Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE / *NASIONALE SENIOR SERTIKAAT*

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2021

TECHNICAL MATHEMATICS P1/*TEGNIESE WISKUNDE V1*
MARKING GUIDELINE/*NASIENRIGLYN*
(EXEMPLAR/*EKSEMPLAAR*)

MARKS/*PUNTE*: 150

MARKING CODES/*NASIENKODES*	
A	Accuracy/*Akkuraatheid*
AO	Answer only/*Slegs antwoord*
CA	Consistent accuracy/*Volgehoue akkuraatheid*
M	Method/*Metode*
R	Rounding/*Afronding*
NPR	No penalty for rounding/*Geen penalisering vir afronding nie*
NPU	No penalty for units omitted/*Geen penalisering vir eenhede weggelaat nie*
S	Simplification/*Vereenvoudiging*
F	Correct formula/*Korrekte formule*
SF	Substitution in correct formula/*Vervanging in korrekte formule*

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

QUESTION/ *VRAAG* 1				
1.1	1.1.1	$x^2-8x-33=0$ $(x-11)(x+3)=0$ **or / of** $x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)-33}}{2(1)}$ $\therefore x=11$ or/*of* $x=-3$	✓ Factors/ *Faktore* **SF A** ✓ $x=11$ ✓ $x=-3$ **CA**	(3)
	1.1.2	$x^2-7x=10(-3x-1)$ $x^2-7x=-30x-10$ $x^2+23x+10=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(23)\pm\sqrt{(23)^2-4(1)(10)}}{2(1)}$ $x=\frac{-23\pm\sqrt{489}}{2}$ $x=-0,44$ or / *of* $x=-22,56$	✓**S A** ✓ **SF CA** ✓both values of x / *beide waardes van x* **CA**	(3)
	1.1.3	$-2x^2+9x+5<0$ $(-2x-1)(x-5)<0$ C.V / *K.W* : $-\frac{1}{2}$ and / *en* 5 Solution / *oplossing* : $x<-\frac{1}{2}$ or / *of* $x>5$	✓ Factors/ *Faktore* **SF A** ✓Critical Values / *Kritiese waardes* **CA** ✓ $x<-\frac{1}{2}$ **CA** ✓ $x>5$ **CA**	(4)

1.2	$P = 2(l + w)$ $90 = 2(l + x)$ $45 - x = l$ $41^2 = x^2 + (45 - x)^2$ $1681 = x^2 + 2025 - 90x + x^2$ $0 = 2x^2 - 90x + 344$ $0 = x^2 - 45x + 172$ $x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(1)(172)}}{2(1)}$ $x = 40,78$ or/*of* $4,22$ $\therefore$ width / *wydte* is $4,22$ cm **OR/*OF*** $l = \sqrt{41^2 - x^2}$ from/*vanaf* Pythagoras $90^2 = 2\left(x + \sqrt{41^2 - x^2}\right)$ $45 = x + \sqrt{1681 - x^2}$ $45 - x = \sqrt{1681 - x^2}$ $(45 - x)^2 = 1681 - x^2$ $2025 - 90x + x^2 = 1681 - x^2$ $0 = 2x^2 - 90x + 344$ $0 = x^2 - 45x + 172$ $x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(1)(172)}}{2(1)}$ $x = 40,78$ or/*of* $4,22$ $\therefore$ width/*wydte* is $4,22$ cm	✓length in terms of x / *lengte in terme van x* **A** ✓**M** (Pyth.) **CA** ✓**S** **CA** ✓**S** **CA** ✓**SF** **CA** ✓width / *wydte* **CA** **OR / *OF*** ✓length in terms of x / *lengte in terme van x* **A** ✓**M** (Pyth.) **CA** ✓**S** **CA** ✓**S** **CA** ✓**SF** **CA** ✓width / *wydte* **CA**	(6)
1.3	$x = y + 3$ and / *en* $y - x^2 = -2x - 3$ substitute / *vervang* x into / *in* $y - x^2 = -2x - 3$ $y - (y + 3)^2 = -2(y + 3) - 3$ $y - y^2 - 6y - 9 = -2y - 6 - 3$ $-y^2 - 3y = 0$ $(-y)(y + 3) = 0$ $y = 0$ or / *of* $y = -3$ $x = 0$ or / *of* $x = 3$	✓ Substitution/*Vervanging* **A** ✓**S** **CA** ✓ Factors/*Faktore* **SF** **CA** ✓ Both y-values//*beide y-waardes* **CA** ✓ Both x-values /*beide x-waardes* **CA**	

