{52}{5}\right)$ OR $WR=\sqrt{(-4-(-4))^2+(4-\left(-\frac{52}{5}\right))^2}$ $\therefore WR=\frac{72}{5}$ units or $WR=14\frac{2}{5}$ units **OR** WR = ST – SK $=\frac{1}{2}x+6-\left(-\frac{2}{5}x-12\right)$ $=\frac{9}{10}x+18$ $=\frac{9}{10}(-4)+18$ $=14{,}4$ units	✓ substitution ✓ y value ✓ method or subst into distance formula ✓ answer (4) ✓ substitution ✓ simplification ✓ subst $x=-4$ ✓ answer (4)
3.5	$m_{SK}=-\frac{2}{5}$ $\beta=158{,}19...^\circ$ (Ref. $\angle$ =21, 801…°) $M\hat{N}S=21{,}80...^\circ$ $m_{ST}=\frac{1}{2}$ $N\hat{M}S=26{,}56...^\circ$ $\theta=21{,}80...^\circ+26{,}56...^\circ$ [ext $\angle$ of Δ] $\theta=48{,}366...^\circ=48{,}37^\circ$	✓ m_{SK} ✓ size of β ✓ size of $N\hat{M}S$ ✓ method ✓ answer (5)
3.6	In ΔSRW: $\perp h=-4-(-20)$ $\perp h=16$units Area $\Delta SRW=\frac{1}{2}(\perp h)(WR)$ $=\frac{1}{2}(16)\left(\frac{72}{5}\right)$ $=115{,}2$ square units Area SWRL = 2Area ΔSRW =2(115,2) = 230,4square units **OR**	✓ $\perp h$ ✓ substitution ✓ area Δ ✓ answer (4)

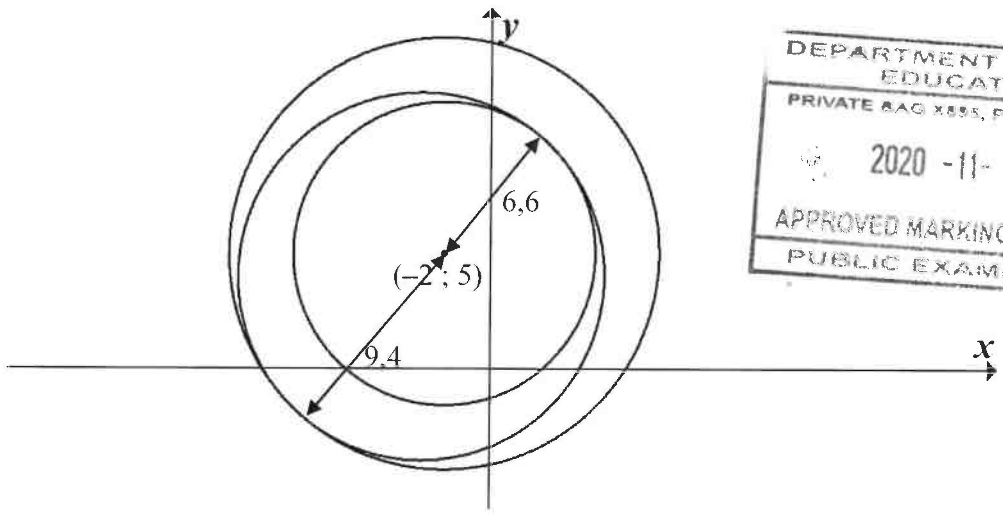
	In ΔSRW: $\perp h = -4-(-20)$ $\perp h = 16$units Area SWRL $= 16 \times \frac{72}{5}$ $= 230{,}40$ square units	✓ $\perp h$ ✓ ✓ substitution ✓ answer (4)
	OR $\text{SW} = \sqrt{(-20+4)^2+(-4-4)^2} = 8\sqrt{5} = 17{,}89$ $\text{SR} = \sqrt{(-20+4)^2+\left(-4+10\frac{2}{5}\right)^2} = \frac{16\sqrt{29}}{5} = 17{,}23$ Area SWRL = $2\times$ Area ΔSRW $=2\left(\frac{1}{2}\text{SW}\times\text{SR}\sin\theta\right)$ $=2\left(\frac{1}{2}8\sqrt{5}\times\frac{16\sqrt{29}}{5}\sin 48{,}37°\right)$ $=230{,}41$square units	✓$\text{SW} = 8\sqrt{5}$ ✓$\text{SR} = \frac{16\sqrt{29}}{5}$ ✓substitution ✓answer (4)
		[21]

