Province of the EASTERN CAPE EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2022

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 13 pages, including a 1-page information sheet and a special answer book.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.

6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.

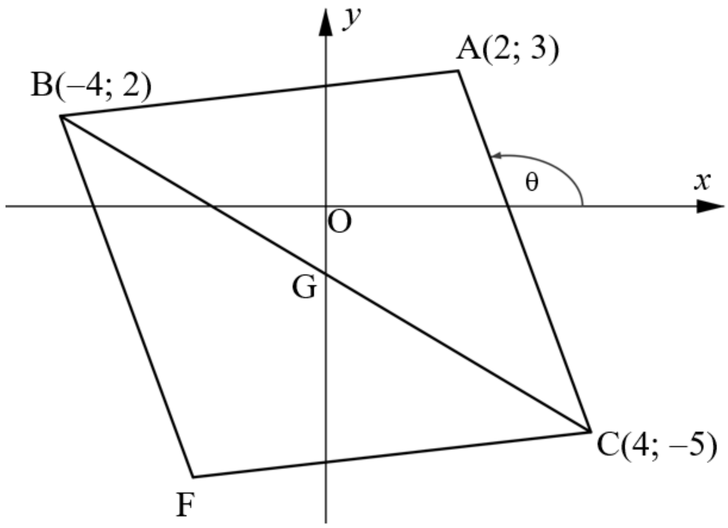
7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

In the diagram below ABFC is a parallelogram with vertices A(2; 3), B(−4; 2), F and C(4;−5).
G is the midpoint of BC and θ is the inclination angle.



Determine:

1.1 The length of AC (leave your answer in simplified surd form.) (2)

1.2 The equation of straight-line AC in the form $y = \ldots$ (4)

1.3 The size of θ (3)

1.4 The coordinates of G (2)

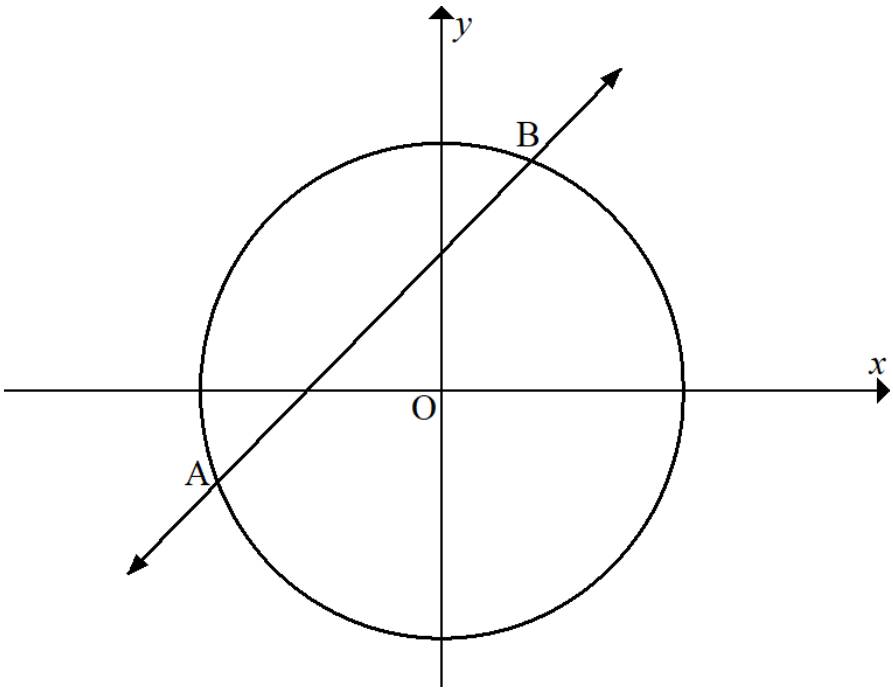
1.5 Hence, the coordinates of F (3)

1.6 If BC $\perp$ AG. Show ALL calculations (4)

[18]

QUESTION 2

2.1 In the diagram below, the straight line $y = x + 3$, intersect with the circle $x^2 + y^2 = 29$ at A and B.



2.1.1 Determine the coordinates of A and B. (7)

2.1.2 Given: the point C $(-5; 2)$.

(a) Show that C lies on the circle. (2)

