Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2023

TECHNICAL MATHEMATICS P2 (DEAF)

MARKS: **150**

TIME: **3 hours**

This question paper has 16 pages, including a
2-page information sheet and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the **instructions**. **Answer** the **questions**.

1. This question paper has **11 questions**.

2. **Answer ALL** the **questions**.
   Write in the **SPECIAL ANSWER BOOK**.

3. **Show ALL calculations**, **diagrams**, **graphs**, etc. that you used in your calculations.

4. **Answers only** will **NOT always** get **full marks**.

5. You **may use** a prescribed **calculator**.
   Some questions will **tell** you **NOT** to use a **calculator**.

6. **Round off** answers to **TWO decimal places**.
   Some questions will **tell** you **how** to **round off**.

7. **Diagrams** are **NOT** always drawn to **scale**.

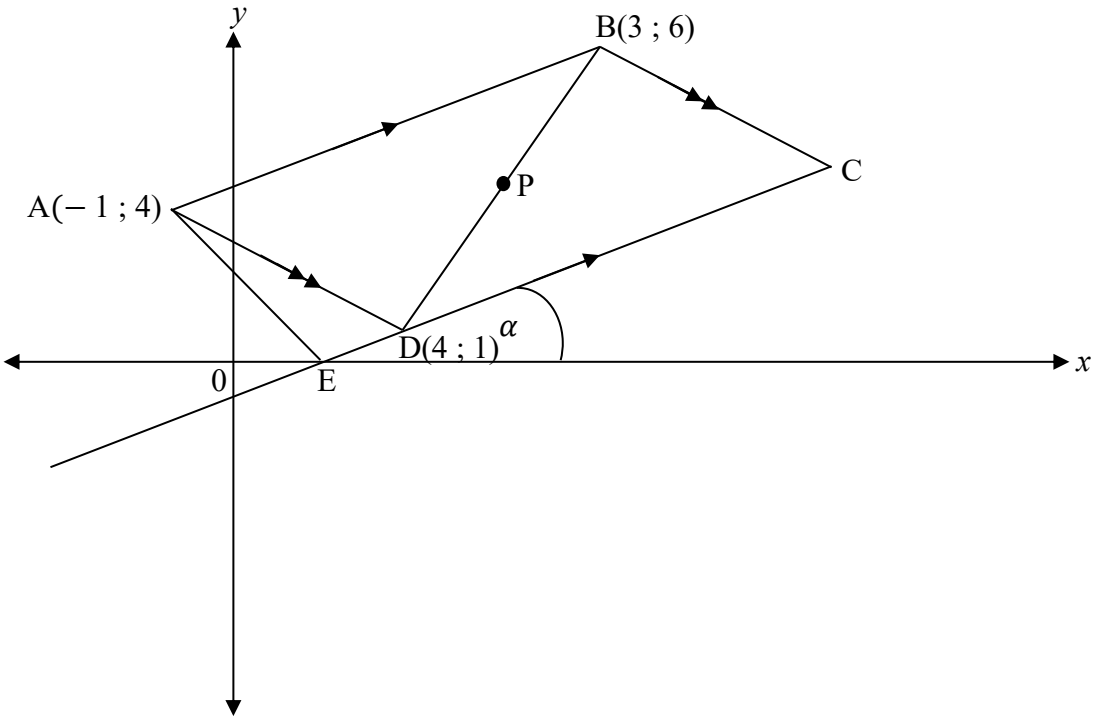
8. An **information sheet** with formulae is at the **end** of the **question paper**.

9. Write **neatly**.
   Your **answers** must be **easy** to **read**.

QUESTION 1

Diagram:

ABCD is a **parallelogram** with **vertices** A(− 1 ; 4), B(3 ; 6), C and D(4 ; 1).
E is the ***x*-intercept** of the **line CD extended**.
α is the **inclination angle** of **line CD**.



