



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2010

MATHEMATICS – PAPER 1

MARKS: 150

TIME: 3 hours



This question paper consists of 10 pages, a diagram sheet and a formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions. Answer ALL the questions.
2. Show clearly ALL calculations, diagrams, graphs, etcetera, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. Answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number the answers correctly according to the numbering system used in this question paper.
6. Diagrams are NOT necessarily drawn to scale.
7. It is in your own interest to write legibly and to present your work neatly.
8. An information sheet with formulae is attached.
9. A diagram sheet is supplied for QUESTION 11. Write your name in the space provided and then hand the diagram sheet in with your ANSWER sheets.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(2x - 5) = 0$ (2)

1.1.2 $(3 - x)(2x - 1) = 1$ (5)

1.1.3 $x^2 - 2x \leq 15$ (4)

1.2 Calculate the coordinates of the points of intersection of the graphs of $y = x^2 - 2x - 3$ and $y = -4x + 5$. (5)

1.3 Simplify the following without using a calculator. Show all working.

$$\frac{6^{2010} \times 10^{2011}}{4^{2010} \times 15^{2009}}$$

(4)
[20]**QUESTION 2**

Consider the following sequence: 11 ; 7 ; 3 ; -1 ; -5 ; ...

2.1 If this pattern continues consistently, determine the n^{th} term of the sequence (3)

2.2 Show that -293 is a term of the sequence. (2)

2.3 Using the given sequence above, a new sequence is formed as follows:

The first term is the same as the first term of the given sequence.

The second term equals the sum of the first 2 terms of the given sequence.

The third term is the sum of the first 3 terms of the given sequence.

The fourth term is the sum of the first 4 terms of the given sequence and so on.

Write down the first 4 terms of the new sequence. (1)

2.4 Determine an expression for the n^{th} term, T_n , for this new sequence if it continues in the way described in QUESTION 2.3. (5)2.5 An arithmetic sequence: $a ; a + d ; a + 2d ; a + 3d$ is given.A new sequence is formed with a as first term, the sum of the first 2 terms of the given sequence as second term, the sum of the first 3 terms of the given sequence as third term and the sum of the first 4 terms of the given sequence as fourth term and it continues in this way.

Prove that the new sequence is a quadratic sequence. (4)

[15]

QUESTION 3

- 3.1 Express $-4 - 8 - 16 - 32$ in sigma notation. (3)
- 3.2 The geometric series $30 + 24 + 19,2 + 15,36 + \dots$ is given.
- 3.2.1 Calculate the common ratio. (1)
- 3.2.2 Calculate the sum of the first eight terms. (3)
- 3.2.3 Calculate the limit toward which the series converges. (3)
- 3.2.4 The terms of the series represent the mass of plastic (in kg) recovered from a quantity of waste material during successive repetitions of an industrial recovery process. For the process to be economical at least 3,6 kg must be recovered during a repetition. Calculate the maximum number of times the process can be repeated before it becomes uneconomical. (5)
- [15]**