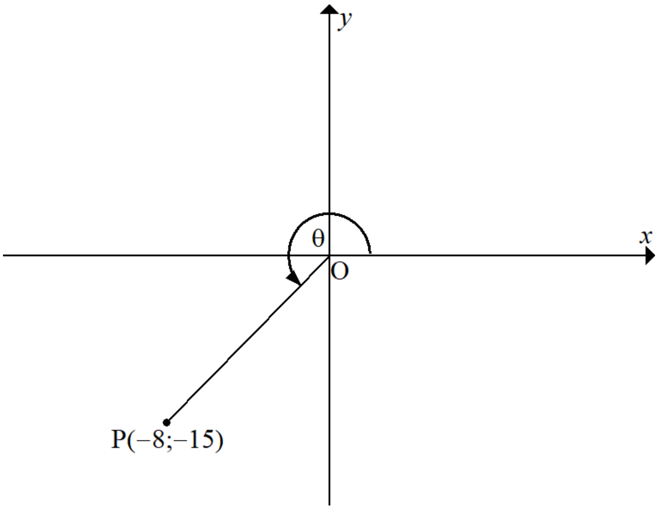
(b) Determine the equation of the tangent to the circle at point C in the form $y = ...$ (4)

2.2 Sketch the graph of $\frac{x^2}{40} + \frac{y^2}{64} = 1$. Clearly indicate the intercepts. (3)

[16]

QUESTION 3

3.1 In the diagram below, P(-8; -15) is a point on the Cartesian plane. OP forms a reflex angle θ with the positive x-axis.



Determine the following, WITHOUT using a calculator:

3.1.1 The length of OP (3)

3.1.2 $\tan\theta$ (1)

3.1.3 $\operatorname{cosec}^2\theta - 1$ (3)

3.2 If $a = 135,5°$ and $b = 15,7°$, determine the numerical value of the following, rounded off to THREE decimal digits:

3.2.1 $\sin\left(\frac{\pi}{2} - b\right)$ (2)

3.2.2 $\sec(a+b)$ (2)

3.3 Solve for x, rounded off to ONE decimal digit:

3.3.1 $\sin x + 1 = 0,587$ for $x \in [0°; 360°]$ (4)

3.3.2 $\cot 2x = 2,114$ for $x \in [90°; 360°]$ (4)

3.4 If $\cos 36° = a$, determine the following in terms of a:

3.4.1 $\tan 36°$ (3)

3.4.2 $\sec^2 144°$ (2)

[24]

QUESTION 4

4.1 Simplify: $\sec^2 2x - \tan^2 2x$ (1)

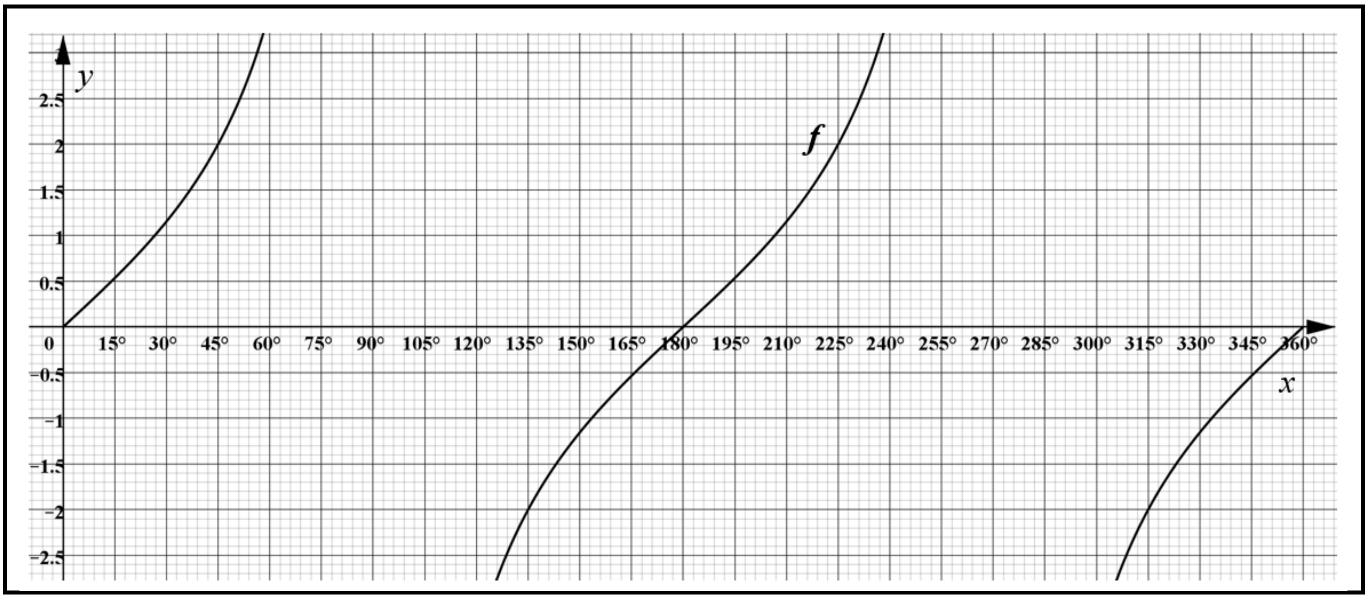
4.2 Simplify: $\dfrac{\sec x}{\cos(360° - x)} + \dfrac{\tan^2(180° - x)}{\sin(180° + x)\,\text{cosec}(180° - x)}$ (8)