QUESTION/*VRAAG* 4

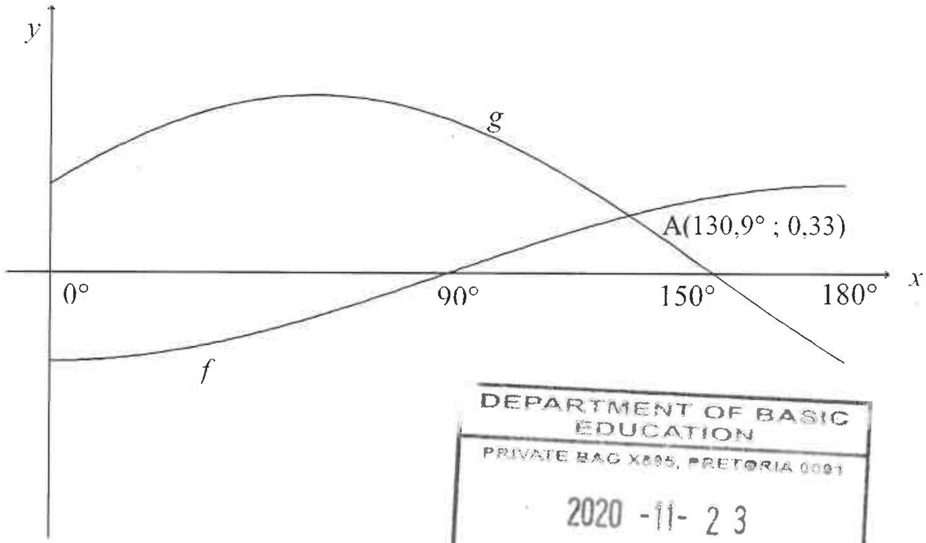


4.1	$x^2 + y^2 = r^2$ $\therefore r^2 = (-3)^2 + (4)^2 = 25$ $x^2 + y^2 = 25$	✓ substitution ✓ answer (2)
4.2	TM ⊥ TN [tangent ⊥ radius] T(–11 ; 4) $r = -3 - (-11) = 8$ $(x+3)^2 + (y-4)^2 = 64$	✓ $x_T = -11$ ✓ LHS ✓ RHS (3)
4.3	O (0 ; 0) and M(–3; 4) $m_{OM} = \frac{4-0}{-3-0} = -\frac{4}{3}$ OR $\frac{0-4}{0-(-3)} = -\frac{4}{3}$ $m_{NM} = \frac{3}{4}$ $y - 4 = \frac{3}{4}(x-(-3))$ **OR** $y = \frac{3}{4}x + c$ $y - 4 = \frac{3}{4}x + \frac{9}{4}$ $\quad$ $4 = \frac{3}{4}(-3) + c$ $\therefore y = \frac{3}{4}x + \frac{25}{4}$ $\quad$ $c = \frac{25}{4}$ $y = \frac{3}{4}x + \frac{25}{4}$	✓ $m_{OM} = -\frac{4}{3}$ ✓ $m_{NM} = \frac{3}{4}$ ✓ substitution of m and M ✓ equation (4)

4.4	$N(-11\,;p)$ $y=\frac{3}{4}x+\frac{25}{4}$ $p=\frac{3}{4}(-11)+\frac{25}{4}$ OR $\frac{4-p}{-3-(-11)}=\frac{3}{4}$ $p=-2$ $\therefore N(-11;-2)$ $\frac{-3+x_S}{2}=0$ and $\frac{4+y_S}{2}=0$ $\therefore S(3;-4)$ $SN=\sqrt{(-11-3)^2+(-2-(-4))^2}$ $=10\sqrt{2}$ units or 14,14 units	✓ subst $x=-11$ into eq or gradient ✓ $p=-2$ ✓ x_S ✓ y_S ✓ answer (CA) (5)
4.5	$B(-2;\,5)$ $BM=\sqrt{2}$ units Radius of circle centred at M = 8 units $k=8-\sqrt{2}$ or $k=8+\sqrt{2}$ = 6,59 units = 9,41 units = 6,6 units = 9,4 units	✓ $\sqrt{2}$ ✓✓ $k=6,6$ ✓✓ $k=9,4$ (5)
		[19]

