Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2023

MATHEMATICS P1

MARKS: 150

TIME: 3 hours

Font size 18

This question paper consists of 16 pages, including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.

3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4. Answers only will not necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

7. Number the answers correctly according to the numbering system used in this question paper.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + x - 30 = 0$ (3)

1.1.2 $x(2x - 6) = -3$ (correct to TWO decimal places) (4)

1.1.3 $x^2 - 2x + 1 > 0$ (3)

1.1.4 $2x - 1 = \sqrt{4 - 5x}$ (4)

1.2 Solve simultaneously for x and y:

$y - 2x = -1$ and $2y^2 + 4xy = 6x^2$ (6)

1.3 Given the quadratic equation: $2x^2 - px + 1 = 0, x \in \mathbb{R}$.

Determine the possible value(s) of p, such that the equation has two **unequal** real roots. (5)

[25]

QUESTION 2

2.1 The tenth and the seventeenth terms of an arithmetic sequence are 21 and 49 respectively.

2.1.1 Determine the common difference of the sequence. (3)

2.1.2 Calculate: $T_1 + T_{18}$ (3)

2.2 Given: $\sum_{n=1}^{m}(4n - 19) = 1189$

2.2.1 Write down the first three terms of the series. (1)

2.2.2 Calculate the value of m. (4)

2.3 $-78; -76; -72; -66; \ldots$ is a quadratic number pattern.

2.3.1 Write down the next two terms of the number pattern. (1)

2.3.2 Determine the n^{th} term of the number pattern in the form, $T_n = an^2 + bn + c$. (4)

2.3.3 A constant k is added to T_n such that all the terms of the quadratic number pattern become positive. Determine the value(s) of k. (2)

[18]

QUESTION 3

3.1 The first term of a geometric sequence is 81 and the common ratio is r. The sum of the first and third terms of the same geometric sequence is 117. Calculate the value of r. (4)

3.2 Given the convergent geometric series:
$3^x + 9^x + 27^x + 81^x + \ldots$

3.2.1 Write down the common ratio in terms of x. (1)

3.2.2 Calculate the value of x, if $S_\infty = \frac{1}{2}$. (3)

[8]

QUESTION 4

Given: $f(x)=\dfrac{2}{x-5}+3$

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Write down the range of f. (1)

4.3 Determine the coordinates of the x- and y-intercepts of f. (3)

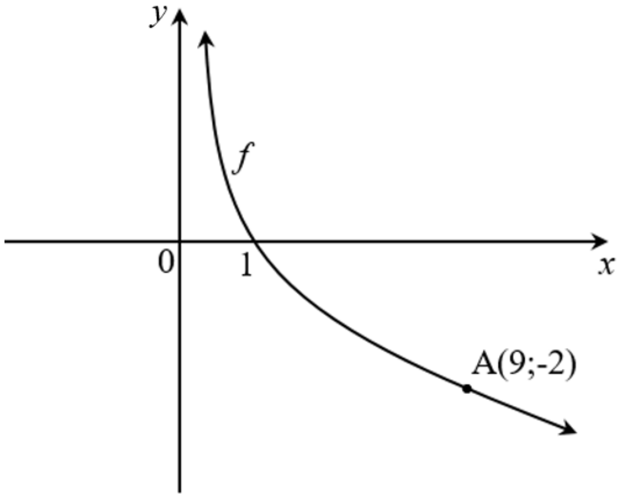
4.4 Sketch the graph of f, clearly showing all asymptotes and intercepts with the axes. (4)

4.5 Describe the transformation that the graph of f has to undergo to form the graph of h, where $h(x)=-\frac{2}{x-5}-5$. (3)

[13]

QUESTION 5

The diagram below shows the graph of $f(x) = log_b\, x$, where b is a constant. f passes through the point $A(9; -2)$.



