Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2023

TECHNICAL MATHEMATICS P2

MARKS: 150

TIME: 3 hours

This question paper consists of 16 pages, including a 2-page information sheet and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.

6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.

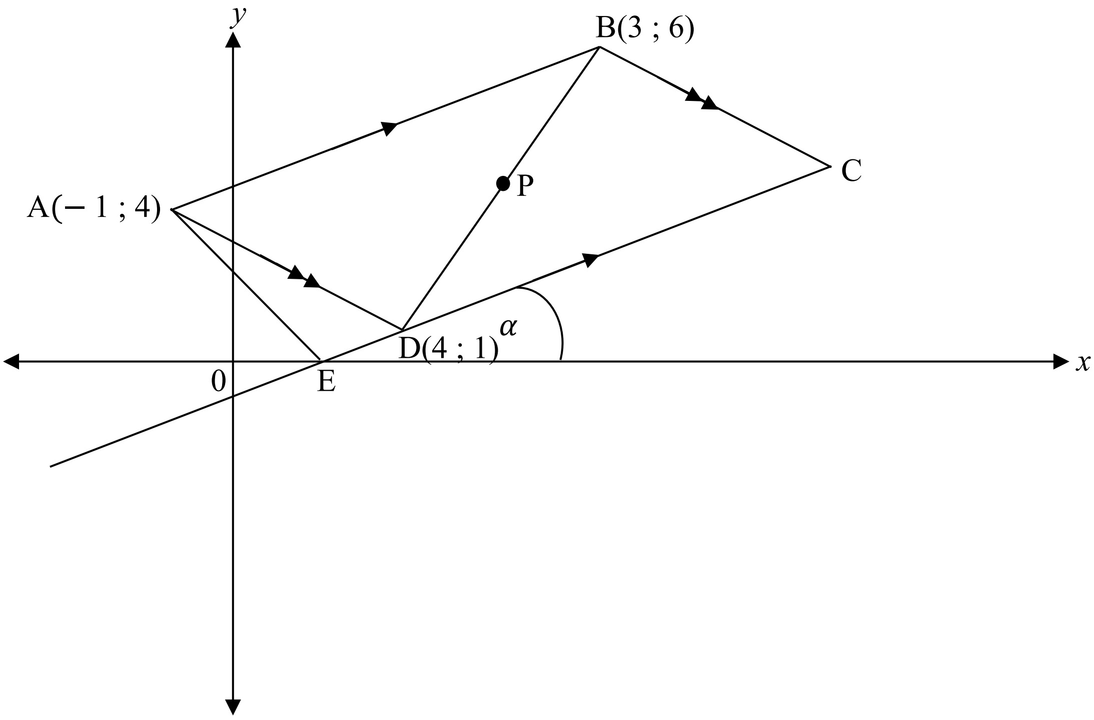
7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

In the diagram below ABCD is a parallelogram with vertices A(− 1 ; 4), B(3 ; 6), C and D(4 ; 1), where E is the x-intercept of the line CD extended and α is the inclination angle of line CD.

