Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE/ *GRADE* 12

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MATHEMATICS P1 (*WISKUNDE V1*) MEMORANDUM

MARKS/ *PUNTE*: **150**

This memorandum consists of 11 pages.
Hierdie memorandum bestaan uit 11 bladsye.

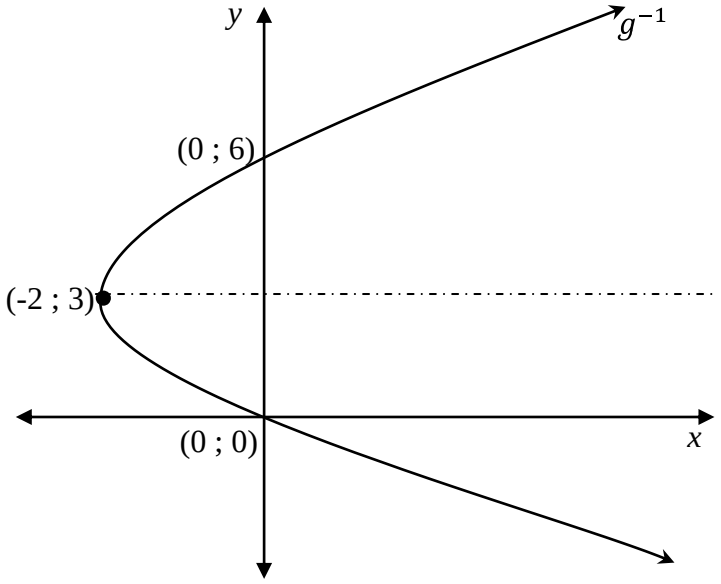
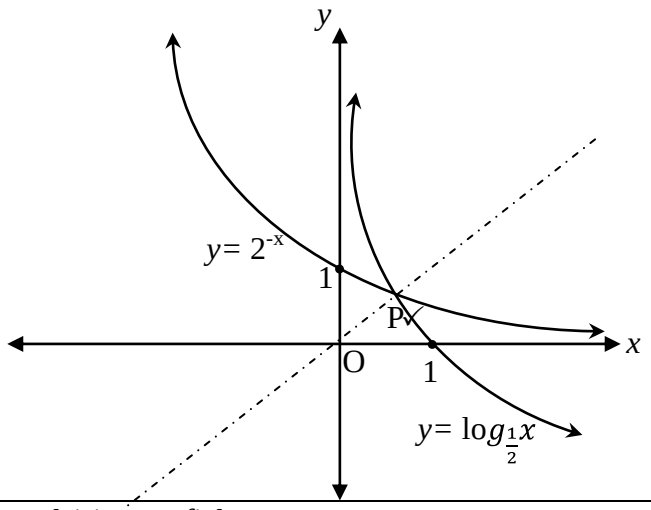
QUESTION/*VRAAG* 1			
1.1	1.1.1	$a + \frac{1}{a} = 2$ $a^2 + 1 = 2a$ $a^2 - 2a + 1 = 0$ $\therefore (a - 1)(a - 1) = 0$ $\therefore a = 1$	✓simplification *vereenvoudiging* ✓standard form *standaard vorm* ✓factorisation *faktorisering* ✓value of a *waarde van a* (4)
	1.1.2	$1 - 2x + \sqrt{5x - 1} = 0$ $\therefore \sqrt{5x - 1} = 2x - 1$ $\therefore 5x - 1 = 4x^2 - 4x + 1$ $\therefore 4x^2 - 9x + 2 = 0$ $\therefore (4x - 1)(x - 2) = 0$ $\therefore x = \frac{1}{4}$ or/*of* $x = 2$ Correct solution/*Korrekte oplossing*: $x = 2$	✓transpose and square both sides *bring om en kwadreer beide kante* ✓standard form *standaard vorm* ✓ factorise *faktoriseer* ✓ both values of x *beide waardes van x* ✓ correct value of x *korrekte waarde van x* (5)
	1.1.3	$\frac{x}{x-3} \leq 2$ $\therefore \frac{x}{x-3} - 2 \leq 0$ $\therefore \frac{x-2(x-3)}{x-3} \leq 0$ $\therefore \frac{x-2x+6}{x-3} \leq 0$ $\therefore \frac{-x+6}{x-3} \leq 0$ $\therefore x < 3$ or/*of* $x \geq 6$	✓simplification *vereenvoudiging* ✓simplification *vereenvoudiging* ✓✓ one mark for each value of x *een punt vir elke waarde van x* (4)
1.2		$y - 2x + 1 = 0$ $\therefore y = 2x - 1$ (1) Substitute/*Vervang* (1) in $xy = 2y + x^2 + 3x - 10$ $\therefore x(2x - 1) = 2(2x - 1) + x^2 + 3x - 10$ $\therefore 2x^2 - x = 4x - 2 + x^2 + 3x - 10$ $\therefore x^2 - 8x + 12 = 0$ $\therefore (x - 6)(x - 2) = 0$ $\therefore x = 6$ or/*of* $x = 2$ and/*en* $y = 2(6) - 1$ or/*of* $y = 2(2) - 1$ $= 11$ $= 3$	✓$y = 2x - 1$ ✓substitution *vervanging* ✓standard form *standaard vorm* ✓both values of x *beide waardes van x* ✓both values of y *beide waardes van y* (5)