5.1 Show that $b = \frac{1}{3}$. (2)

5.2 Determine the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

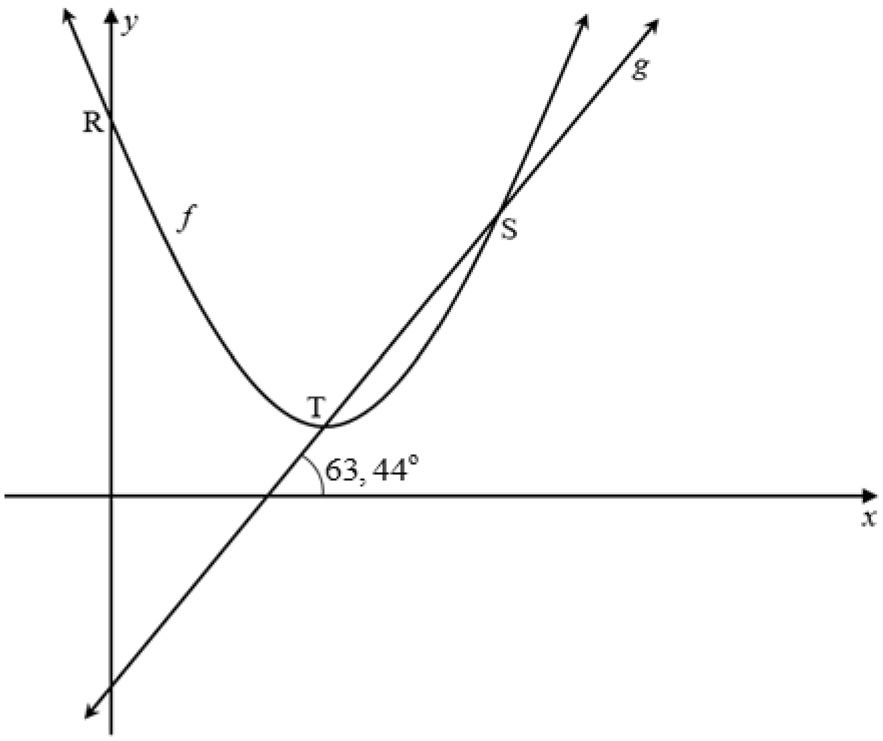
5.3 For which values of x is $f(x) \geq 0$? (2)

5.4 Write down the equation of the asymptote of g, if $g(x) = f^{-1}(x + 1)$. (2)

[8]

QUESTION 6

The diagram below shows the graphs of $f(x) = x^2 - 6x + 11$ and $g(x) = ax + b$. The graphs of f and g intersect at S and T, where T the turning point of f. The angle of inclination of g is $63{,}44^o$.



6.1 Calculate the coordinates of T. (4)

6.2 Determine the equation of g in the form $y = mx + c$. (3)

6.3 Hence, or otherwise determine the coordinates of S. (4)

6.4 Determine the values of:

6.4.1 x, for which $f(x) \leq 6$ (2)

6.4.2 k, for which $f(x) + k$ will have real roots (2)

[15]

QUESTION 7

7.1 Lufezo deposited R97 000 into an account that offered interest at 9,1% p.a. compounded quarterly. Calculate how many years it took for his investment to reach R166 433. (4)

7.2 On 1 January 2018 a school bought a new bus for R482 000. On that day they also started a sinking fund to make provision for a new bus in 5 years' time.

7.2.1 Over the next 5 years the value of the bus depreciated at 14,7% p.a. on the reducing-balance method. Calculate the trade-in value of the bus after 5 years. (2)

7.2.2 The price of these buses increases by 8,1% per year. Calculate the price of a new bus on 1 January 2023, i.e. after 5 years. (2)

7.2.3 The bank offered an interest rate of 7,3% p.a., compounded monthly, for the sinking fund. The first payment, x rands, was made in the fund on 1 January 2018 and thereafter the same amount was deposited on the first day of every month. The last payment was made on 1 December 2022.

On 31 December 2022 the school bought a new bus and used the trade-in value of the old bus as a deposit.

Calculate the monthly payment into the sinking fund. (6)

[14]

QUESTION 8

8.1 Determine $f^{/}(x)$ from first principles if $f(x) = 1 - x^2$. (5)

8.2 Determine:

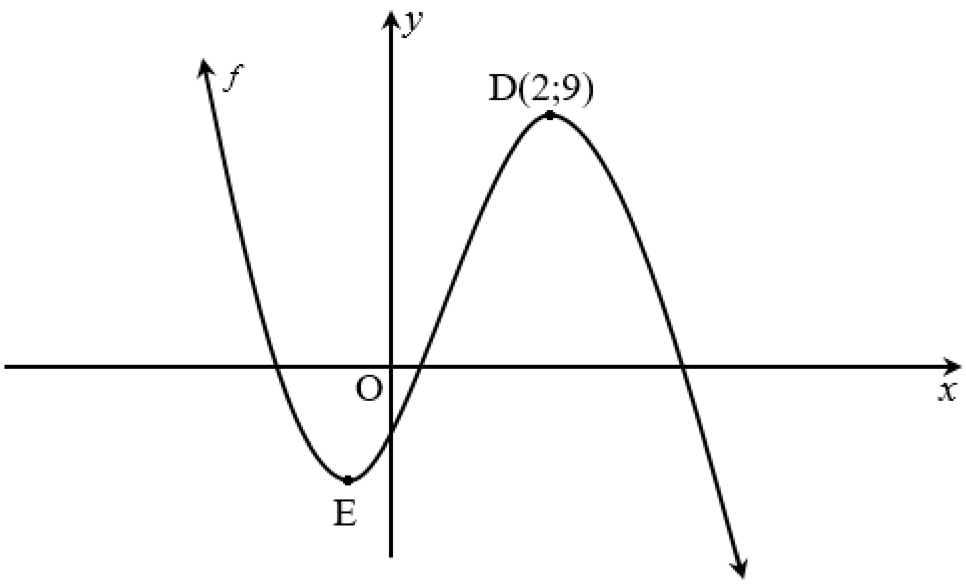
8.2.1 $D_x\left(x - \frac{1}{x}\right)^2$ (3)