4.3 Prove that: $\sin(360° - x)\cot(180° - x) = \cos x$ (4)

[13]

QUESTION 5

In the diagram below the graph of $f(x) = a\tan x$ is given for the interval $x \in [0°; 360°]$.



5.1 Write down the value of *a*. (1)

5.2 Write down the equations of the asymptotes of *f*. (2)

5.3 Write down the period of *f*. (1)

5.4 On the same axes given in your SPECIAL ANSWER BOOK draw the graph of $g(x) = \cos(x - 60°)$. Clearly show the intercepts with the axes, turning points and endpoints. (3)

5.5 Write down the amplitude of *g*. (1)

5.6 Use your graphs to determine for which values of *x* is:

5.6.1 $f(x).g(x) = 0$ (2)

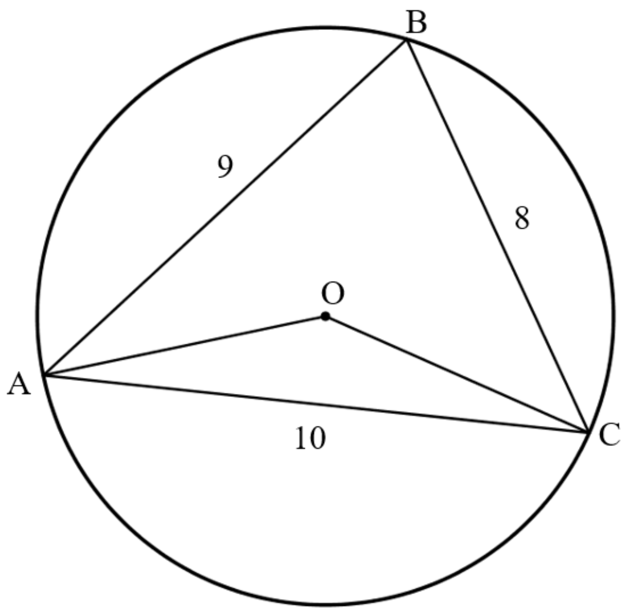
5.6.2 $f(x) \geq 0$ (4)

[14]

QUESTION 6

6.1 Complete the cosine rule for $\triangle PQR$. (1)

6.2 In the diagram below, O is the centre of the circle. AB = 9 units, BC = 8 units and AC = 10 units.



Determine:

6.2.1 The size of $\hat{B}$ (4)

6.2.2 Hence, the size of $A\hat{O}C$, stating a reason (2)

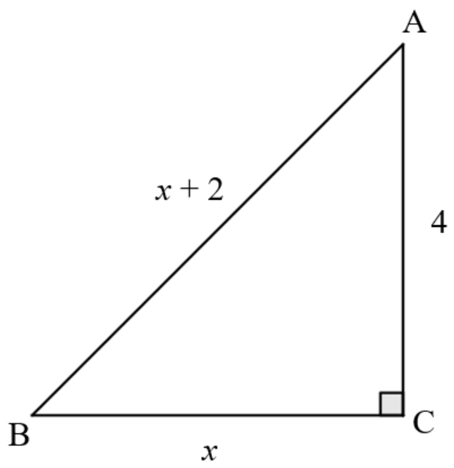
6.2.3 The length of the diameter of the circle (5)