	OR/*OF*		**OR/*OF***	
	$x = y + 3$ and / *en* $y - x^2 = -2x - 3$		✓ equating/*gelykstelling y* **A**	
	$x - 3 = y$ and / *en* $y = x^2 - 2x - 3$			
	equate / *gelykstelling y*: $x - 3 = x^2 - 2x - 3$			
	$0 = x^2 - 3x$		✓ **S** **CA**	
	$0 = x(x - 3)$		✓ Factors/*Faktore* **CA**	
	$x = 0$ or / *of* $x = 3$		✓ *x*-values/*waardes* **CA**	
	$y = 0$ or / *of* $y = -3$		✓ *y*-values/*waardes* **CA**	(5)
1.4	1.4.1	K = 8 + 32 + 1 = 41	✓ value of/*waarde van* K **A**	(1)
	1.4.2	41 = 1 0 1 0 0 1_2	✓ 1 0 1 0 0 1_2 **CA** **From/*vanaf* 1.4.1**	(1)
				[23]

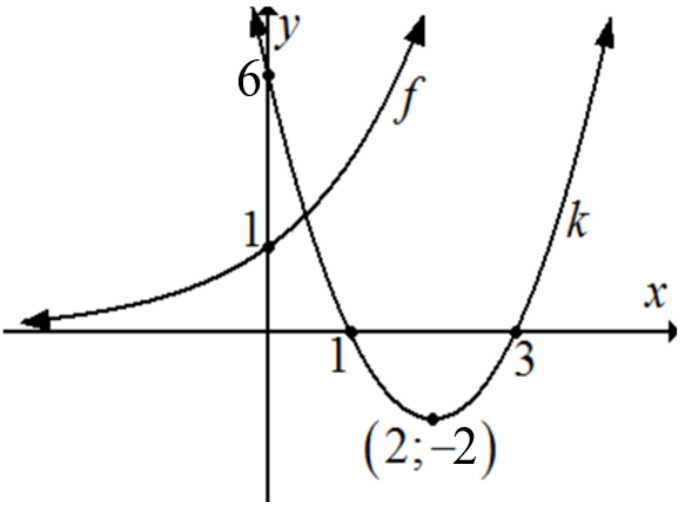
QUESTION/*VRAAG* 2			
2.1	$\Delta = b^2 - 4ac < 0$ $4 + 8m < 0$ $m < -\frac{1}{2}$	✓Discriminant / *diskriminant* < 0 **A** ✓**S** **CA**	(2)
2.2	$\Delta = b^2 - 4ac$ $= (-b)^2 - 4(a)\left(-\frac{1}{a}\right)$ $= 0 + 4 = 4$ Roots are real, rational and unequal *Wortels is reël, rasionaal en ongelyk*	✓ **SF** **A** ✓ **S** **CA** ✓Real and unequal / *Reël en ongelyk* **CA** ✓rational roots / *rasionale wortels* **CA**	(4)
			[6]