Determine:

1.1 The **gradient** of **AB** (2)

1.2 The **coordinates** of **P**, the **midpoin**t of **BD** (2)

1.3 The **equation** of **CD** (3)

1.4 The **coordinates** of **E**, if **E** is the ***x*-intercept** of **line CD extended** (2)

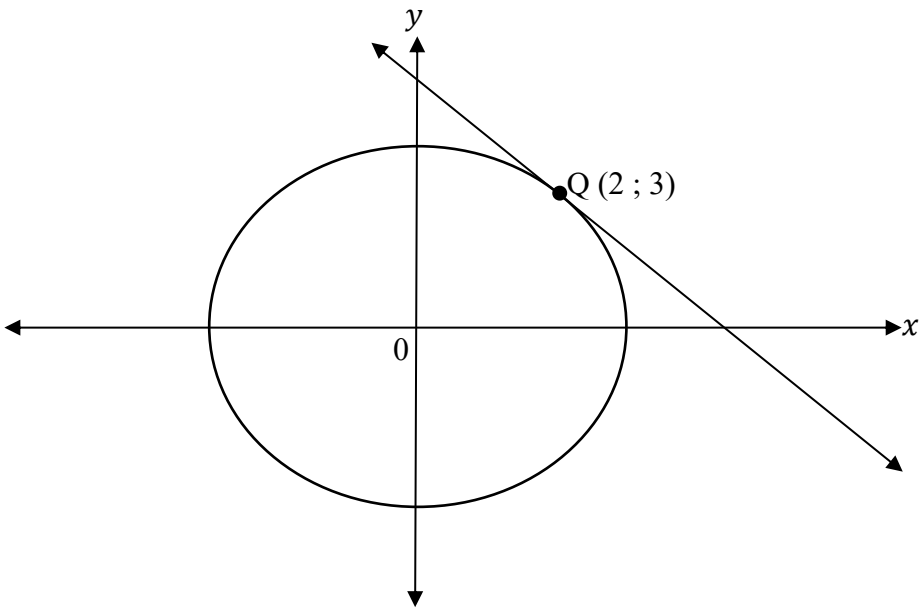
1.5 The **inclination angle** of **line AE** (4)

1.6 The **size** of $A\hat{E}D$ (4)
[17]

QUESTION 2

2.1 **Diagram**:

It shows a **circle** with **equation** $x^2 + y^2 = 13$.
There is **tangent line touching** at **point Q(2 ; 3)**.



2.1.1 **Determine** the **gradient** of **OQ**. (2)

2.1.2 Hence, or otherwise, **determine** the e**quation** of the **tangent line** in the **form** $y = \ldots$ (3)

2.2 **Given**:

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Sketch the given **graph**, in your **SPECIAL ANSWER BOOK**.
Indicate all **intercepts** with the **axis**. (3)
[8]

QUESTION 3

3.1 Given: P = 128,2˚ and S = 204,7˚

Determine:

3.1.1 $\cos(P + S)$ (2)

3.1.2 $\text{cosec}(S - P)$ (3)

3.2 If **$\cos 75° = k$, express** the following in **terms** of **k**.

3.2.1 $\sin 15°$ (3)

3.2.2 $\tan 255°$ (3)

3.3 **Solve** for θ, **rounded off** to **ONE decimal digit**, if $\theta \in (90° ; 180°)$:

$\sec\theta = -1{,}583$ (4)

[15]

QUESTION 4

4.1 **Simplify:**

$$\text{cosec}^2(180° + \theta) + \frac{\sin(180° - \theta) \cdot \cot^2(180° + \theta) \cdot \sin 270°}{\cos(360° - \theta) \cdot \tan(180° + \theta)} \quad (9)$$

4.2 **Prove** that:

$$\frac{1}{(1 - \sin\theta)(1 + \sin\theta)} = \sec^2\theta \quad (2)$$

[11]

QUESTION 5

Given $f(x) = \tan x$ and $g(x) = \sin x - 1; x \in (0°; 360°)$

5.1 **Use** the **grid** in the SPECIAL ANSWER BOOK,
Draw sketch graphs of $f(x) = \tan x$ and $g(x) = \sin x - 1$ on the **same set of axes**.