6.2.4 The area of $\triangle ABC$ (3)

[15]

QUESTION 7

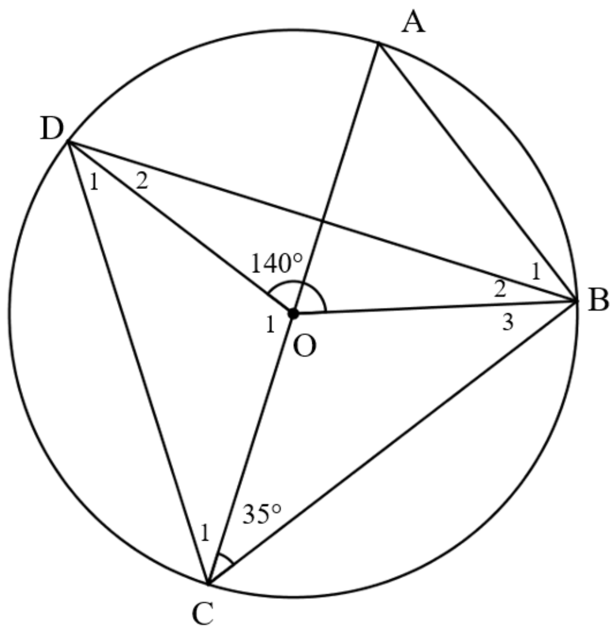
7.1 ΔABC is a right-angled triangle in the diagram below. AB = (x + 2) units, AC = 4 units and BC = x units in length.



7.1.1 Determine the length of BC. (3)

7.1.2 Give a reason why AB is a diameter of the circle through A, B and C. (1)

7.2 In the diagram below, AC is a diameter of the circle with centre O. $D\hat{O}B = 140°$ and $A\hat{C}B = 35°$.



7.2.1 Determine, stating reasons, the size of $\hat{O}_1$. (3)

7.2.2 Determine, stating reasons, the size of $\hat{B}_1$. (3)

7.2.3 Show that AC bisect $D\hat{C}B$. (2)

[12]

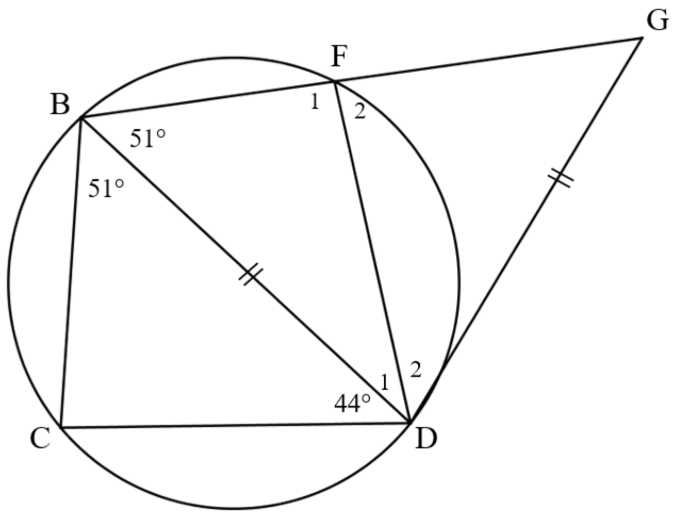
QUESTION 8

8.1 Complete the following theorem statement:

The exterior angle of a cyclic quadrilateral is equal to the … (1)