QUESTION/*VRAAG* 3				
3.1	3.1.1	$81^{\frac{3}{4}} - 8^{\frac{2}{3}} + \sqrt[4]{1296}$ $= 3^{4\times\frac{3}{4}} - 2^{3\times\frac{2}{3}} + 6^{4\times\frac{1}{4}}$ or/*of* $3^{4\times\frac{3}{4}} - 2^{3\times\frac{2}{3}} + (2\times 3)^{4\times\frac{1}{4}}$ $= 27 - 4 + 6 = 29$	✓ Exponential form / *Eksponensiële vorm* **A** ✓ **S** **CA**	(2)
	3.1.2	$\log\sqrt{65} + \log\sqrt{260} - \log 13$ $= \log\frac{\left(\sqrt{65}\times\sqrt{260}\right)}{13}$ $= \log\frac{\left(\sqrt{5\times 13\times 5\times 4\times 13}\right)}{13}$ $= \log\frac{\left(5^2.13^2.2^2\right)^{\frac{1}{2}}}{13}$ $= \log\frac{(5.13.2)}{13}$ $= \log 10 = 1$ **OR/*OF*** $\log\sqrt{65} + \log\sqrt{260} - \log 13$ $= \log\sqrt{13\times 5} + \log\sqrt{13\times 5\times 2} - \log 13$ $= \log\sqrt{13} + \log\sqrt{5} + \log\sqrt{13} + \log\sqrt{5} + \log\sqrt{2} - \log 13$ $= 2\log\sqrt{13} + 2\log\sqrt{5} + \log 2 - \log 13$ $= \log 13 + \log 5 + \log 2 - \log 13$ $= \log(5\times 2)$ $= \log 10 = 1$	✓Log property /*eienskap* **A** ✓Exponential form / *Eksponensiële vorm* **CA** ✓**S** **CA** ✓Log property/*eienskap* **CA** **OR/*OF*** ✓Log property/*eienskap* **A** ✓**S** **CA** ✓Log property/*eienskap* **CA** ✓Log property/*eienskap* **CA**	(4)
3.2	$\frac{-2(\log 25 - \log 4)}{\log 2 - \log 5} = 4$ RHS $= \frac{-2\left(\log 5^2 - \log 2^2\right)}{\log 2 - \log 5}$ $= \frac{-2\times 2(\log 5 - \log 2)}{-(\log 5 - \log 2)}$ $= 4 = \text{LHS}$		✓Exponential form/ *Eksponensiële vorm* **A** ✓Log property/*eienskap* **CA** ✓Factors/*Faktore* **CA**	(3)

3.3	3.3.1	$64x^{\frac{3}{2}} = 27x^{\frac{-3}{2}}$ $\frac{x^{\frac{3}{2}}}{x^{-\frac{3}{2}}} = \frac{27}{64}$ $x^3 = \left(\frac{3}{4}\right)^3$ $x = \frac{3}{4}$	✓**M** **A** ✓Exponential property/ *Eksponensiële eienskap* **CA** ✓Exponential property/ *Eksponensiële eienskap* **CA**	(3)
	3.3.2	$\log_3(x-3) - \log_3 5 = 1$ $\log_3 \frac{(x-3)}{5} = \log_3 3$ $\frac{(x-3)}{5} = 3$ $x - 3 = 15$ $x = 18$ **OR/*OF*** $\log_3(x-3) - \log_3 5 = 1$ $\log_3(x-3) = \log_3 3 + \log_3 5$ $\log_3(x-3) = \log_3(3\times 5)$ $x - 3 = 15$ $x = 18$	✓Log property/*eienskap* **A** ✓Log property/ *eienskap* **CA** ✓**S** **CA** **OR/*OF*** ✓Log property/ *eienskap* **A** ✓Log property *eienskap* **CA** ✓**S** **CA**	(3)
3.4		$x - 3(5i+2) = 4 - 3i + yi$ $x - yi = 3(5i+2) + 4 - 3i$ $= 15i + 6 + 4 - 3i$ $= 10 + 12i$ $x = 10$ $y = -12$ **OR/*OF*** $x - 3(5i+2) = 4 - 3i + yi$ $x - 15i - 6 = 4 + i(-3+y)$ $x - 6 - 15i = 4 + i(-3+y)$ $x - 6 = 4$ and/*en* $-15 = -3 + y$ $x = 10$ and/*en* $-12 = y$	✓**S** **A** ✓*x*-value/*waarde* **CA** ✓*y*-value/*waarde* **CA** **OR/*OF*** ✓**S** **A** ✓*x*-value /*waarde* **CA** ✓*y*-value/*waarde* **CA**	(3)

3.5	$V = 110,4 + 46,1i$ $r = \sqrt{(110,4)^2 + (46,1)^2}$ $= \sqrt{10062,95}$ $= 119,64$ $\tan\theta = \frac{46,1}{110,4}$ $\theta = 22,7^\circ$ or/*of* $\theta = 180^\circ + 22,7^\circ = 202,7^\circ$ $z = 119,64\,cis\left(22,7^\circ\right)$ or/*of* $z = 119,64\,cis\,202,7^\circ$	✓Substitution/*vervanging* **A** ✓**S** **CA** ✓Ratio/*verhouding* **CA** ✓ θ **CA** ✓Polar form/*Polêre vorm* **CA**	(5)
			[23]