Show ALL:
- The intercepts with the axes,
- Turning points and
- Asymptote(s). (7)

5.2 **Write** down the **range** of $\boldsymbol{g}$. (2)

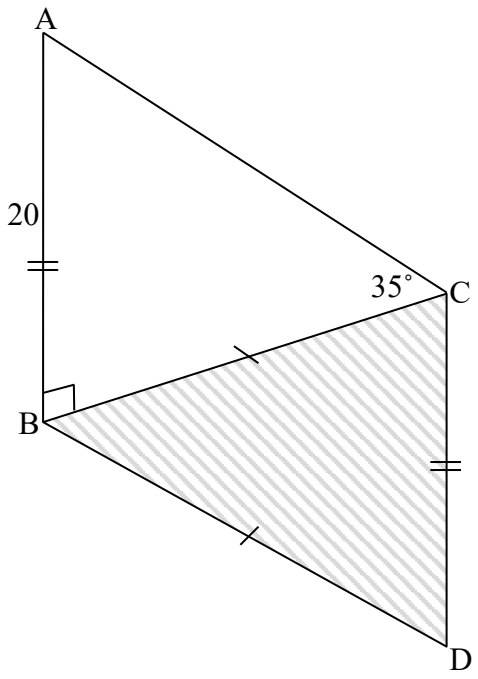
5.3 **State** the **period** of $\boldsymbol{f}$. (1)

5.4 **Use** your **graphs**.
State for which **values** of $x \in (90°; 270°)$, that $f(x).g(x) < 0$. (2)
[12]

QUESTION 6

Diagram:

AB is a **tower**, **anchored** at **point C**, **forming** an **angle** of **elevation 35°**.
B, **C** and **D** are in the **same horizontal plane**.
AB = **CD** = **20 units** and **BC** = **BD**.



6.1 **Determine** the **length BC**. (3)

6.2 **Determine** the **size** of $C\widehat{B}D$.
Rounded off to the **nearest degree**. (4)

6.3 **Determine** the **area** of the **isosceles** Δ**BCD**. (3)
[10]

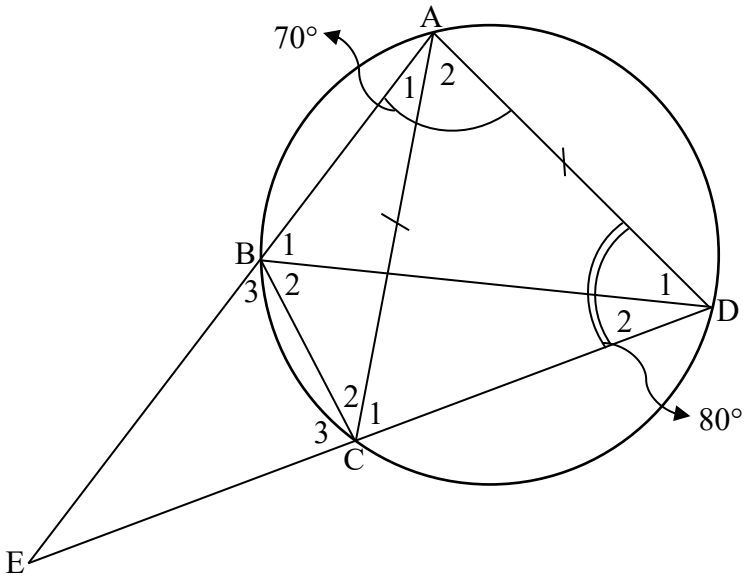
Give reasons for your statements in QUESTIONS 7, 8 and 9.

QUESTION 7

7.1 **Fill in** the **missing word(s)** in the **theorem** statement:

"The **exterior angle** of a **cyclic quadrilateral** is … to the **interior opposite angle**." (1)

7.2 **ABCD** is a **cyclic quadrilateral** with **AD** = **AC** and **A$\hat{D}$C** = 80°.
AB and **DC** are **produced** to **meet** at **E**.



7.2.1 Name THREE other **angles equal** to **80°**.
Give reasons. (6)

7.2.2 It is **given** that **B$\hat{A}$D = 70°**.
Calculate with **reasons**, the sizes of:

(a) $\hat{C}_3$ (2)

(b) $\hat{E}$ (2)

(c) $\hat{D}_1$ (3)

7.2.3 **Prove** that **AD** is a **tangent** to the **circle DBE** at **D**. (2)
[16]