QUESTION 4

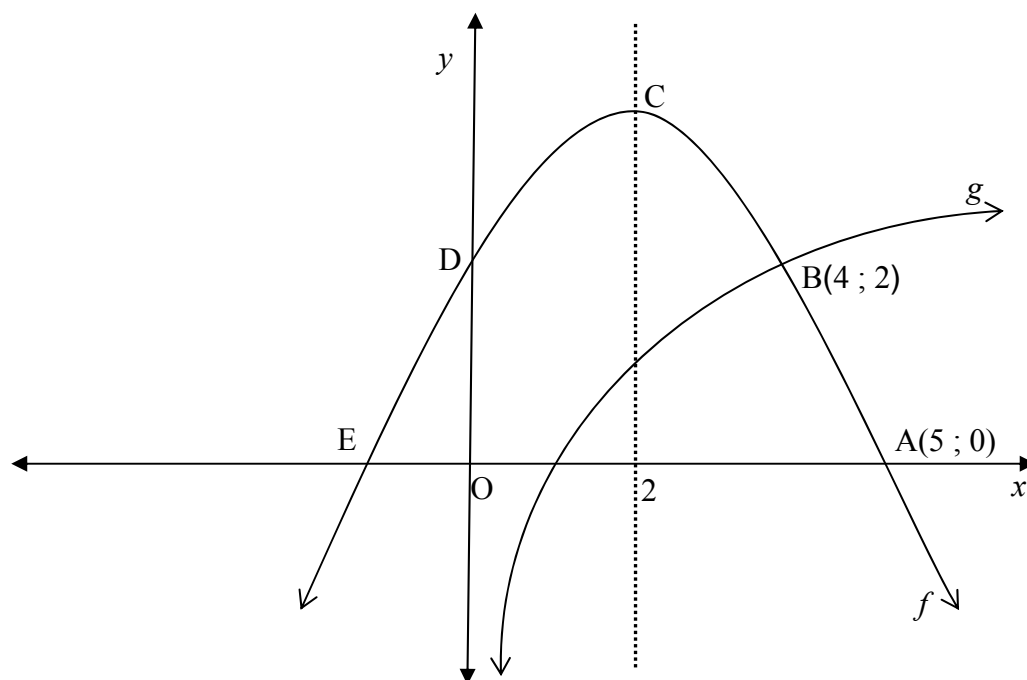
Given $g(x) = \frac{-3}{x+2} + 1$

- 4.1 Write down the equations of the asymptotes of g . (2)
- 4.2 Calculate the intercepts of g with the axes. (3)
- 4.3 Draw the asymptotes and make a neat sketch of g . Indicate the intercepts with the axes on your sketch. (3)
- 4.4 Write down the equation of the vertical asymptote of $h(x) = g(x + 2)$. (1)
- [9]**

QUESTION 5

In the sketch, the graphs of the functions given by $f(x) = ax^2 + bx + c$ and $g(x) = \log_m x$ are represented.

$A(5; 0)$ is an x -intercept of f and $x = 2$ is the axis of symmetry of f .
 $B(4; 2)$ is the point of intersection of f and g .