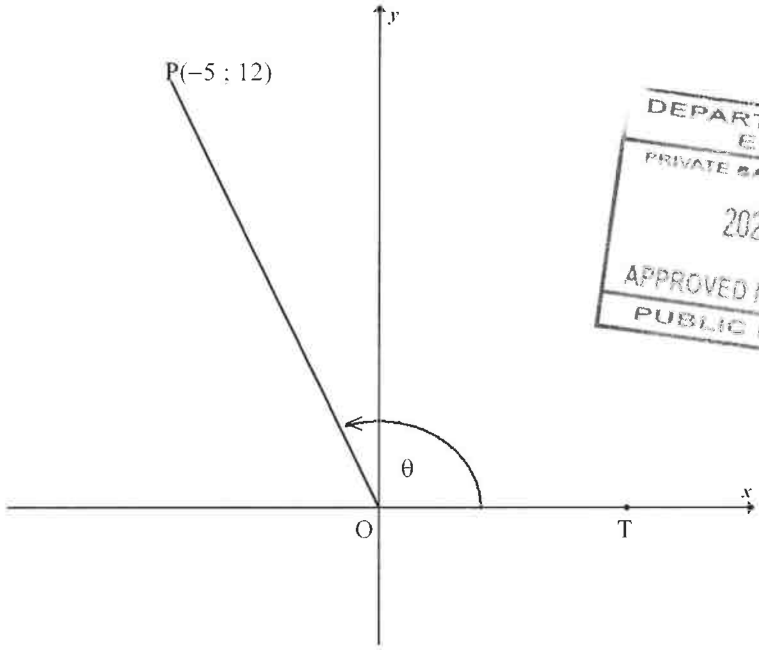


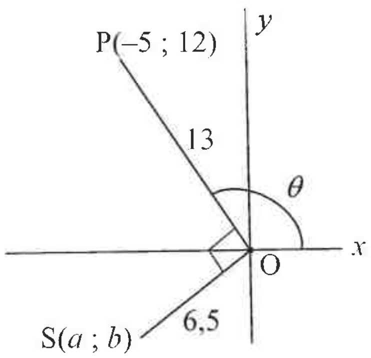
QUESTION/*VRAAG* 5



5.1	Period of $g = 360°$	✓ answer (1)
5.2	Amplitude of $f = \frac{1}{2}$	✓ answer (A) (1)
5.3	$f(180°) - g(180°)$ $= \frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 1$	✓ 1 (1)
5.4.1	$x = 140{,}9°$	✓ $x = 140{,}9°$ (1)
5.4.2	$\sqrt{3}\sin x + \cos x \geq 1$ $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x \geq \frac{1}{2}$ $\sin x\cos 30° + \cos x\sin 30° \geq \frac{1}{2}$ $\sin(x + 30°) \geq \frac{1}{2}$ $\sin(x + 30°) = \frac{1}{2}$ at $x = 0°$ or $x = 120°$ $\therefore x \in [0°;120°]$ **OR** $0° \leq x \leq 120°$	✓ dividing by 2 ✓ $\cos 30°; \sin 30°$ ✓ $\sin(x + 30°) \geq \frac{1}{2}$ ✓ interval (4)
		[8]

QUESTION/*VRAAG* 6

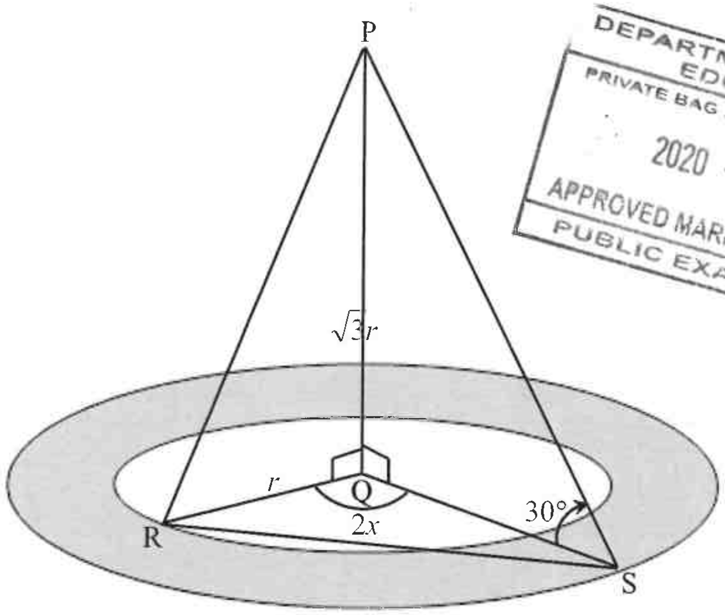


6.1.1	$\tan\theta = -\frac{12}{5}$ or $-2\frac{2}{5}$	✓answer (1)
6.1.2	$(\text{OP})^2 = (-5)^2 + (12)^2$ $\text{OP} = 13$ $\cos\theta = -\frac{5}{13}$	✓ Pythagoras ✓ OP ✓ answer (3)
6.1.3	$\sin(\theta + 90°) = \frac{b}{6{,}5}$ $\cos\theta = \frac{b}{6{,}5}$ $\frac{-5}{13} = \frac{b}{6{,}5}$ $b = -\frac{5}{2}$  **OR** $\cos(90° + \theta) = \frac{a}{6{,}5}$ $-\sin\theta = \frac{a}{6{,}5}$ $-\frac{12}{13} = \frac{a}{6{,}5}$ $\therefore a = -6$ $b = \sqrt{(6{,}5)^2 - (-6)^2} = -\frac{5}{2}$	✓ $\sin(\theta + 90°) = \frac{b}{6{,}5}$ ✓ $\cos\theta$ ✓ $\frac{-5}{13} = \frac{b}{6{,}5}$ ✓ value of b (4) ✓ $\cos(\theta + 90°) = \frac{a}{6{,}5}$ ✓ $-\sin\theta$ ✓ value of a ✓ value of b (4)

6.2	$\dfrac{\sin 2x.\cos(-x)+\cos 2x.\sin(360°-x)}{\sin(180°+x)}$ $=\dfrac{\sin 2x\ \cos x\ +\ \cos 2x(-\sin x)}{-\sin x}$ $=\dfrac{\sin(2x-x)}{-\sin x}$ $=\dfrac{\sin x}{-\sin x}$ $=-1$	✓ $\cos(-x)=\cos x$ ✓ $\sin(360°-x)=-\sin x$ ✓ $\sin(180°+x)=-\sin x$ ✓ numerator = $\sin x$ ✓ answer (5)
6.3	$6\sin^2 x+7\cos x-3=0$ $6(1-\cos^2 x)+7\cos x-3=0$ $6-6\cos^2 x+7\cos x-3=0$ $6\cos^2 x-7\cos x-3=0$ $(3\cos x+1)(2\cos x-3)=0$ $\cos x=-\dfrac{1}{3}$ or $\cos x=\dfrac{3}{2}$ (N/A) $\therefore\ x=109{,}47°+k.360°;\ k\in Z$ or $x=250{,}53°+k.360°;\ k\in Z$	✓ identity ✓ standard form ✓ factors ✓ both solutions of $\cos x$ ✓ $x=109{,}47°\ \&\ 250{,}53°$ ✓ $+k.360°;\ k\in Z$ (6)
6.4	$x+\dfrac{1}{x}=3\cos A$ $(3\cos A)^2=\left(x+\dfrac{1}{x}\right)^2$ $9\cos^2 A=x^2+\dfrac{1}{x^2}+2$ $9\cos^2 A=2+2$ $\cos^2 A=\dfrac{4}{9}$ $\cos 2A=2\cos^2 A-1$ $=2\left(\dfrac{4}{9}\right)-1$ $=-\dfrac{1}{9}$ **OR**	✓squaring both sides ✓ $9\cos^2 A=x^2+\dfrac{1}{x^2}+2$ ✓ $\cos^2 A=\dfrac{4}{9}$ ✓ $\cos 2A=2\cos^2 A-1$ ✓ answer (5)