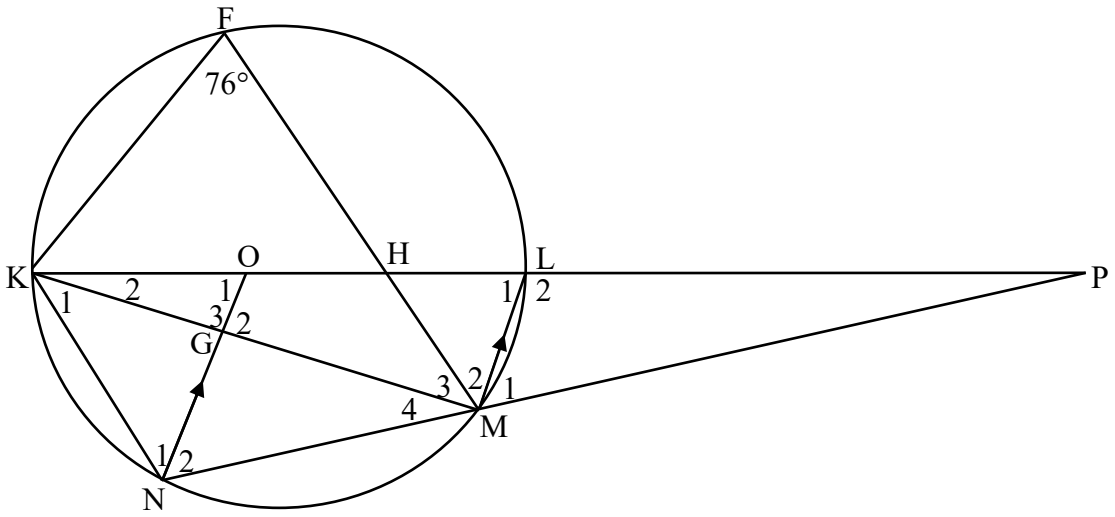
QUESTION 8

8.1 **Fill in** the **missing word(s)** in the **theorem** statement:

"The **angle subtended** by an **arc** at the **centre** of a **circle** is … the **size** of the **angle subtended** by the **same arc** at the **circumference** of the **circle**." (1)

8.2 **Diagram:**

O is the **centre** of the **circle** and **diameter**.
KL is **produced** to **meet NM produced** at **P**.
ON || LM and $\hat{F}$ **= 76°**.



Calculate.
Give reasons:

8.2.1 $\hat{L}_1$ (2)

8.2.2 $\hat{O}_1$ (2)

8.2.3 $\hat{M}_4$ (2)

8.2.4 $\hat{N}_1 + \hat{N}_2$ (2)
[9]

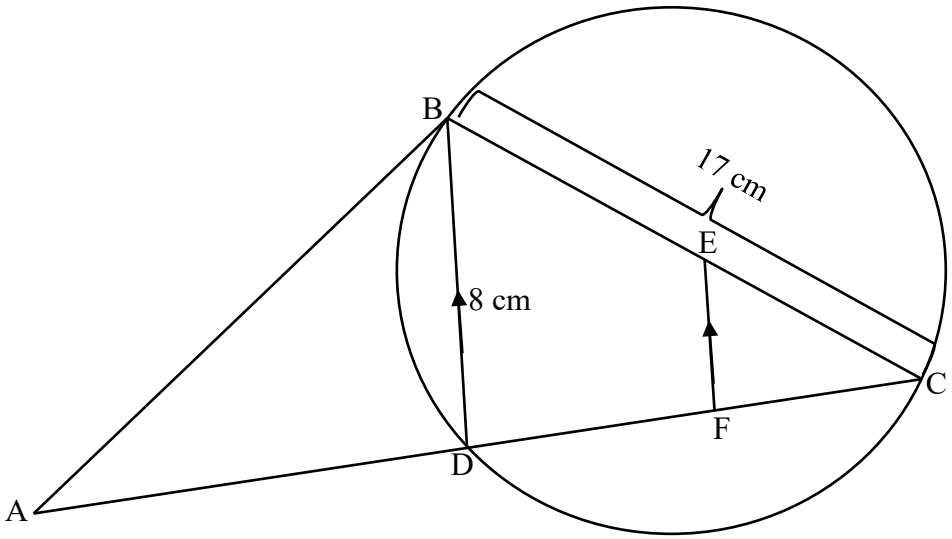
QUESTION 9

9.1 **Fill in** the **missing word(s)** in the **theorem** statement:

"A **line drawn parallel** to **one side** of a **triangle** … the **other two sides proportionally**." (1)

9.2 **Diagram:**

In the diagram, **BC** = **17 cm**.
BC is a **diameter** of the **circle**.
The **length** of the **chord BD** is **8 cm**.
The **tangent** at **B** meets **CD produced** at **A**.



