



**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2023

**MATHEMATICS P2
(DEAF)**

MARKS: 150

TIME: 3 hours

This question paper has 16 pages, including a 1-page
information sheet, and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the instructions. Answer the questions.

1. This **question paper** has **10 questions**.
2. **Answer ALL** the questions.
Write in the SPECIAL ANSWER BOOK.
3. **Show ALL calculations, diagrams, graphs, etc.** which you have **used in your answers**.
4. **Answers only** will **NOT** always get **full marks**.
5. You **may use** a prescribed **calculator**.
Some questions will **tell** you **NOT** to use a **calculator**.
6. **Round off** answers correct to **TWO decimal places**.
Some questions will **tell** you **how to round off**.
7. **Diagrams** are **NOT** always **drawn to scale**.
8. An **information sheet** with formulae is **included** at the **end** of the question paper.
9. Write **neatly**.
Your work must be **easy to read**.

QUESTION 1

- 1.1 A school's hockey team recorded the number of push-ups each player completed in a minute.

The numbers for the seven players were:

29 27 24 31 22 19 30

- 1.1.1 Calculate the:

(a) Mean (2)

(b) Standard deviation (1)

- 1.1.2 How many players were within one deviation of the mean? (3)

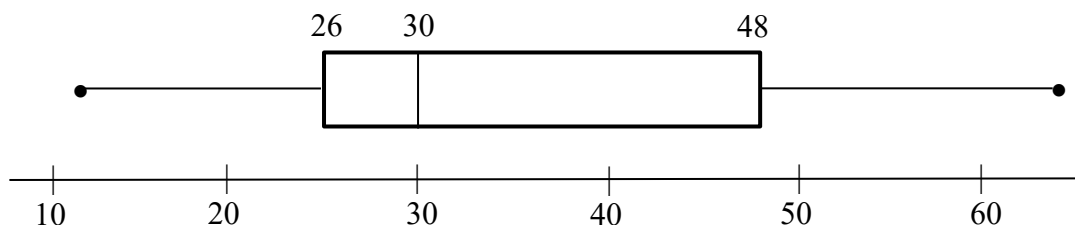
- 1.1.3 Seven players in the school's rugby team also recorded the number of push-ups they completed in a minute.

Their numbers gave a mean of 26 and a standard deviation of 3,2.

Use the standard deviations and the means.

Compare the number of push-ups of the players in the rugby and hockey teams. (2)

- 1.2 The number of points scored by a rugby team in each of 10 matches is represented in the box and whisker diagram below.
The scores of the 10 matches were different.



- 1.2.1 In what percentage of the matches did the team score over 30 points? (1)

- 1.2.2 Which of the mean or median is likely to be greater?

Give a reason for your answer.

(2)
[11]