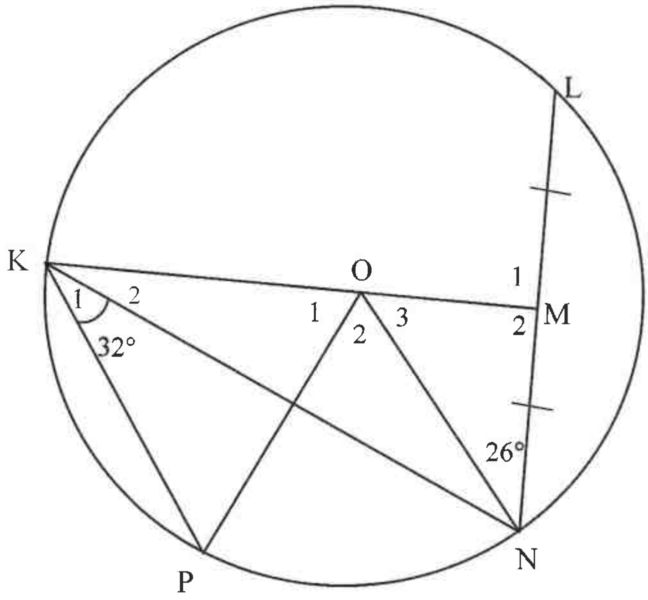
	$x^2 - 2 + \frac{1}{x^2} = 0$ $\left(x - \frac{1}{x}\right)^2 = 0$ $x^2 = 1$ $x = \pm 1$ $3\cos A = 2$ or $3\cos A = -2$ $\cos A = \frac{2}{3}$ or $\cos A = -\frac{2}{3}$ $\cos 2A = 2\cos^2 A - 1$ $= 2\left(\pm\frac{2}{3}\right)^2 - 1$ $= -\frac{1}{9}$	✓ $x = \pm 1$ ✓ $\cos A = \frac{2}{3}$ ✓ $\cos A = -\frac{2}{3}$ ✓double angle identity ✓ answer (5)
		[24]

QUESTION/*VRAAG* 7



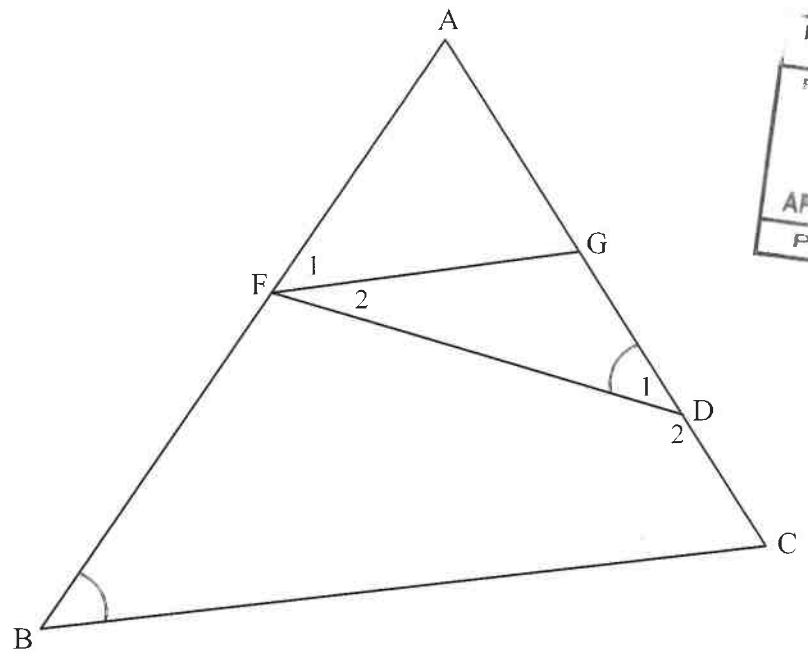
7.1	$\tan 30° = \frac{\sqrt{3}r}{QS}$ **OR** $\tan 60° = \frac{QS}{\sqrt{3}r}$ $QS = \frac{\sqrt{3}r}{\tan 30°}$ $\sqrt{3} = \frac{QS}{\sqrt{3}r}$ $= \frac{\sqrt{3}r}{\frac{1}{\sqrt{3}}}$ *or* $\frac{\sqrt{3}r}{\frac{\sqrt{3}}{3}}$ $QS = 3r$ $= 3r$	✓✓ trig ratio ✓ QS subject (3)
7.2	Area of flower garden $= \pi(3r)^2 - \pi r^2$ $= 9\pi r^2 - \pi r^2$ $= 8\pi r^2$	✓ substitution into difference of areas ✓ answer (2)
7.3	$RS^2 = r^2 + (3r)^2 - 2(r)(3r)\cos 2x$ $= r^2 + 9r^2 - 6r^2 \cos 2x$ $= 10r^2 - 6r^2 \cos 2x$ $= r^2(10 - 6\cos 2x)$ $RS = r\sqrt{10 - 6\cos 2x}$	✓substitution into cosine rule correctly ✓ $10r^2 - 6r^2 \cos 2x$ ✓ $r^2(10 - 6\cos 2x)$ (3)
7.4	$RS = 10\sqrt{10 - 6\cos 2(56)}$ $= 34{,}9966...$ ≈ 35 m	✓substitution ✓ answer (2)
		[10]