QUESTION/*VRAAG* 4				
4.1	4.1.1	$g(x) = \sqrt{r^2 - x^2}$ $3 \quad = \sqrt{r^2 - 1^2}$ $9 + 1 = r^2$ $\sqrt{10} = r$ and / *en* $h(x) = \frac{a}{x}$ or / *of* $xy = a$ $3 = \frac{a}{1}$ $\quad 3 \times 1 = a$ $\therefore 3 = a$	✓ **SF** **A** ✓value of /*waarde van r* **CA** ✓value of / *waarde van a* **A**	**(3)**
	4.1.2	$0 \le y \le \sqrt{10}$	✓ 0 and / *en* $\sqrt{10}$ **CA** **from/*vanaf* 4.1.2** ✓Correct notation/ *korrekte notasie* **A**	(2)
	4.1.3	$B\left(-\sqrt{10};0\right)$	✓ Coordinates of / *koördinate van* B **CA** **from/*vanaf* 4.1.1**	(1)
	4.1.4	$y = 0$ $x = 0$	✓ $x = 0$ **A** ✓ $y = 0$ **A**	(2)
4.2	4.2.1	$k(x) = 2(x-2)^2 - 2$ $k(x) = 2(x-2)^2 - 2$ $k(0) = 2(0-2)^2 - 2$ $= 6$ y - int. $= 6$ **OR / *OF*** $k(x) = 2(x-2)^2 - 2$ $= 2(x^2 - 4x + 4) - 2$ $= 2x^2 - 8x + 6$ y - intercept / *afsnit* $= 6$	✓Substitution/ *vervanging* **A** ✓y-int/*afsnit* **CA** **OR/*OF*** ✓**S** **A** ✓y-int. **CA**	(2)
	4.2.2	$TP(2;-2)$	✓x-coordinate/ *x-koördinaat* **A** ✓y-coordinate / *y-koördinaat* **A**	(2)

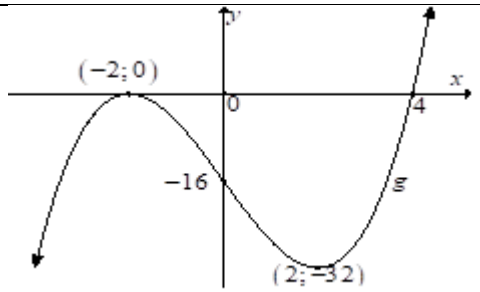
	4.2.3	$k(x) = 2(x-2)^2 - 2$ $= 2(x^2 - 4x + 4) - 2$ $= 2x^2 - 8x + 6$ x-int / *afsnitte.*; $k(x) = 0$ $0 = x^2 - 4x + 3$ $= (x-1)(x-3)$ $x = 1$ or / *of* $x = 3$	✓**S** **A** ✓Equate to 0 / *gelykstel aan 0* **CA** ✓Factors / *faktore* **CA** ✓Both x-values / *beide x-waardes* **CA**	(4)
	4.2.4	$x \in \mathbf{R}$	✓ $x \in \mathbf{R}$ **A**	(1)
	4.2.5		*f*: ✓Shape/*vorm* **A** ✓y-intercept/*afsnit* **CA** *k*: ✓Shape/*vorm* **A** ✓both x-intercepts / *beide x-afsnitte* **CA** ✓y- intercept *afsnit* **CA** ✓Turning point / *draaipunt* **CA**	(6)
	4.2.6	$y \geq -2$	✓ $y \geq -2$ **CA**	(1)
				[24]