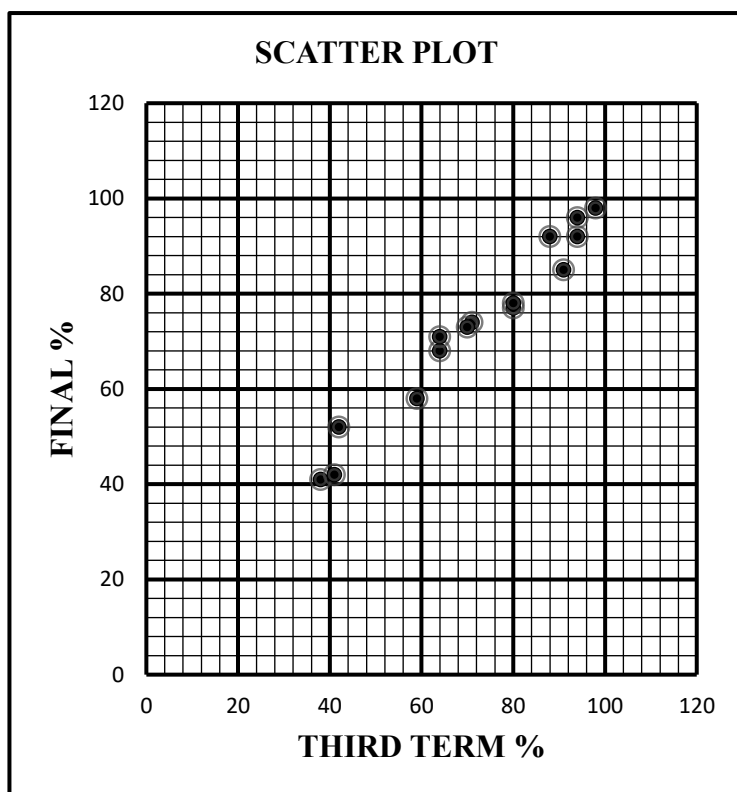
QUESTION 2

Table.

Shows the percentages scored by a sample of 15 candidates in the third term and final examinations of 2022.

The table and the scatter plot below represent these marks.

Third	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71



- 2.1 Determine the equation of the least squares regression line for the data.
Round off your answers to 3 decimal places. (3)
- 2.2 Write the value of the correlation coefficient, r , between the 3rd term and final exam percentages. (1)
- 2.3 A candidate scored 48% in the third term.
 - 2.3.1 Use the equation of the least squares regression line to predict his final percentage.
Round off your answer to the nearest whole number. (2)
 - 2.3.2 Give a reason why the prediction can be regarded as reliable. (1)

2.4 The **least squares regression line** is **used** to **predict** that the **final percentage** of a **candidate** who scored **50%** in the **third term** is **80%**.

2.4.1 **Why** would this **prediction** be **unreliable**? (1)

2.4.2 **Would adding** the **point (20;10)** to the **original data set** **increase** or **decrease** the **gradient** of the **least squares regression line**? (1)

[9]