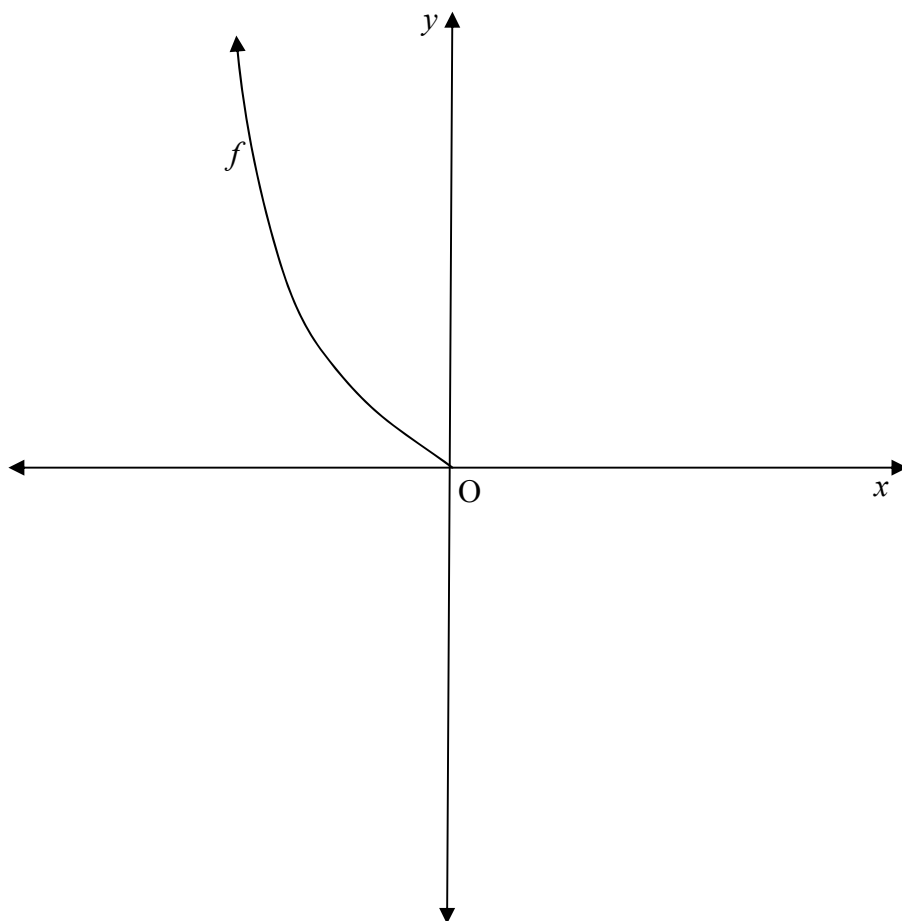


- 5.1 Determine the value of m . (2)
- 5.2 Write down the domain of g . (1)
- 5.3 Determine the equation of g^{-1} , in the form $y = \dots$. (2)
- 5.4 Write down the equation of h if h is obtained by shifting g^{-1} 2 units to the left. (1)
- 5.5 Determine the equation of the parabola f and hence show that $a = -\frac{2}{5}$; $b = \frac{8}{5}$ and $c = 2$. (4)
- 5.6 Write f in the form $f(x) = a(x - p)^2 + q$. (3)
- 5.7 Use your answer to QUESTION 5.6 and the graph to explain why the equation $f(x) - 4 = 0$ will have no real roots. (3)
- 5.8 Use the sketch to determine the values of x for which $f(x) \cdot g(x) > 0$. (2)

[18]

QUESTION 6

In the diagram, the graph of $f(x) = x^2$ is drawn for $x \leq 0$.



Use the given graph to answer the following questions:

- 6.1 Write down the equation of the inverse of f . (2)
- 6.2 Write down the domain of f^{-1} , the inverse of f . (1)
- 6.3 Write down the range of f^{-1} . (1)
- 6.4 Explain why the inverse of f is a function when the domain of f is restricted as in the given graph. (2)
- [6]**

QUESTION 7

- 7.1 R10 000 is invested into an account offering a nominal interest rate of 12% per year, compounded monthly.
- 7.1.1 Calculate how long it will take to have R15 000 in the account. (4)
- 7.1.2 Calculate the effective interest rate. (3)
- 7.2 A father deposited a sum of money into a savings account on the 1st January to enable his son to make 36 withdrawals of R500 per month at the end of each month for 3 years.
- 7.2.1 If the account pays interest at a rate of 8% per annum compounded monthly, what was the amount deposited? (3)
- 7.2.2 What will be the balance in the account after the 6th withdrawal of R500? (3)
- 7.2.3 Immediately after the sixth withdrawal of R500, the interest rate went up to 12% per annum, compounded monthly. What equal monthly amount can the son draw at the end of each of the next 30 months? (3)
- [16]**

QUESTION 8

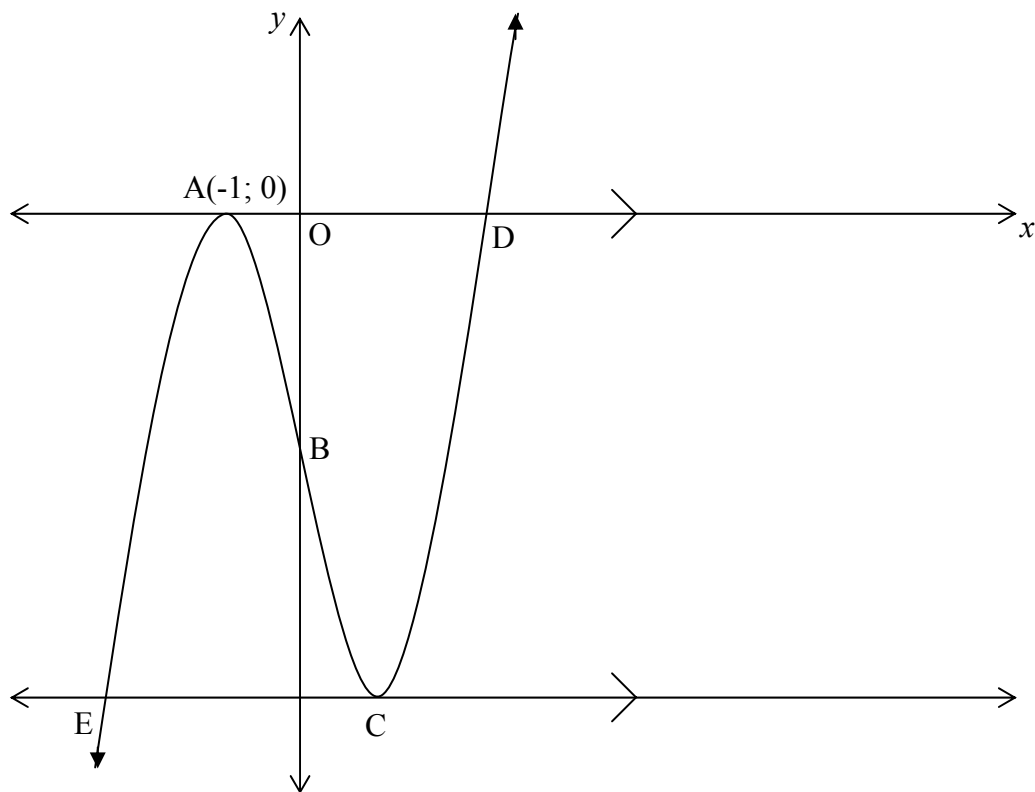
- 8.1 What happens to $5h$ as $h \rightarrow 2$? (1)
- 8.2 Determine from first principles the derivative of f with respect to x if:
- $$f(x) = \frac{5}{x} \quad (5)$$
- 8.3 Determine $D_x [8x^4 - 6\sqrt{x} + \frac{5}{x}]$ (4)
- [10]**