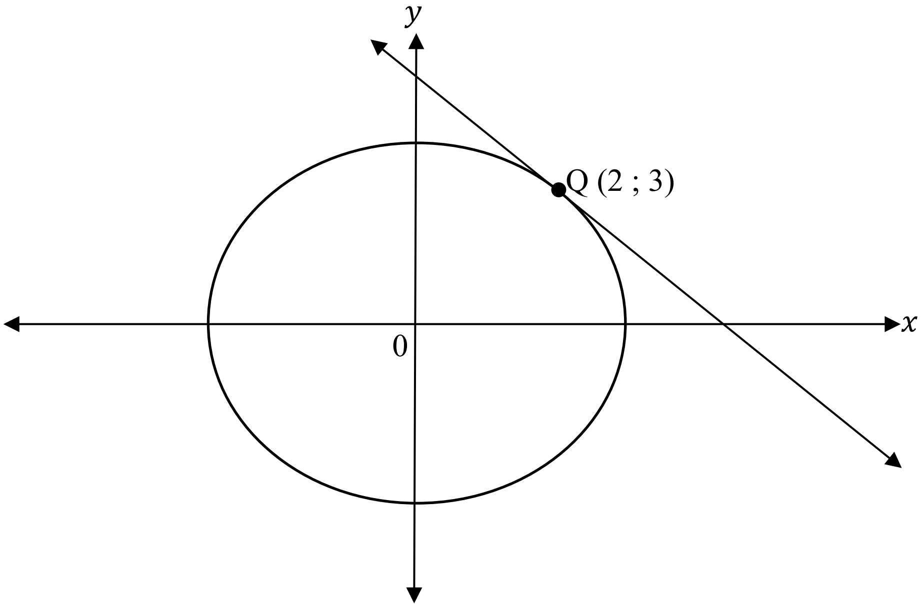


Determine:

1.1	The gradient of AB	(2)
1.2	The coordinates of P, the midpoint of BD	(2)
1.3	The equation of CD	(3)
1.4	The coordinates of E, if E is the x-intercept of line CD extended	(2)
1.5	The inclination angle of line AE	(4)
1.6	The size of $A\hat{E}D$	(4)
		[17]

QUESTION 2

2.1 The diagram below shows a circle with equation $x^2 + y^2 = 13$ and a tangent line touching at point Q(2 ; 3).



2.1.1 Determine the gradient of OQ. (2)

2.1.2 Hence, or otherwise, determine the equation of the tangent line in the form $y = \ldots$ (3)

2.2 Given:

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Sketch the given graph, in your SPECIAL ANSWER BOOK, clearly indicating all intercepts with the axis. (3)

[8]

QUESTION 3

3.1 Given: P = 128,2° and S = 204,7°

Determine the following:

3.1.1 $\cos(\text{P} + \text{S})$ (2)

3.1.2 $\text{cosec}(\text{S} - \text{P})$ (3)

3.2 If $\cos 75° = k$, express the following in terms of k.

3.2.1 $\sin 15°$ (3)

3.2.2 $\tan 255°$ (3)

3.3 Solve for θ, rounded off to ONE decimal digit, if $\theta \in (90° ; 180°)$:

$\sec\theta = -1{,}583$ (4)

[15]

QUESTION 4

4.1 Simplify:

$$\text{cosec}^2(180° + \theta) + \frac{\sin(180° - \theta)\,.\cot^2(180° + \theta)\,.\sin 270°}{\cos(360° - \theta).\tan(180° + \theta)}$$

(9)

4.2 Prove that:

$$\frac{1}{(1 - \sin\theta)(1 + \sin\theta)} = \sec^2\theta$$

(2)

[11]

QUESTION 5

Given $f(x) = \tan x$ and $g(x) = \sin x - 1$; $x \in (0°; 360°)$

5.1 On the same axes, given in your SPECIAL ANSWER BOOK, draw the graphs of $f(x) = \tan x$ and $g(x) = \sin x - 1$. Clearly show the intercepts with the axes, turning points and asymptotes. (7)

5.2 Write down the range of g. (2)

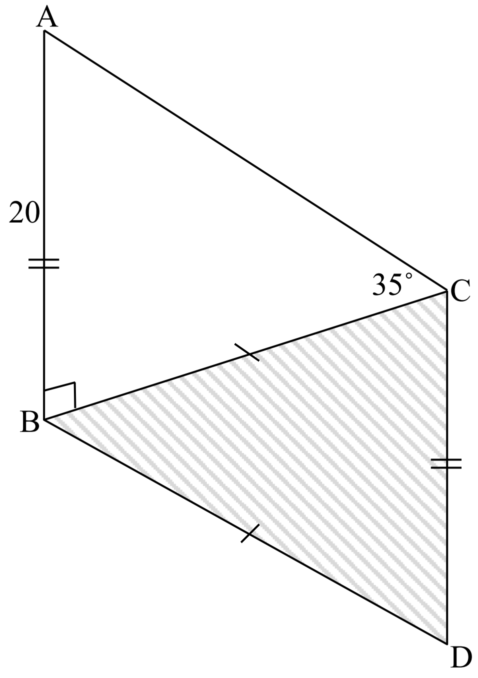
5.3 State the period of f. (1)