QUESTION/*VRAAG* 8



8.1.1(a)	$\hat{O}_2 = 64°$ [$\angle$ at centre = 2 × $\angle$ at circumference/ *Middelpts* $\angle$ = 2 × $\angle$ *omtreks* $\angle$]	✓ S ✓ R	(2)
8.1.1(b)	$\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $K\hat{O}N = 90° + 26° = 116°$ [ext $\angle$ of Δ/*buite* $\angle$ *van* Δ] $\hat{O}_1 = 116° - 64° = 52°$ **OR** $\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $\hat{O}_3 = 64°$ [sum of $\angle$s in Δ] $\hat{O}_1 = 52°$ [$\angle$s on straight line/*op 'n reguitlyn*]	✓ S ✓ R ✓ S ✓ answer ✓ S ✓ R ✓ S ✓ answer	(4) (4)
8.1.2	$P\hat{K}O + \hat{P} = 128°$ [sum of $\angle$s in Δ/*som* $\angle$*e van* Δ] $P\hat{K}O = \hat{P}$ [$\angle$s opp = sides/$\angle$*e teenoor = sye*] $= 64°$ $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$ **OR** $\hat{K}_2 = K\hat{N}O$ [$\angle$s opp = sides/$\angle$*e teenoor = sye*] $\hat{K}_2 + K\hat{N}O = 64°$ [sum of $\angle$s in Δ/*som* $\angle$*e van* Δ] $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$	✓ S ✓ S ✓ S ✓ S ✓ S ✓ S	(3) (3)

8.2

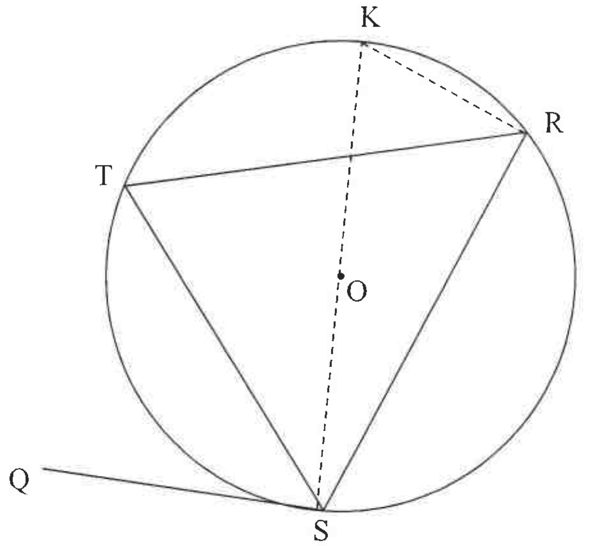


8.2.1	$\hat{F}_1 = \hat{D}_1$ [tan chord theorem/*raaklyn koordst*] $\hat{D}_1 = \hat{B}$ [Given/*Gegee*] $\therefore \hat{F}_1 = \hat{B}$ $\therefore FG \parallel BC$ [corresp $\angle$s =/*Ooreenkomstige*$\angle$*e* =]	✓ S ✓ R ✓ $\hat{F}_1 = \hat{B}$ ✓ R (4)
8.2.2	$\frac{GC}{AC} = \frac{FB}{AB}$ [line ‖ one side of Δ/*lyn* ‖ *een sy v* Δ] $\frac{x+9}{2x-6} = \frac{5}{7}$ $7x + 63 = 10x - 30$ $3x = 93$ $x = 31$ **OR** $AG = 2x - 6 - (x+9) = x - 15$ $\frac{AG}{GC} = \frac{AF}{FB}$ [line ‖ one side of Δ/*lyn* ‖ *een sy v* Δ] $\frac{x-15}{x+9} = \frac{2}{5}$ $5x - 75 = 2x + 18$ $3x = 93$ $x = 31$ **OR**	✓ S ✓ R ✓ substitution ✓ answer (4) ✓ S ✓ R ✓ substitution ✓ answer (4)

	$\frac{AF}{AB}=\frac{AG}{AC}$ [line ∥ one side of Δ /lyn ∥ een sy vΔ] $\frac{2}{7}=\frac{x-15}{2x-6}$ $7x-105=4x-12$ $3x=93$ $x=31$	✓ S ✓ R ✓ substitution ✓ answer (4)
		[17]