QUESTION 9

The sketch represents the function $f(x) = x^3 - 3x^2 - 9x - 5$.

A and C are the local maximum and minimum respectively and A is the point $(-1; 0)$.

B is the y -intercept and D is an x -intercept. $EC \parallel AD$

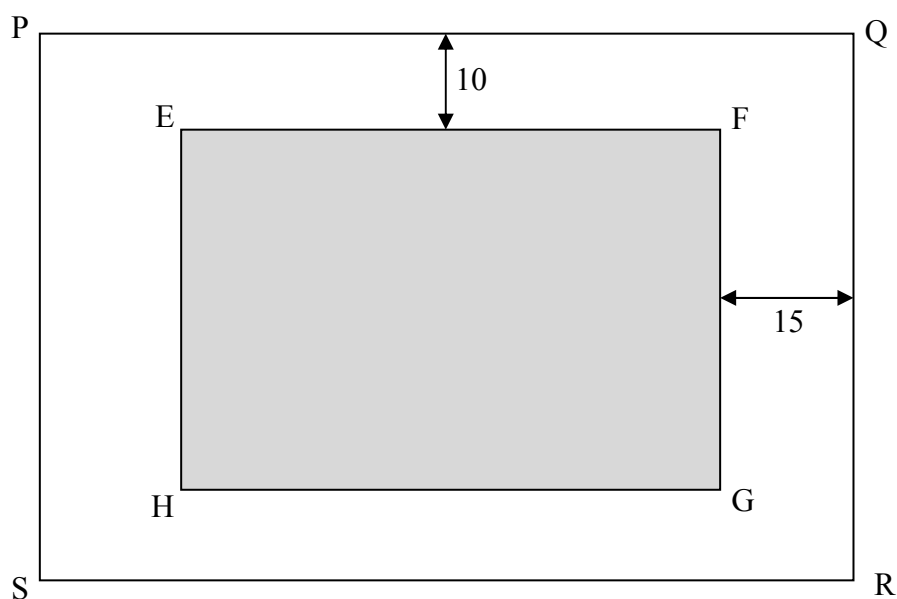


- 9.1 Determine the gradient of the tangent to the curve of f at D. (6)
- 9.2 Determine the equation of the straight line CE. (5)
- 9.3 For which values of x is $f(x)$ decreasing? (2)
- 9.4 Determine the average gradient between B and D. (3)
- 9.5 Determine the x -coordinate of the point at which the concavity of the graph changes and hence state the values of x for which the graph is concave downward. (3)

[19]

QUESTION 10

A page PQRS has to contain printed matter EFGH.



Open spaces of 10 mm should be left at the top and bottom of the page and 15 mm spaces on each side.

The printed part must cover $5\,400\text{ mm}^2$.

- 10.1 If $HG = x\text{ mm}$, write down the lengths of SR and PS in terms of x and hence show that the area of $PQRS$ is given by $6000 + 20x + 162000x^{-1}$. (4)
- 10.2 Determine the length of SR such that the area of the page $PQRS$ will be a minimum. (5)

[9]

QUESTION 11

A retailer needs to buy a certain quantity of television sets.

He can buy either LCD sets for R3 000 each or plasma sets for R6 000 each.

R90 000 has been budgeted for the purchase of the television sets.

However, at least 4 of each type must be purchased by the retailer.

The above information can be expressed algebraically by 3 constraint inequalities.

If x is the number of LCD television sets and y the number of plasma television sets, then one of the inequalities is: $x \geq 4$.