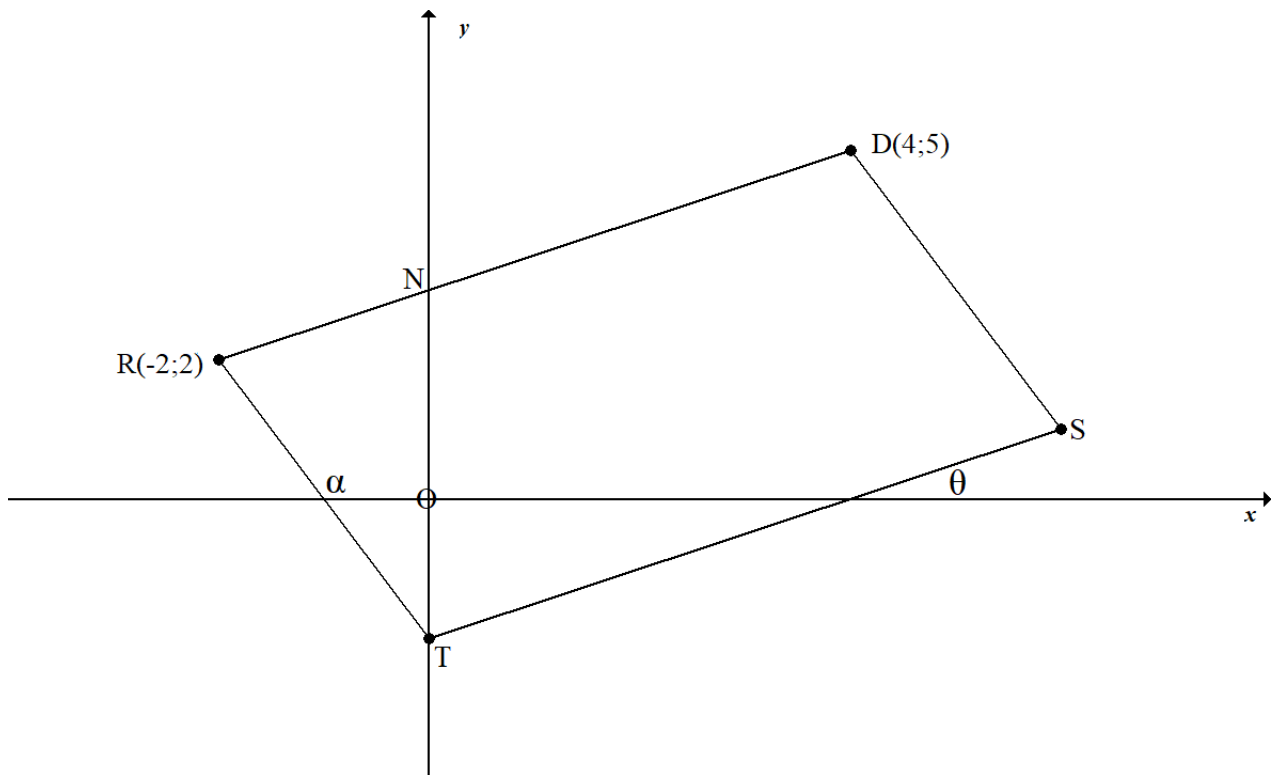
QUESTION 3

Diagram:

D(4; 5), R(-2; 2), T and S form a quadrilateral.

RD cuts the y-axis at N.

T is a point on the y-axis.

The inclinations of RT and TS are α and θ respectively.RD \parallel TS and the equation of TS is $y = \frac{1}{2}x - 2$.

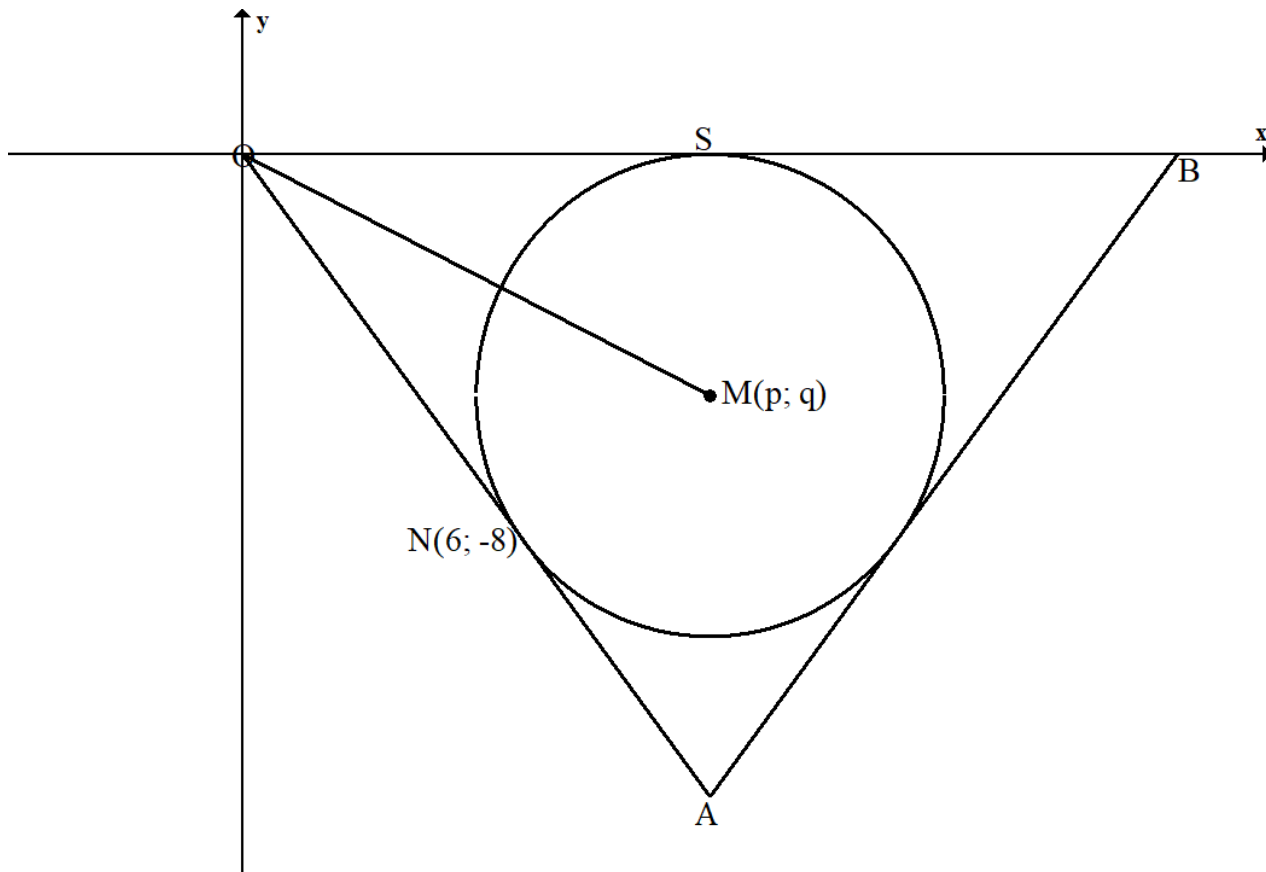
- 3.1 Write down the **coordinates** of T. (1)
- 3.2 Calculate: (2)
- 3.2.1 The **gradient** of RT (2)
- 3.2.2 The **size** of \widehat{RTS} (5)
- 3.3 Determine the **equation** of RD in the **form** $y = mx + c$. (3)
- 3.4 If $RT \parallel DS$, calculate the **coordinates** of M, the **midpoint** of RS. (3)
- 3.5 Calculate the **area** of $\triangle RTN$. (4)
- [18]