9.2.1 **Calculate** the **length** of **DC**.
Give reasons. (4)

9.2.2 **E** is a **point** on **BC.**
BE : EC = 3 : 1.
EF is parallel to BD with F on DC.

(a) **Calculate** the **length** of **CF**.
Give reasons. (4)

(b) **Prove** that **ΔBAC ||| ΔFEC**. (5)

(c) **Determine** the **length** of **AD**. (4)
[18]

QUESTION 10

10.1 A **train moves** on a **circular track** with a **diameter** of **14 6425 km**.
It takes **50 minutes** to **complete one revolution**.

10.1.1 **Determine** the **rotational frequency per minute**. (1)

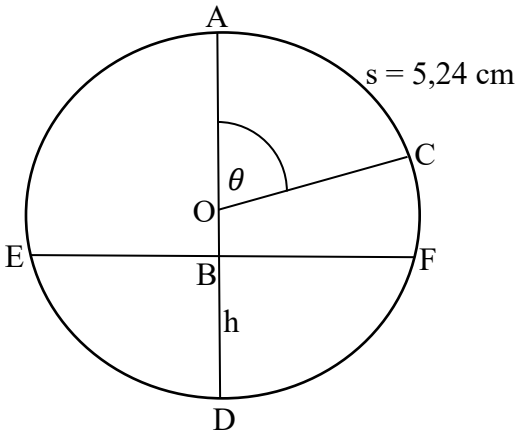
10.1.2 **Convert** the **length** of the **diameter** to **metre**. (1)

10.1.3 Hence, **calculate** the **circumferential velocity** of the **train** in **metres per min**. (3)

10.2 A **wheel rotates** at **15 revolutions per second**.
Calculate the **angular velocity** of the **wheel** in **radians per minute**. (4)

10.3 **Diagram:**

The **circle** below with **centre O**, has a **chord EF** of **length 80 mm**.
It has a **diameter AD** that **equals 10 cm**.
Arc AC subtends a **central angle θ**.



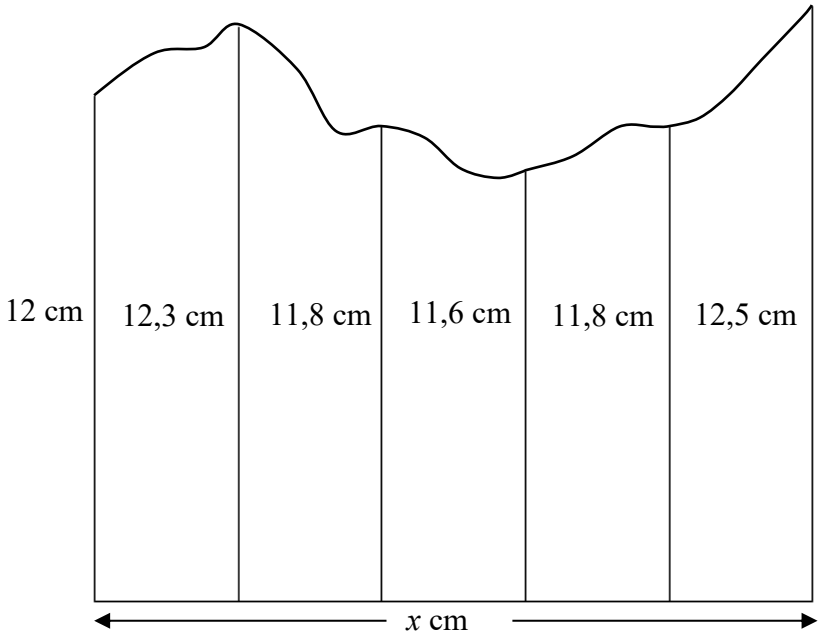
10.3.1 **Calculate** the **height** of the **minor segment**, **h (BD),** in **cm**. (5)