QUESTION/*VRAAG* 5			
5.1	$A = P(1+ni)$ $= 35(1+9\times 0,15)$ $= R82,25$	✓**SF** **A** ✓**S** **CA**	(2)
5.2	$A = P(1+i)^n$ $= 13\,565(1+0,065)^8$ $= 22\,450$	✓**F** **A** ✓**SF** **CA** ✓**S** **CA**	(3)
5.3	Year / *Jaar* 1 $A = P\left(1+\frac{i}{m}\right)^n$ $= 120\,000\left(1+\frac{0,09}{12}\right)^{12}$ $= R131\,256,8277$ $R131\,256,8277 + R50\,000 = R181\,256,8277$ Year / *Jaar* 2 $= R181\,256,8277\left(1+\frac{0,085}{4}\right)^4$ $= R197161,7449$ $R197161,7449 - R35\,000 = R162\,161,7449$ The remaining 3 years *Oorblywende* 3 *jare* $= R162\,161,7449\left(1+\frac{0,085}{4}\right)^{12}$ $= R208\,705,19$ **OR/*OF*** $A = P\left(1+\frac{i}{m}\right)^n$ $= 120\,000\left(1+\frac{0,09}{12}\right)^{12}\left(1+\frac{0,085}{4}\right)^{16}$ $= R183\,753,2447$ $R50\,000\left(1+\frac{0,085}{4}\right)^{16} = R69\,997,59473$ $R35\,000\left(1+\frac{0,085}{4}\right)^{12} = R45\,045,65215$ Investment after 5 years / *Belegging na* 5 *jaar* $= R183\,753,2447 + R69\,997,59473 - R45\,045,65215$ $= R208\,705,19$	✓ **SF** **A** ✓**S** **CA** ✓ Sum /*Som* **CA** ✓ **SF** **A** ✓**S** **CA** ✓Difference /*Verskil* **CA** ✓**SF** **CA** ✓**S** **CA** ✓ **SF** **A** ✓**S** **CA** ✓ **SF** **A** ✓ **S** **CA** ✓**SF** **A** ✓**S** **CA** ✓**S** **CA** ✓**S** **CA**	(8)
			[13]

QUESTION/*VRAAG* 6

6.1	$f(x) = -2x + \frac{1}{4}$ $f(x+h) = -2(x+h) + \frac{1}{4}$ $= -2x - 2h + \frac{1}{4}$ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{-2x - 2h + \frac{1}{4} - \left(-2x + \frac{1}{4}\right)}{h}$ $= \lim_{h\to 0} \frac{-2x - 2h + \frac{1}{4} + 2x - \frac{1}{4}}{h}$ $= \lim_{h\to 0} \frac{-2h}{h}$ $= \lim_{h\to 0} -2$ $= -2$	✓**F** **A** ✓**SF** **CA** ✓**S** **CA** ✓**S** **CA** ✓ $f'(x) = -2$ **CA**	(5)
6.2	6.2.1	$D_a\left[\frac{3}{2}a^2 - a^{-5}\right]$ $= 3a + a^{-4}$	✓$3a$ **A** ✓a^{-4} **CA**	(2)
	6.2.2	$f(x) = \frac{x^3 - 3x^5}{4x}$ $= \frac{x^2}{4} - \frac{3x^4}{4}$ $f'(x) = \frac{x}{2} - 3x^3$	✓**S** **A** ✓ $\frac{x}{2}$ **CA** ✓ $-3x^3$ **CA**	(3)
	6.2.3	$S = \frac{1}{2}ft^2$ $\frac{ds}{dt} = ft$ $= \pi t$	✓ ft **A** ✓ πt **CA**	(2)

	6.3.3	$f(x) = 3x^2$ $f'(x) = 6x$ Av.grad. $= \frac{f(8) - f(2)}{8 - 2}$ $= \frac{192 - 12}{8 - 2}$ $6x = 30$ $x = 5$	✓$6x$ **A** ✓**S** **CA** ✓Equating derivative and av. gradient / *Gelykstelling van afgeleide en gemid. gradiënt* **CA** ✓ $x = 5$ **CA**	(4)
				[16]