QUESTION 4

Diagram:

A circle centred at $M(p; q)$, touches the x -axis at S .

Line OA is a tangent to the circle at $N(6; -8)$.



4.1 Calculate the:

4.1.1 Length of ON (2)

4.1.2 Value of p (2)

4.1.3 Gradient of NM (3)

4.1.4 Value of q (2)

4.2 Determine the equation of the circle in the form: $(x - a)^2 + (y - b)^2 = r^2$. (3)

4.3 $x = k$ is a tangent to the circle.
Write the value(s) of k . (2)

4.4 Line $y = -\frac{4}{3}x + t$ cuts the circle at two different points.
Determine the values of t . (6)

4.5 Another circle with equation $(x - 10)^2 + (y - 6)^2 = 25$ is given.

Will the two circles touch, cut or not?

Give a reason.

(2)

[22]

QUESTION 5

5.1 **Do NOT use a calculator.**

If $\sin 54^\circ = p$, express each in terms of p .

$$5.1.1 \quad \sin 594^\circ \quad (2)$$

$$5.1.2 \quad \cos 36^\circ \quad (2)$$

$$5.1.3 \quad \cos 18^\circ \quad (4)$$

5.2 **Do NOT use a calculator.**

Simplify the following:

$$\frac{\cos 140^\circ - \sin(90 - \theta)}{\sin 410^\circ + \cos(-\theta)} \quad (6)$$

5.3 **Do NOT use a calculator.**

Determine the value of the trigonometric expression.

$$\cos(x + 65^\circ) \cdot \cos(x + 20^\circ) - \sin(x + 245^\circ) \cdot \sin(x + 20^\circ) \quad (4)$$

$$5.4 \quad \text{Determine the general solution of: } \cos^2 x - \sin^2 x = \frac{1}{2} \quad (4)$$

5.5 **Given the identity:**

$$\frac{\sin 2\theta \cdot \tan \theta}{\cos 2\theta + 1} = \tan^2 \theta$$

$$5.5.1 \quad \text{Prove the identity.} \quad (4)$$

$$5.5.2 \quad \text{Determine the values of } \theta \text{ for which the identity is undefined if } 0^\circ \leq \theta \leq 180^\circ. \quad (4)$$