5.4 Use your graphs to state for which values of $x \in (90°; 270°)$, that $f(x).g(x) < 0$. (2)

[12]

QUESTION 6

AB is a tower, anchored at point C, forming an angle of elevation 35°.
B, C and D are in the same horizontal plane.
AB = CD = 20 units and BC = BD.



6.1 Determine the length BC. (3)

6.2 Determine the size of $C\hat{B}D$, rounded off to the nearest degree. (4)

6.3 Determine the area of the isosceles Δ BCD. (3)

[10]

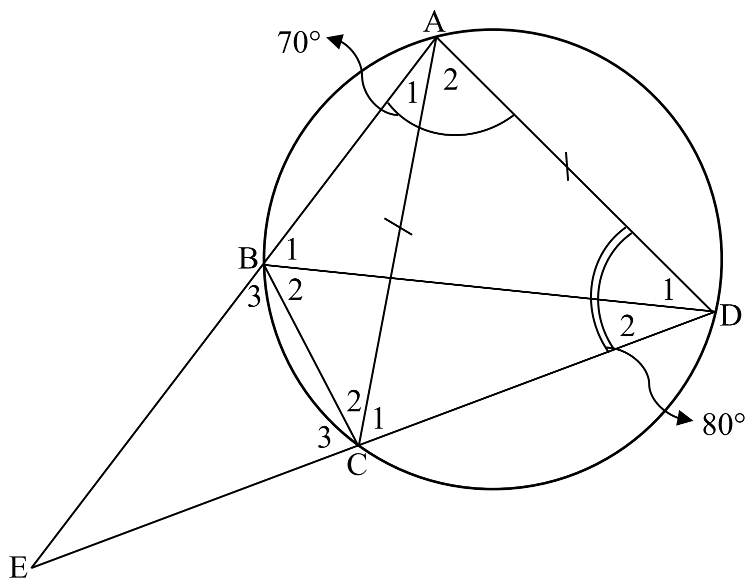
Give reasons for your statements in QUESTIONS 7, 8 and 9.

QUESTION 7

7.1 Complete the following theorem statement:

"The exterior angle of a cyclic quadrilateral is … to the interior opposite angle." (1)

7.2 ABCD is a cyclic quadrilateral with AD = AC and $A\hat{D}C = 80°$. AB and DC are produced to meet at E.



7.2.1 Name, with reasons, three other angles equal to 80°. (6)

7.2.2 If it is given that $B\hat{A}D = 70°$, calculate with reasons, the sizes of:

(a) $\hat{C}_3$ (2)

(b) $\hat{E}$ (2)

(c) $\hat{D}_1$ (3)