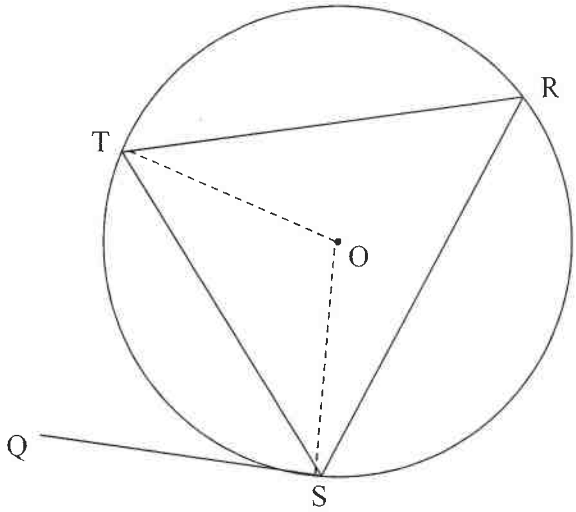
QUESTION/*VRAAG* 9

9.1



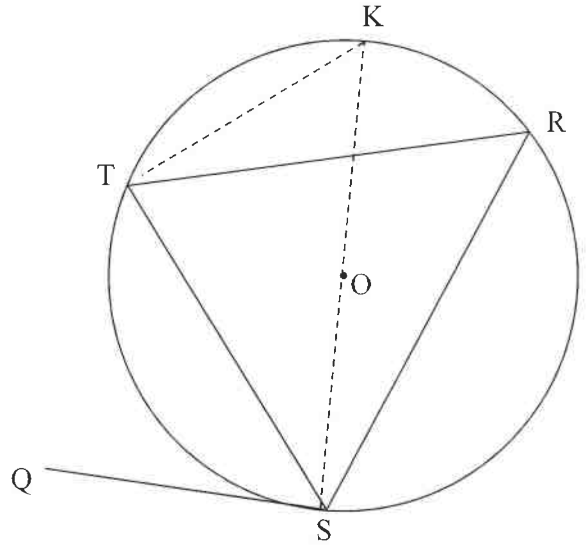
9.1	Construction: Draw diameter KS and draw KR *Konstruksie: Trek middellyn KS en verbind KR* $Q\hat{S}T = 90° - T\hat{S}K$ [radius ⊥ tangent/*raaklyn*] $S\hat{R}K = 90°$ [∠ in semi circle/*halfsirkel*] $\therefore S\hat{R}T = 90° - K\hat{R}T$ $T\hat{S}K = T\hat{R}K$ [∠s same segment/∠*e dieselfde segment*] $\therefore Q\hat{S}T = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



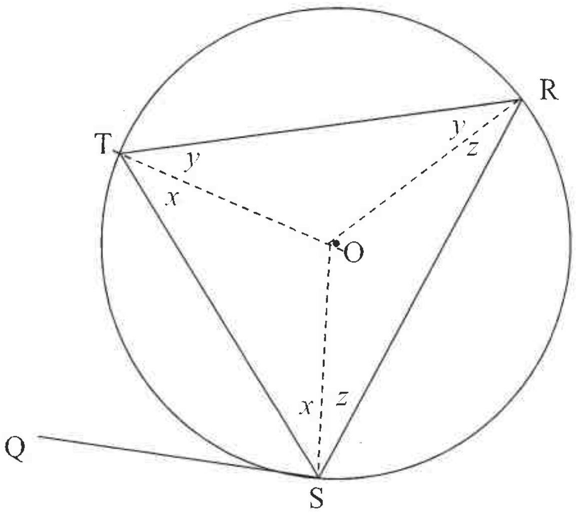
9.1	Construction: Draw radii OS and OT *Konstruksie: Trek radii OS en OT* $Q\hat{S}T = 90° - O\hat{S}T$ [radius ⊥ tangent/*raaklyn*] $O\hat{S}T = S\hat{T}O$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore S\hat{O}T = 180° - 2O\hat{S}T$ [∠s of Δ/*∠e van Δ*] $\hat{R} = 90° - O\hat{S}T$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore Q\hat{S}T = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



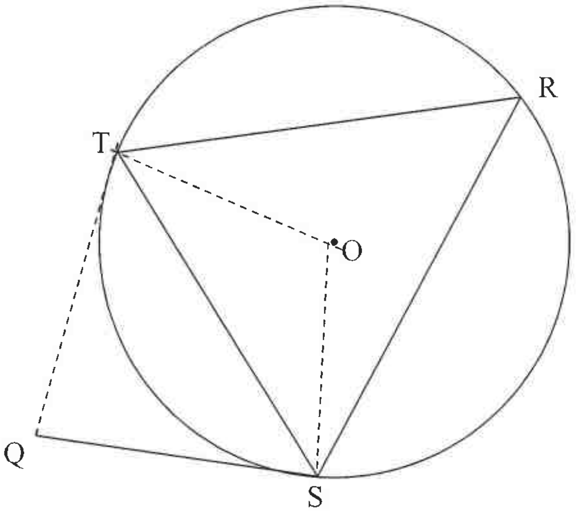
9.1	Construction: Draw diameter KS and join K to T. *Konstruksie: Trek middellyn KS en verbind K tot T*K $Q\hat{S}T = 90° - T\hat{S}K$ [radius ⊥ tangent/*raaklyn*] $S\hat{T}K = 90°$ [∠ in semi circle/*halfsirkel*] $\therefore \hat{K} = 90° - T\hat{S}K$ $\therefore Q\hat{S}T = \hat{K}$ but $\hat{R} = \hat{K}$ [∠s same segment/*∠e dieselfde segment*] $\therefore Q\hat{S}T = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