[30]

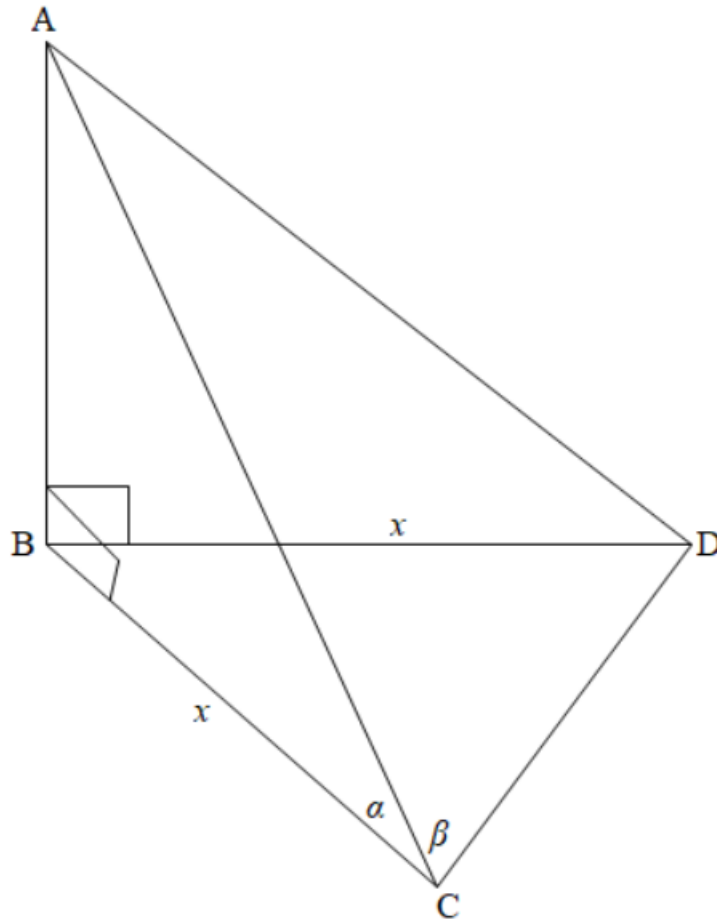
QUESTION 6

Diagram:

B, C and D are points on the same horizontal plane.

AB is a vertical tower with the angle of elevation from **C** to **A** equal to α .

$\hat{ACD} = \beta$. $BD = BC = x$.