7.2.3 Prove that AD is a tangent to the circle DBE at D. (2)

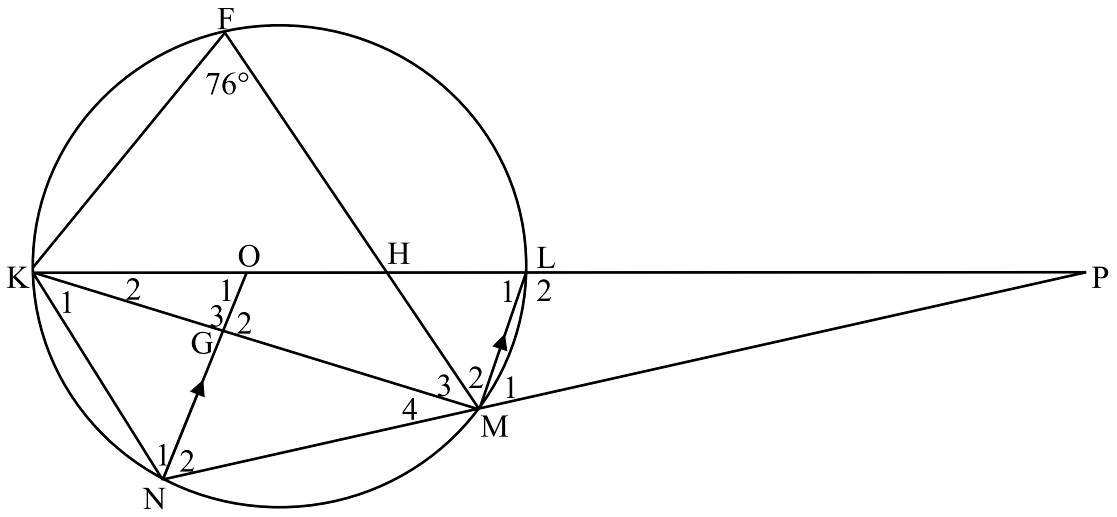
[16]

QUESTION 8

8.1 Complete the following theorem statement:

"The angle subtended by an arc at the centre of a circle is … the size of the angle subtended by the same arc at the circumference of the circle." (1)

8.2 O is the centre of the circle and diameter KL is produced to meet NM produced at P. ON || LM and $\hat{F} = 76°$.



Calculate the following, with reasons:

8.2.1 $\hat{L}_1$ (2)

8.2.2 $\hat{O}_1$ (2)

8.2.3 $\hat{M}_4$ (2)

8.2.4 $\hat{N}_1 + \hat{N}_2$ (2)

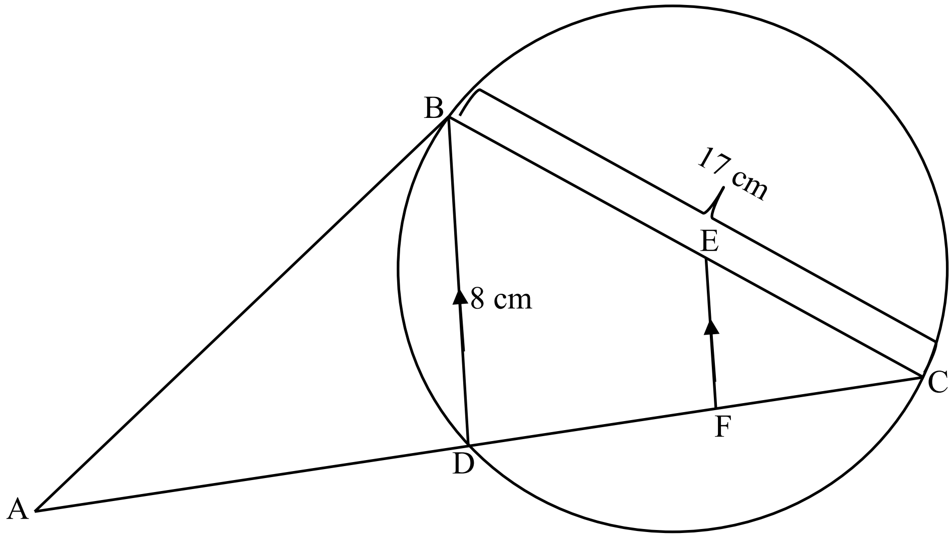
[9]

QUESTION 9

9.1 Complete the following theorem statement:

"A line drawn parallel to one side of a triangle … the other two sides proportionally." (1)

9.2 In the diagram, BC = 17 cm, where BC is a diameter of the circle. The length of the chord BD is 8 cm. The tangent at B meets CD produced at A.