8.2 In the diagram below BCDF is a cyclic quadrilateral with BF extended to meet DG in G.

$F\hat{B}D = 51^\circ = D\hat{B}C$
$B\hat{D}C = 44^\circ$
$BD = DG$



8.2.1 Show, stating reasons, that $\hat{D}_2 = 44^\circ$. (5)

8.2.2 Hence, show that $\Delta GFD \equiv \Delta BCD$, stating reasons. (5)

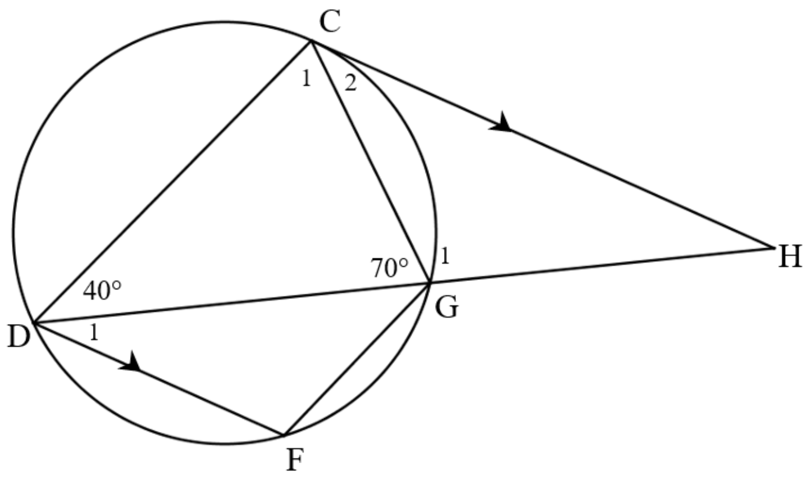
[11]

QUESTION 9

9.1 Complete the following theorem statement:

The tangent to a circle is perpendicular to the … of the circle at the point of contact. (1)

9.2 In the diagram below, CH is a tangent to the circle at C.
DG extended meets the tangent in H.
CH ∥ DF
$C\hat{D}G = 40°$
$C\hat{G}D = 70°$



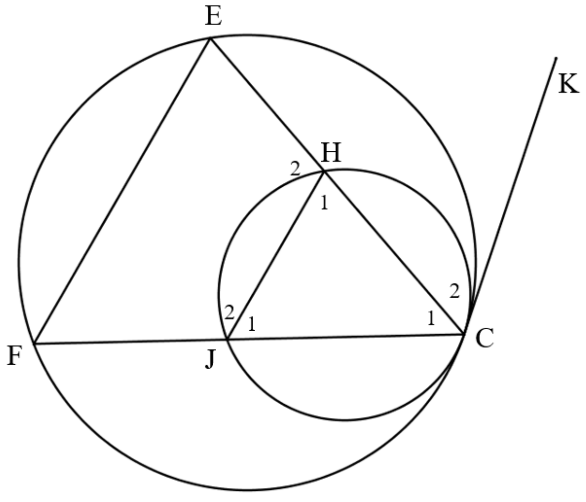
Stating reasons, calculate the size of the following angles:

9.2.1 $\hat{C}_2$ (2)

9.2.2 $\hat{F}$ (3)

9.2.3 $\hat{D}_1$ (2)

9.3 In the diagram below, CK is a common tangent.



Show, stating reasons, that EF ∥ JH. (4)

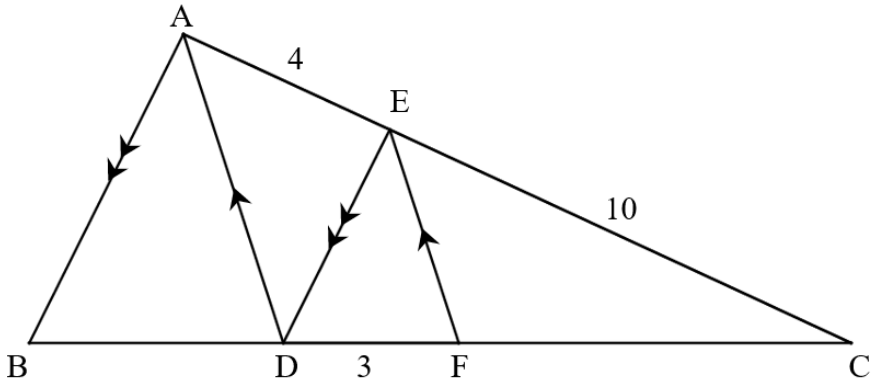
[12]

QUESTION 10

10.1 Complete the following theorem statement:

If a line divides two sides of a triangle in the same proportion, then the line is … (1)

10.2 In ΔABC below, E, D and F is on the sides of the triangle such that $AB \parallel DE$ and $AD \parallel FE$. $AE = 4$ units, $EC = 10$ units and $DF = 3$ units.