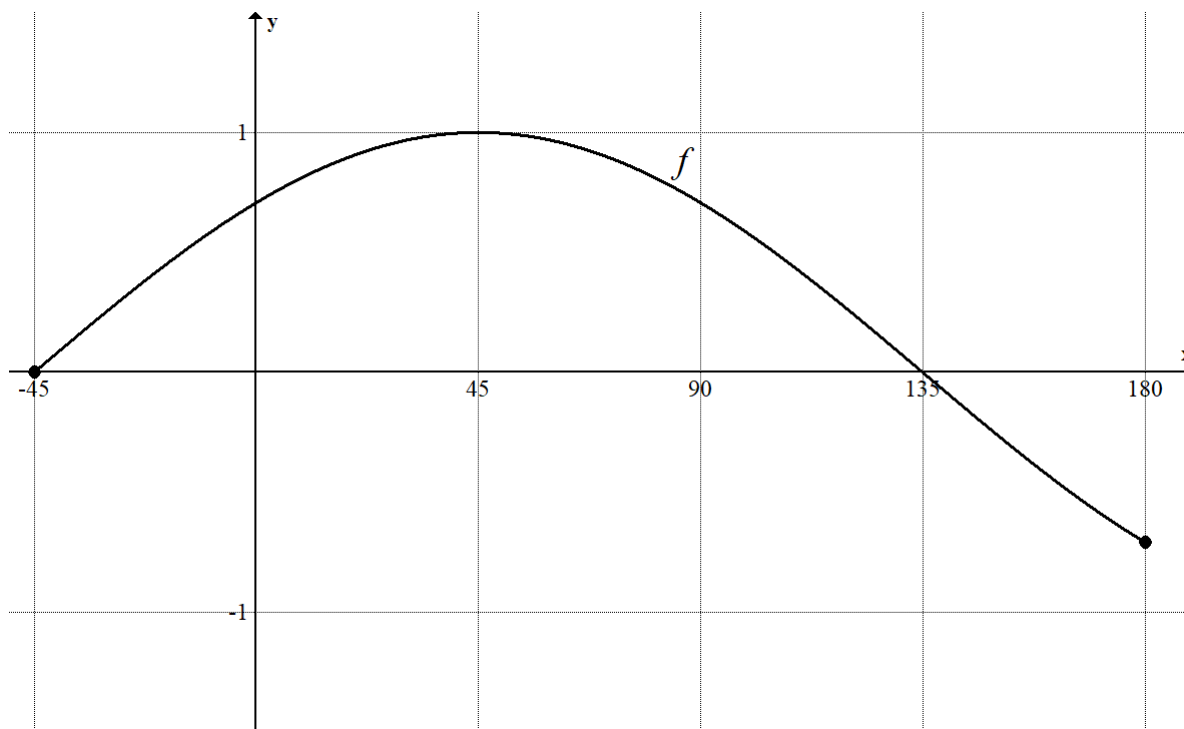
- 6.1 Why is $AC = AD$? (1)
- 6.2 Write AC in terms of x and α . (2)
- 6.3 Show that $CD = \frac{2x \cos \beta}{\cos \alpha}$ (4)
- 6.4 Hence, determine the length of CD if $x = 25 \text{ cm}$, $\alpha = 30^\circ$ and $\beta = 65, 62^\circ$. (2)
- [9]

QUESTION 7

Graph:

Sketched is a graph of $f(x) = \cos(x - 45^\circ)$ for $-45^\circ \leq x \leq 180^\circ$.

Use the **graph**. Answer the **questions**.



- 7.1 Write down the range of f , for the given interval. (2)
- 7.2 Draw the graph of $h(x) = \sin 2x$, for $x \in [-45^\circ; 180^\circ]$ on the same set of axes as f in the ANSWER BOOK.
Indicate the coordinates of all intercepts with the axes as well as turning points. (3)
- 7.3 State the period of h . (1)
- 7.4 Use your graph to determine the values of x for which f and h are both increasing. (2)
- 7.5 Determine the values of x for which $f(x) - h(x) = 1$. (2)
- 7.6 The graph of f is translated 60° to the left to form the graph of g .
Write down the equation of g in the form: $g(x) = \underline{\hspace{2cm}}$. (1)

[11]