9.2.1 Calculate, with reasons, the length of DC. (4)

9.2.2 E is a point on BC, such that BE : EC = 3 : 1. EF is parallel to BD with F on DC.

(a) Calculate, with reasons, the length of CF. (4)

(b) Prove that ΔBAC ||| ΔFEC. (5)

(c) Determine the length of AD. (4)

[18]

QUESTION 10

10.1 A train moving on a circular track with a diameter of 14 6425 km, takes 50 minutes to complete one revolution.

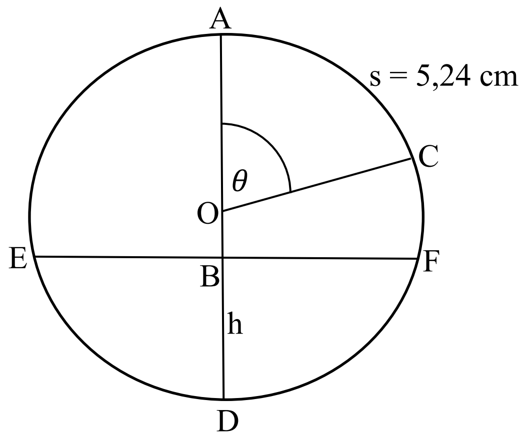
10.1.1 Determine the rotational frequency per minute. (1)

10.1.2 Convert the length of the diameter to metre. (1)

10.1.3 Hence, calculate the circumferential velocity of the train in metres per min. (3)

10.2 A wheel rotates at 15 revolutions per second. Calculate the angular velocity of the wheel in radians per minute. (4)

10.3 The circle below with centre O, has a chord EF of length 80 mm and diameter AD equals 10 cm. Arc AC subtends a central angle θ.



10.3.1 Calculate the height of the minor segment, h (BD), in cm. (5)

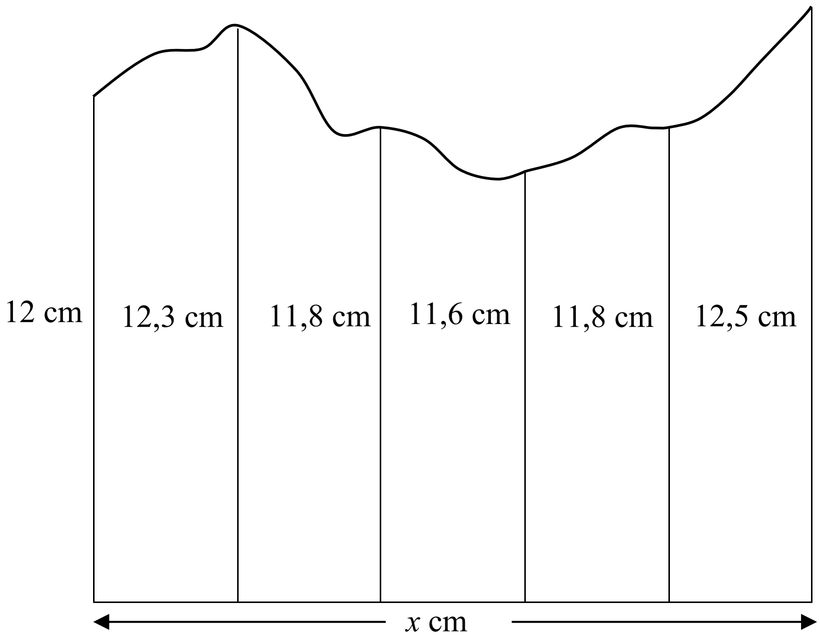
10.3.2 If the arc length, AC, of the circle is 5,24 cm, calculate the central angle, θ, to the nearest degree. (4)

10.3.3 Hence, determine the area of the minor sector AOC of the circle. (3)

[21]