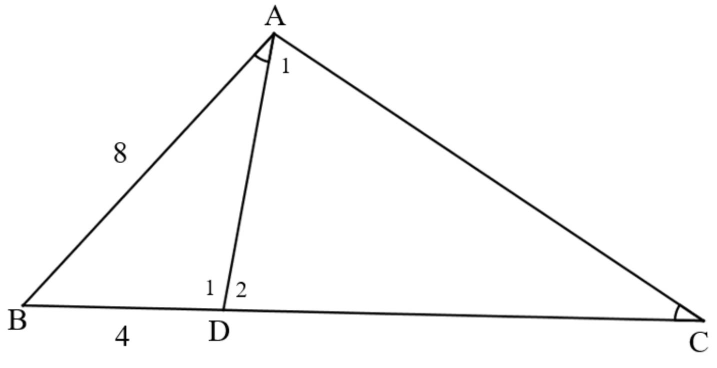


Calculate, giving reasons, the lengths of the following:

10.2.1 FC (3)

10.2.2 BD (4)

10.3 In the diagram below, D is a point on BC such that $B\hat{A}D = \hat{C}$. $BD = 4$ units and $AB = 8$ units.



10.3.1 Prove that $\Delta ABD \parallel\!| \Delta CBA$ (3)

10.3.2 Calculate the length of DC. (4)

[15]

TOTAL: 150

INFORMATION SHEET

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad$ $x=-\frac{b}{2a}$ $\quad$ $y=\frac{4ac-b^2}{4a}$

$a^x=b\Leftrightarrow x=\log_a b$ $a>0$, $a\neq 1$ and $b>0$

$A=P(1+ni)$ $\quad$ $A=P(1-ni)$

$A=P(1+i)^n$ $\quad$ $A=P(1-i)^n$ $\quad$ $i_{eff}=\left(1+\frac{i^m}{m}\right)^m-1$

$d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad$ $y-y_1=m\left(x-x_1\right)$ $\quad$ $m=\frac{y_2-y_1}{x_2-x_1}$

$m=\tan\theta$ $\quad$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $\quad$ $\int x^n dx=\frac{x^{n+1}}{n+1}+C$, $n\neq -1$

$\int\frac{1}{x}dx=\ln(x)+C,\quad x>0$ $\quad$ $\int a^x dx=\frac{a^x}{\ln a}+C$, $a>0$

$\pi rad=180^\circ$

Angular velocity $=\omega=2\pi n=360^\circ n$ where n = rotation frequency

Circumferential velocity $=v=\pi Dn$ where D = diameter and n = rotation frequency

$s=r\theta$ where $r=$ radius and $\theta=$ central angle in radians

$4h^2-4dh+x^2=0$ where $h=$ height of segment, $d=$ diameter of circle and $x=$ length of chord

Area of a sector $=\frac{rs}{2}=\frac{r^2\theta}{2}$ where $r=$ radius, $s=$ arc length and $\theta=$ central angle in radians

In ΔABC:

$\frac{a}{\sin \mathrm{A}}=\frac{b}{\sin \mathrm{B}}=\frac{c}{\sin \mathrm{C}}$ $\quad$ $a^2=b^2+c^2-2bc.\cos \mathrm{A}$ $\quad$ Area $=\frac{1}{2}ab.\sin \mathrm{C}$

$\sin^2\theta+\cos^2\theta=1$ $\quad$ $1+\tan^2\theta=\sec^2\theta$ $\quad$ $\cot^2\theta+1=\cos ec^2\theta$

$\mathrm{A}_T=a\left(\frac{o_1+o_n}{2}+o_2+o_3+o_4+...+o_{n-1}\right)$ where a = width of equal parts, o_i= i^{th} ordinate and n = number of ordinates

OR

$\mathrm{A_T}=a\left(m_1+m_2+m_3+...+m_{n-1}\right)$ where a = width of equal parts, $m_i=\frac{o_i+o_{i+1}}{2}$ and n = number of ordinates; i = 1;2;3; …;n-1