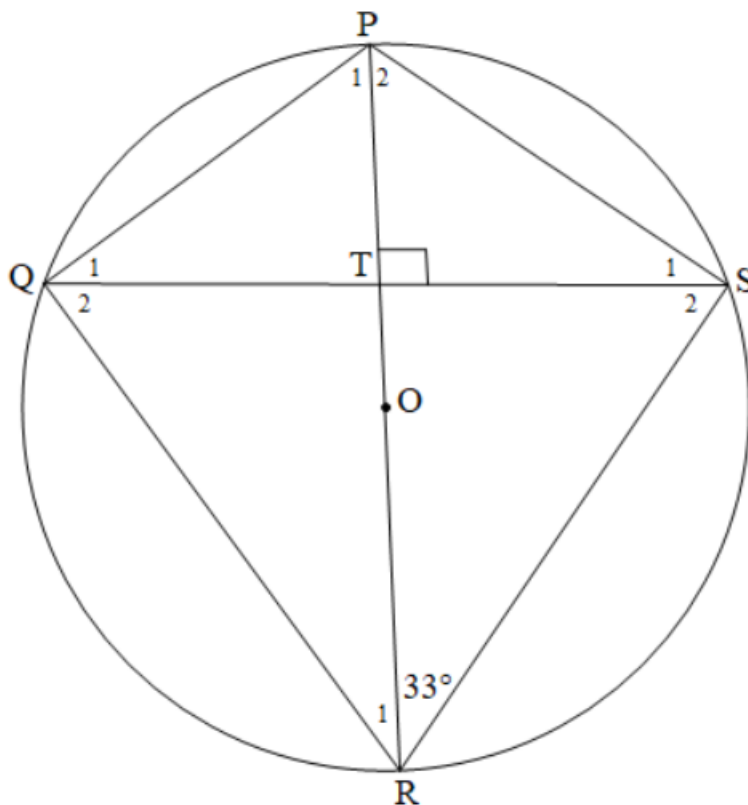
QUESTION 8

Diagram:

PR is a **diameter** of circle **PQRS** with **centre O**.

PR intersects with **chord QS** at **T** such that $\widehat{PTS} = 90^\circ$.

$\widehat{PRS} = 33^\circ$.



8.1 Determine, with reasons, the size of:

8.1.1 $\widehat{P_1}$ (3)

8.1.2 $\widehat{Q_2}$ (2)

8.2 **QS = 16 cm** and **PR = 20 cm**.

Determine, with reasons, the length of **TO**. (4)

[9]

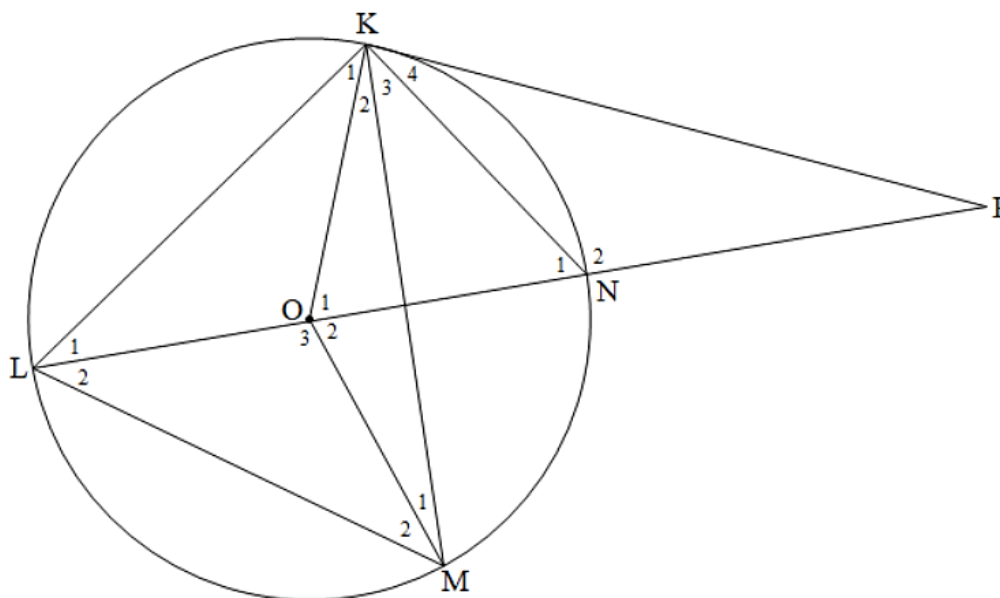
QUESTION 9

Diagram:

O is the **centre** of the **circle** and **KP** is a **tangent** to the **circle**.

LN, the **diameter** of the **circle**, is **extended** to meet **KP** at **P**.

Straight lines **OK**, **OM**, **KM** and **KN** are **drawn**.