8.2.2 $\frac{dy}{dx}$ if $y = \frac{x^5}{10} - \frac{2}{\sqrt{x}}$ (3)

[11]

QUESTION 9

The diagram below shows the graph of
$f(x) = -2x^3 + ax^2 + bx - 3$.
D(2 ; 9) and E are the turning points of f.



9.1 Determine the values of a and b. (5)

9.2 If $f(x) = -2x^3 + 5x^2 + 4x - 3$, calculate the coordinates for E. (3)

9.3 Determine the values of x for which:

9.3.1 $f^/(x) < 0$ (2)

9.3.2 The graph of f is concave down (3)

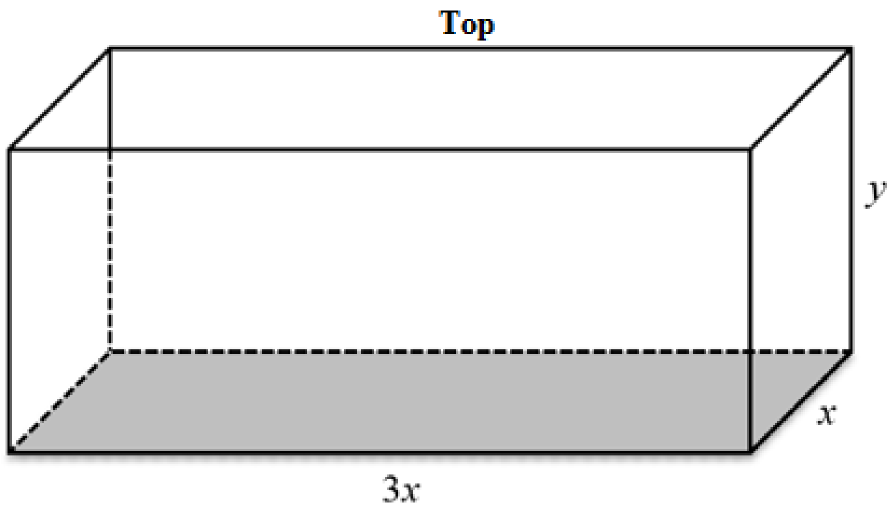
9.4 Determine the equation of the tangent to the graph of f at $P(-1; 0)$, in the form $y = mx + c$. (4)

[17]

QUESTION 10

The wooden box in the diagram is a rectangular prism and it is open at the top. The dimensions of the base are $3x$ metres by x metres and the height is y metres.

The total surface area is $147\ m^2$.



10.1 Show that $y = \frac{147-3x^2}{8x}$. (2)

10.2 Calculate the value of x for which the volume of the box is a maximum. (5)

[7]

QUESTION 11

11.1 A survey was carried out among 210 people to determine whether they prefer watching rugby or soccer on TV. The results are shown in the contingency table below.

	WATCH SOCCER	**WATCH RUGBY**	**TOTAL**
Female	72	a	120
Male	54	36	90
Total	b	84	210

11.1.1 Determine the values of a and b. (2)

11.1.2 Give the probability that an individual chosen at random is a female preferring to watch soccer. (2)

11.1.3 Are the events 'being male' and 'watch rugby' independent? Justify your answer with calculations. (4)

11.2 The password of a computer consists of 3 letters and 3 digits, in that order.
All 10 digits and 26 letters of the alphabet may be used, without repetition.

Example:

A	B	C	1	2	3

11.2.1 How many different passwords can be formed out of the 10 digits and 26 letters? (2)

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11.2.2 Calculate the probability that the first letter of a password formed is a vowel and the last digit of the password is a factor of 9. (4)

[14]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$

$A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $\quad r \neq 1$

$S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$$cos(\alpha + \beta) = cos\,\alpha\,.\,cos\,\beta - sin\,\alpha\,.\,sin\,\beta$$

$$cos(\alpha - \beta) = cos\,\alpha\,.\,cos\,\beta + sin\,\alpha\,.\,sin\,\beta$$

$$cos\,2\,\alpha = \begin{cases} cos^2\,\alpha - sin^2\,\alpha \\ 1 - 2\,sin^2\,\alpha \\ 2\,cos^2\,\alpha - 1 \end{cases}$$

$$sin\,2\,\alpha = 2\,sin\,\alpha\,.\,cos\,\alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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