10.3.2 If the **arc length**, **AC**, of the **circle** is **5,24 cm**.
Calculate the **central angle, θ**, to the **nearest degree**. (4)

10.3.3 Hence, **determine** the **area** of the **minor sector AOC** of the **circle**. (3)
[21]

QUESTION 11

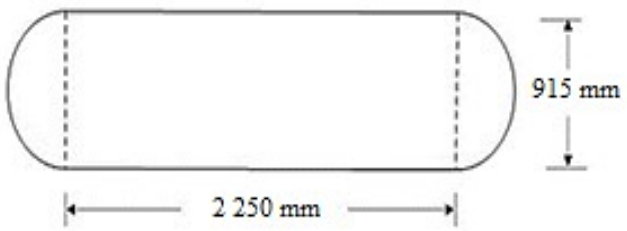
11.1 The **irregular shape**, with **area 149,38 cm^2**, below has a **straight side** of **length *x*** cm.
It has been **divided** into **5 equal parts**.
The **ordinates dividing** the **parts** are: **12 cm, 12,3 cm, 11,8 cm, 11,6 cm, 11,8 cm** and **12,5 cm respectively**.



Determine the **value** of ***x***, the **length** of the **side** of the **irregular shape**. (4)

11.2 The **picture** below is of a **Liquefied Petroleum Gas (LPG) storage tank**.
The **diagram** next to it **shows** the **diameter** of the **tank** as **915 mm**.
The **middle part** of the **tank** is **made** of a **cylinder** with **two identical hemispheres at each end**.
The **height** of the **cylindrical tank** is **2 250 mm**.
The **tank** is **filled** with **propane gas**.
It is further given that:

- **1 m = 100 cm**
- **1 litre = 1 000 cm^3**
- **1 kg = 1,96 litre of propane gas**
- **1 ton = 1 000 kg**
- **Volume of a cylinder = $\pi r^2 h$**
- **Volume of a hemisphere = $\frac{1}{2} \times \frac{4}{3} \pi r^3$**



11.2.1 **Convert** the **measurements** in the **diagram** to **centimetres**. (1)

11.2.2 Hence, **determine** the **volume**, in **litres**, of the **propane gas** inside the **storage tank**. (4)

11.2.3 The **total weight** of the **storage tank** is **0,5 ton**.
The **tare weight** of the **tank** is the **weight** of the **empty tank**.
Calculate what **percentage** is the **tare weight** of the **storage tank**. (4)
[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \Leftrightarrow x = \log_a b \;\; a > 0, a \neq 1 \text{ and } b > 0$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n \qquad i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = tan\theta \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C \text{ , } n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$

$\int \frac{k}{x} dx = k \ln(x) + C, x > 0 \; and \; k \in \mathbb{R}; k \neq 0$

$\int ka^{nx} dx = \frac{ka^{nx}}{nlna} + C, a > 0; a \neq 1 \; and \; k, a \; \in \mathbb{R}; k \neq 0$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ where $n =$ rotation frequency

Circumferential velocity $= v = \pi Dn$ where $D =$ diameter and $n =$ rotation frequency

Circumferential velocity = $v = \omega r$ where $\omega =$ angular velocity and $r =$ radius

Arc length = $s = r\theta$ where = radius and $\theta =$ central angle in radians

$4h^2 - 4dh + x^2 = 0$ where $h =$ height of segment, $d =$ diameter of circle and $x =$ length of chord

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where $r =$ radius, $s =$ arc length and $\theta =$ central angle in radians

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.cosA \qquad Area = \frac{1}{2}ab.sinC$$

$$sin^2\theta + cos^2\theta = 1 \qquad 1 + tan^2\theta = sec^2\theta \qquad cot^2\theta + 1 = cosec^2\theta$$

$A_T = a\left(\frac{O_1 + O_n}{2} + O_2 + O_3 + O_4 + \ldots + O_{n-1}\right)$ where $a =$ width of equal parts, $O_i = i^{th}$ ordinate and $n =$ number of ordinates

OR

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ where $a =$ width of equal parts, $m_i = \frac{O_i + O_{i+1}}{2}$ and $n =$ number of ordinates; i = 1;2;3; …;n-1