9.1 Write down two angles equal to 90° . (2)

9.2 If $\hat{K}_4 = x$, write down the angles in terms of x , giving reasons.

9.2.1 \hat{L}_1 (2)

9.2.2 \hat{K}_1 (2)

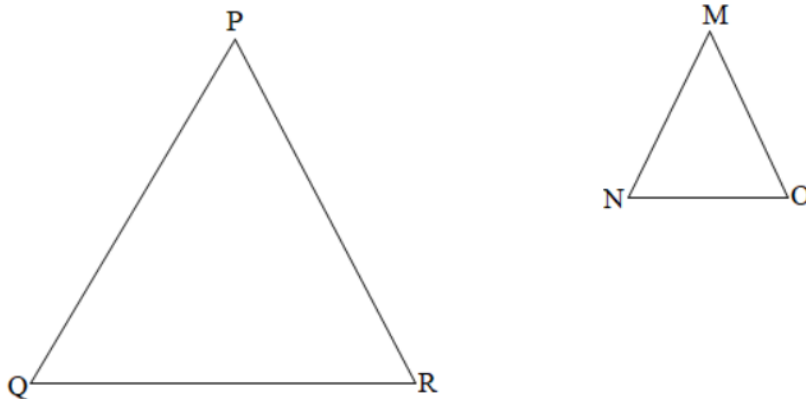
9.2.3 \hat{P} (2)

9.3 Join MP, which is a tangent to the circle.
Hence, prove that KOMP is a cyclic quadrilateral. (3)

[11]

QUESTION 10**10.1 Diagram:**

$\triangle PQR$ and $\triangle MNO$ are given with $\hat{P} = \hat{M}$, $\hat{Q} = \hat{N}$ and $\hat{R} = \hat{O}$.



Use the **diagram** in your **ANSWER BOOK**.

Prove the theorem which states that:

$$\frac{MN}{PQ} = \frac{MO}{PR}$$

(6)

10.2 **Diagram:**

PQ is a **tangent** to the **circle** at **Q**.

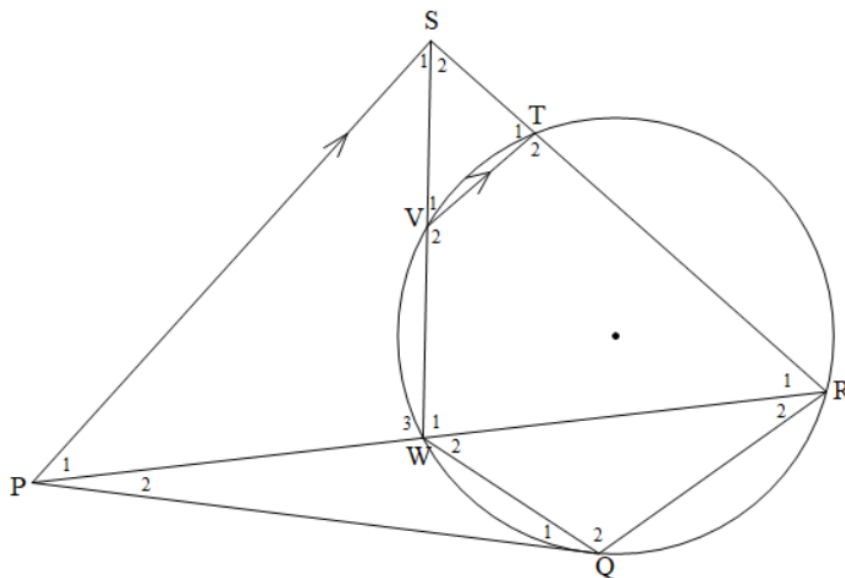
R is a **point** on the **circle** and **S** lies **outside** the **circle**.

PR cuts the **circle**.

W and **RS** cuts the **circle** in **T**.

SW cuts the **circle** in **V**.

VT \parallel **PS**.



Prove that:

$$10.2.1 \quad \hat{S}_1 = \hat{R}_1 \quad (3)$$

$$10.2.2 \quad \triangle PWS \parallel \triangle PSR \quad (3)$$

$$10.2.3 \quad PQ^2 = PW \cdot PR \quad (5)$$

$$10.2.4 \quad PQ = PS \quad (3)$$

[20]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$