QUESTION/*VRAAG* 7			
7.1	$g(x) = x^3 - 12x - 16$ $g(-2) = (-2)^3 - 12(-2) - 16$ $= 0$ $\therefore (x-2)$ is a factor of / *is 'n faktor van* $g(x)$	✓substitution by -2 / *vervanging deur -2* **A** ✓**S** **CA**	(2)
7.2	$g(x) = x^3 - 12x - 16$ $0 = (x+2)(x^2 - 2x - 8)$ $0 = (x+2)(x+2)(x-4)$ $x = -2$ or/*of* $x = 4$	✓Equating to/*Gelykstelling aan* 0 **A** ✓Quadratic factor / *Kwadratiese faktor* **A** ✓Factors of quadratic factor / *faktore van kwadratiese factor* **CA** ✓values of *x / waarde van x* **CA**	(4)
7.3	$(0;-16)$	✓ *y*-intercept /*y-afsnit* **A**	(1)
7.4	$f(x) = x^3 - 12x - 16$ $f'(x) = 3x^2 - 12$ $0 = 3x^2 - 12$ $0 = x^2 - 4$ $0 = (x-2)(x+2)$ $x = 2$ or/*of* $x = -2$ $g(2) = (2)^3 - 12(2) - 16 = -32$ $g(-2) = (-2)^3 - (-2) - 16 = 0$ TP$(-2;0)$ and/*en* $(2;-32)$	✓Derivative /*Afgeleide* **A** ✓0 **CA** ✓Factors/ *Faktore* **CA** ✓Both *x* values /*beide x-waardes* **CA** ✓$(-2;0)$ **CA** ✓ $(2;-32)$ **CA**	(6)
7.5		✓Shape /*vorm* **A** ✓y-intercept /*y-afsnit* **CA** ✓x-intercepts /*x-afsnitte* **CA** ✓Both turning points /*Beide draaipunte* **CA**	(4)
7.6	$h(x) = (x-2)^3 - 12(x-2) - 16$	✓$h(x)$ **A**	(1)
7.7	$-2 > x$ or/*of* $x < 2$	✓ $-2 > x$ **CA** ✓ $x < 2$ **CA**	(2)
			[20]

QUESTION/*VRAAG* 8				
8.1	8.1.1	$q = 820 - p$	✓ $q = 820 - p$ **A**	(1)
	8.1.2	$Z = pq$ $= p(820 - p)$ $= 820p - p^2$	✓Substitution/*Vervanging* **CA** ✓**S** **CA**	(2)
	8.1.3	$Z = 820p - p^2$ $\frac{dZ}{dp} = 820 - 2p$ $0 = 820 - 2p$ $\therefore p = 410$	✓Derivative/*Afgeleide* = 0 **CA** ✓**S** **CA**	(2)
8.2	8.2.1	$R(x) = -50x^2 + 3200x - 1860$ $R(15) = -50(15)^2 + 3200(15) - 1860$ $= R34\,890$	✓Substitution/*Vervanging* **A** ✓**S** **CA**	(2)
	8.2.2	$R(x) = -50x^2 + 3200x - 1860$ $R'(x) = -100x + 3200$ $0 = -100x + 3200$ $100x = 3200$ $x = 32$ $R(x) = -50x^2 + 3200x - 1860$ $R(32) = -50(32)^2 + 3200(32) - 1860$ $= R49\,340$ = artisan's maximum earnings / = *vakman se maksimum verdienste*	✓Derivative/*Afgeleide* = 0 **CA** ✓**S** **CA** ✓Substitution/*Vervanging* **CA** ✓**S** **CA**	(4)
				[11]

QUESTION / *VRAAG* 9				
9.1	9.1.1	$\int\left(\frac{2}{\sqrt{x}}+\pi\right)dx$ $=\int\left(2x^{-\frac{1}{2}}+\pi\right)dx$ $=4x^{\frac{1}{2}}+\pi x+c$	✓ $2x^{-\frac{1}{2}}$ **A** ✓ $4x^{\frac{1}{2}}$ **CA** ✓ πx **A** ✓ c **A**	(4)
	9.1.2	$\int(x+1)(3x-2)dx$ $=\int\left(3x^2+x-2\right)dx$ $=x^3+\frac{x^2}{2}-2x+c$	✓**S** **A** ✓ x^3 **CA** ✓ $\frac{x^2}{2}$ **CA** ✓ $-2x+c$ **CA**	(4)
9.2	$A=\int_{1,5}^{3}\left(4x-x^2\right)dx$ $=\left[2x^2-\frac{x^3}{3}\right]_{1,5}^{3}$ $=\left(2(3)^2+\frac{(3)^3}{3}\right)-\left(2(1,5)^2+\frac{(1,5)^3}{3}\right)$ $A=9-\frac{27}{8}$ $=5,625$ square units / *vierkante eenhede*	✓A definite integral formula/ *'n Bepaalde integrale formule* **A** ✓ $2x^2$ **CA** ✓ $-\frac{x^3}{3}$ **CA** ✓Substitution in A by 3 / *Vervanging in A deur 3* **CA** ✓Substitution in A by 1,5 / *Vervanging in A deur 1,5* **CA** ✓**S** **CA**	(6)
				[14]
			TOTAL/*TOTAAL*:	**150**