QUESTION 11

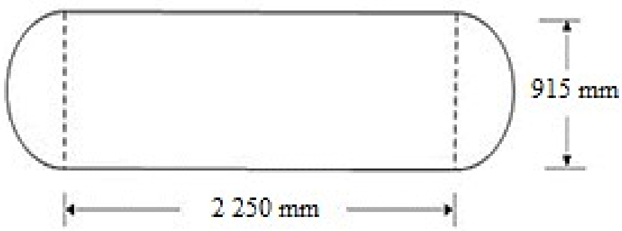
11.1 The irregular shape, with area 149,38 cm^2, below has a straight side of length x cm and has been divided into 5 equal parts. The ordinates dividing the parts are: 12 cm, 12,3 cm, 11,8 cm, 11,6 cm, 11,8 cm and 12,5 cm respectively.



Determine the value of x, the length of the side of the irregular shape. (4)

11.2 The picture below is of a Liquefied Petroleum Gas (LPG) storage tank.
The diagram next to it shows the diameter of the tank as 915 mm.
The middle part of the tank is made of a cylinder with two identical hemispheres at each end.
The height of the cylindrical tank is 2 250 mm.
The tank is filled with propane gas.
It is further given that:

- 1 m = 100 cm
- 1 litre = 1 000 cm^3
- 1 kg = 1,96 litre of propane gas
- 1 ton = 1 000 kg
- Volume of a cylinder = $\pi r^2 h$
- Volume of a hemisphere = $\frac{1}{2} \times \frac{4}{3}\pi r^3$



11.2.1 Convert the measurements in the diagram to centimetres. (1)

11.2.2 Hence, determine the volume, in litres, of the propane gas inside the storage tank. (4)

11.2.3 The total weight of the storage tank is 0,5 ton.
The tare weight of the tank is the weight of the empty tank.
Calculate what percentage is the tare weight of the storage tank. (4)

[13]

TOTAL: 150

INFORMATION SHEET

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\frac{b}{2a}$ $\qquad y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b \ \ a > 0, a \neq 1$ and $b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$

$A = P(1 + i)^n$ $\qquad A = P(1 - i)^n$ $\qquad i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = tan\theta$ $\qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$\int kx^n dx = \frac{kx^{n+1}}{n + 1} + C \text{ , } n, k \in \mathbb{R}$ with $n \neq -1$ and $k \neq 0$

$\int \frac{k}{x} dx = k \ln(x) + C, x > 0 \ and \ k \in \mathbb{R}; k \neq 0$

$\int ka^{nx} dx = \frac{ka^{nx}}{nlna} + C, a > 0; a \neq 1 \ and \ k, a \ \in \mathbb{R}; k \neq 0$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ where $n =$ rotation frequency

Circumferential velocity $= v = \pi Dn$ where $D =$ diameter and $n =$ rotation frequency

Circumferential velocity $= v = \omega r$ where $\omega =$ angular velocity and $r =$ radius

Arc length $= s = r\theta$ where = radius and $\theta =$ central angle in radians

$4h^2 - 4dh + x^2 = 0$ where $h =$ height of segment, $d =$ diameter of circle and $x =$ length of chord

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where $r =$ radius, $s =$ arc length and $\theta =$ central angle in radians

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.cosA \qquad Area = \frac{1}{2}ab.sinC$$

$$sin^2\theta + cos^2\theta = 1 \qquad 1 + tan^2\theta = sec^2\theta \qquad cot^2\theta + 1 = cosec^2\theta$$

$A_T = a\left(\frac{O_1 + O_n}{2} + O_2 + O_3 + O_4 + \ldots + O_{n-1}\right)$ where $a =$ width of equal parts, $O_i = i^{th}$ ordinate and $n =$ number of ordinates

OR

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ where $a =$ width of equal parts, $m_i = \frac{O_i + O_{i+1}}{2}$ and $n =$ number of ordinates; $i = 1;2;3;\ldots;n-1$