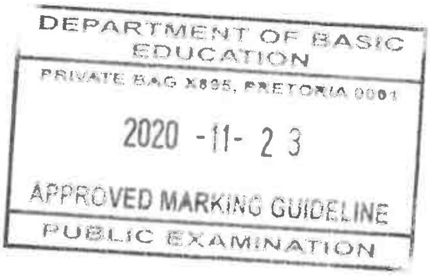
OR



9.1	Construction: Draw radii OT, OR and OS *Konstruksie: Trek radiuse OT, OR en OS* $O\hat{S}T = O\hat{T}S$ [∠s opp = radii/*∠e teenoor = radiuse*] Also: $O\hat{T}R = O\hat{R}T$ and $O\hat{R}S = O\hat{S}R$ $2x + 2y + 2z = 180°$ [∠s of Δ] $x + y + z = 90°$ $y + z = 90° - x$ $O\hat{S}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore\ T\hat{S}Q = 90° - x$ $\therefore\ T\hat{S}Q = y + z = \hat{R}$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5)

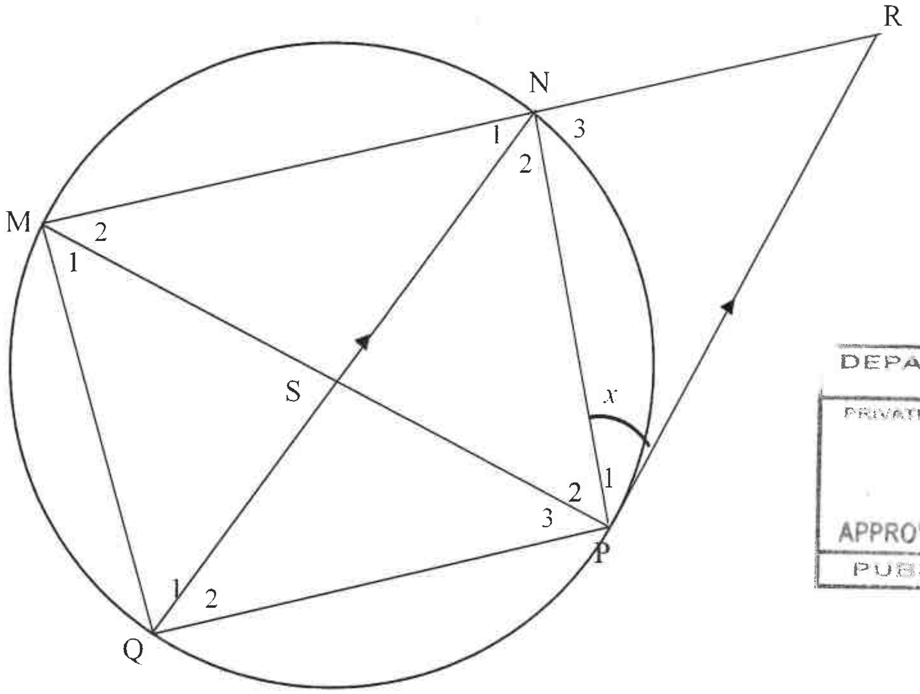
OR





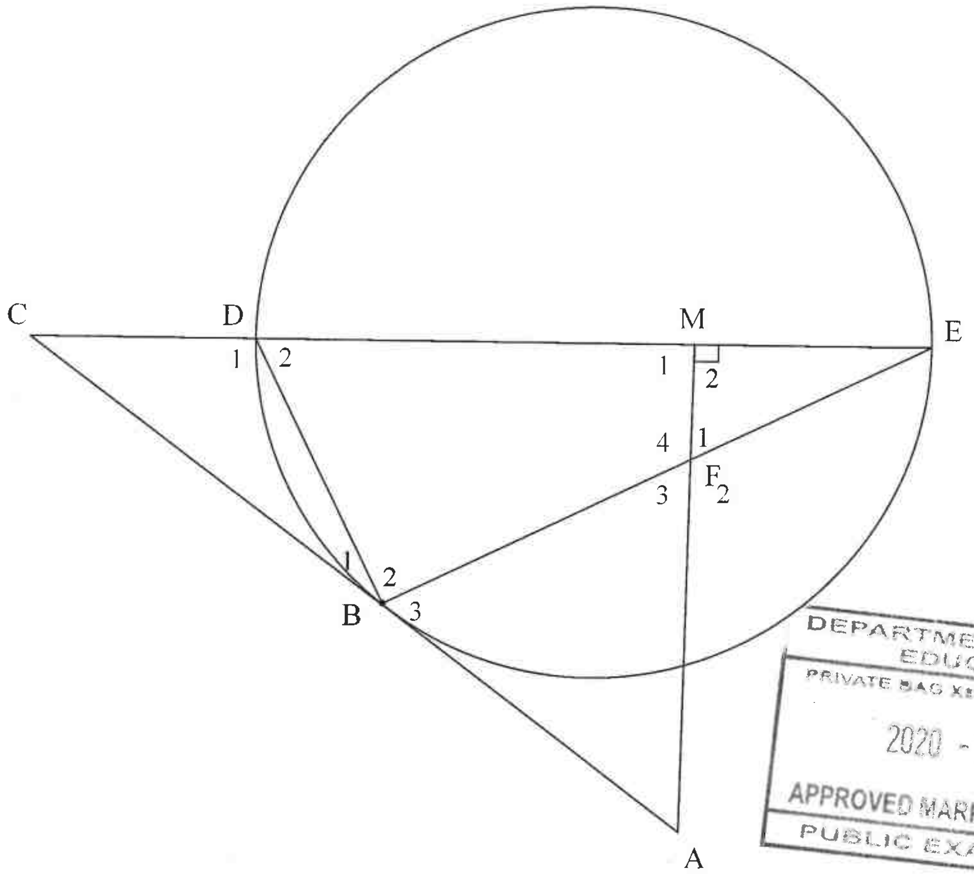
9.1	Construction: Draw radii OT and OS, tangent QT *Konstruksie: Trek radiuse OT en OS, raaklyn QT* $O\hat{S}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore\ T\hat{S}Q = 90° - T\hat{S}O$ $\therefore\ T\hat{S}O = S\hat{T}O$ [∠s opp = radii/*∠e teenoor = radiuse*] $T\hat{O}S = 180° - 2T\hat{S}O$ [∠s of Δ] $\hat{R} = 90° - T\hat{S}O$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore\ T\hat{S}Q = \hat{R}$	✓ construction ✓ S/R ✓ S ✓ S ✓ S/R (5)