Use the information above, and the diagram sheet supplied, to answer the following:

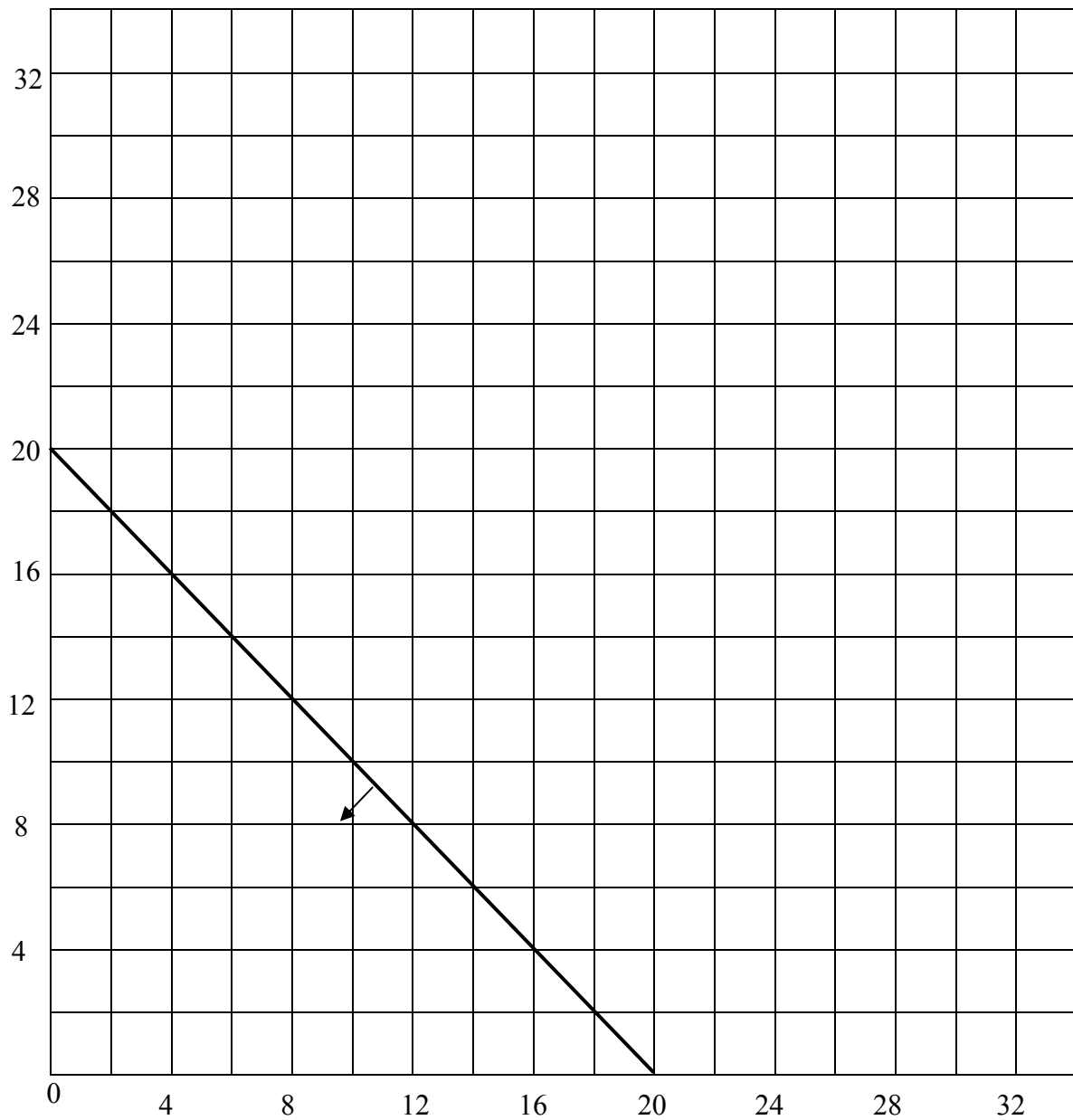
- 11.1 Write down the two inequalities defining the remaining constraints. (2)
- 11.2 A fourth constraint is shown on the diagram which is supplied. Represent the other three constraint inequalities graphically on the same diagram and shade the feasible region. (5)
- 11.3 If the retailer makes a profit of R400 on each LCD set and R1 000 on each plasma set and if he sells all his stock:
- 11.3.1 Write down an equation for profit P . (1)
- 11.3.2 The objective is to maximise profit. Draw a search line in optimum position on the diagram which is supplied. (2)
- 11.3.3 Determine the maximum profit. (3)
- [13]

TOTAL: 150

NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 11



FORMULA SHEET: MATHEMATICS
FORMULEBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ -2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