9.2



9.2.1(a)	$\hat{N}_2 = x$	[alt ∠s; PR ∥ NQ/*verw. ∠e;* PR ∥ NQ]	✓S ✓R (2)
9.2.1(b)	$\hat{Q}_2 = x$ OR $M_2 = x$ $\hat{Q}_2 = x$	[tan chord theorem/*raaklyn koordstelling*] [tan chord theorem/*raaklyn koordstelling*] [∠s in same segment/*∠e in dieselfde segm*]	✓S ✓R (2) ✓S/R ✓S/R (2)
9.2.2	$\frac{MN}{NR} = \frac{MS}{SP}$ $\hat{N}_1 = \hat{N}_2 = x$ $\hat{P}_3 = x$ $\hat{P}_3 = \hat{Q}_2$ $SQ = PS$ $\frac{MN}{NR} = \frac{MS}{SQ}$	[QN ∥ PR; Prop Th] [given] [∠s in same segment/*∠e in dieselfde segm*] [$= x$] [sides opp = ∠/*sye teenoor = ∠e*]	✓S ✓R ✓S ✓S ✓R ✓R (6)
			[15]

QUESTION/*VRAAG* 10



10.1.1	$D\hat{B}E = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\therefore D\hat{M}A = 90°$ [$AM \perp DE$] $\therefore$ FBDM is a cyclic quadrilateral/*koordevh* [converse opp $\angle$s cyclic quad/*omgek teenoorst* $\angle$*e kvh*]	✓S ✓R ✓R (3)
	OR $D\hat{B}E = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\hat{M}_2 = D\hat{B}E = 90°$ $\therefore$ FBDM is a cyclic quadrilateral/*koordevh* [converse ext$\angle$ of cyclic quad/*omgek buite*$\angle$ *van kvh*]	✓S ✓R ✓R (3)

10.1.2	$\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = \hat{D}_2$ [ext ∠ cyc quad/*buite* ∠ *koordevh*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = 90° - x$ [∠ sum in Δ/∠ *van* Δ] $\hat{D}_2 = 90° - x$ [∠ sum in Δ/∠ *van* Δ] $\therefore \hat{F}_1 = D_2$ $\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{B}_3 = 90° - x$ [straight line/*reguitlyn*] $\hat{F}_1 = 90° - x$ [sum of ∠s Δ / *som van* ∠*e van* Δ] $\therefore \hat{B}_3 = \hat{F}_1$	✓S ✓R ✓S ✓R (4) ✓S ✓R ✓ $\hat{F}_1 = 90° - x$ $= \hat{D}_2$ ✓R (4) ✓S ✓R ✓S ✓S (4)
10.1.3	In ΔCDB and ΔCBE $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike* ∠] $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $C\hat{D}B = C\hat{B}E$ [∠ sum in Δ/∠ *van* Δ] ΔCDB ||| ΔCBE **OR** In ΔCDB and ΔCBE $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike* ∠] ΔCDB ||| ΔCBE [∠, ∠, ∠]	✓S ✓S/R ✓R (3) ✓S/R ✓S ✓R (3)
10.2.1	$\frac{BC}{EC} = \frac{DC}{BC}$ [||| Δs] $BC^2 = EC \times DC$ $= 8 \times 2$ $= 16$ $BC = 4$	✓ ratio ✓ substitution ✓answer (3)

10.2.2	$\frac{BC}{EC}=\frac{DB}{BE}$ [ǁǁ Δs] $\frac{DB}{BE}=\frac{4}{8}=\frac{1}{2}$ $BE = 2DB$ $DB^2+BE^2=DE^2$ [Pyth theorem] $DB^2+(2DB)^2=36$ $5DB^2=36$ $DB^2=\frac{36}{5}$ $DB=\frac{6}{\sqrt{5}}=2{,}68$ units	✓ $BE = 2DB$ ✓ substitution into Pyth theorem ✓ $DB^2=\frac{36}{5}$ ✓answer (4)
		[17]

TOTAL/*TOTAAL*: 150