1.3	$\frac{10^n + 4.2^n}{5^{2n} + 4.5^n} = \frac{(2.5)^n + 4.2^n}{5^n.5^n + 4.5^n}$ $= \frac{2^n.5^n + 4.2^n}{5^n.5^n + 4.5^n}$ $= \frac{2^n(5^n + 4)}{5^n(5^n + 4)}$ $= \frac{2^n}{5^n}$	✓write in factors *skryf in faktore* ✓simplification *vereenvoudiging* ✓factorise *faktorisering* ✓simplification *vereenvoudiging* ✓answer *antwoord* (5)
1.4	$\sqrt[n]{\frac{10^n + 4.2^n}{5^{2n} + 4.5^n}} = \sqrt[n]{\frac{2}{5}^n}$ $= \frac{2}{5}$	✓equate values *vergelyk waardes* ✓answer *antwoord* (2)
		[25]
QUESTION/*VRAAG* 2		
2.1	1 ; 4 ; 11 ; 22 ; 37 ; **56** ; **79**	✓✓1 mark for each term *1 punt vir elk term* (2)
2.2	1 4 11 22 37 56 79 3 7 11 15 19 23 4 4 4 4 4 $\therefore T_n = an^2 + bn + c$ and/*en* $2a = 4$ $\therefore a = 2$ $\therefore T_n = 2n^2 + bn + c$ n = 1 $T_1 = 2(1)^2 + b(1) + c$ $\therefore 1 = 2 + b + c$ (1) $\therefore -1 = b + c$ n = 2 $T_2 = 2(2)^2 + b(2) + c$ $4 = 8 + 2b + c$ $-4 = 2b + c$ (2) $\therefore$ (2) – (1): $-3 = b \therefore b = -3$ (1) $-1 = -3 + c$ $\therefore c = 2$ $\therefore T_n = 2n^2 - 3n + 2$	✓Identification of 2nd difference *Identifisering van 2de verskil* ✓value of a *waarde van a* ✓1st equation *1ste vergelyking* ✓2nd equation *2de vergelyking* ✓value of c *waarde van c* ✓formula *formule* (6)
		[8]

QUESTION/*VRAAG* 3			
3.1	3.1.1	$\sum_{k=1}^{n} 4 - 3k = -125$ $T_1 = 1$ $T_2 = -2$ $T_3 = -5$ $\therefore$ sequence is arithmetic/*ry is meetkundig*	✓1st 3 terms *1ste 3 terme* ✓answer *antwoord* (2)
	3.1.2	d = -3	✓ value of d *waarde van d* (1)
	3.1.3	$S_n = \frac{n}{2}[2a + n - 1\ d]$ $\therefore -125 = \frac{n}{2}[2\ 1 + n - 1\ -3\ ]$ $-125 = \frac{n}{2}(2 - 3n + 3)$ $-250 = n(-3n + 5)$ $-250 = -3n^2 + 5n$ $\therefore 3n^2 - 5n - 250 = 0$ $\therefore (3n + 25)(n - 10) = 0$ $\therefore n = \frac{-25}{3}$ or/*of* n = 10 $\therefore$ n = 10 $\quad n \in \mathbb{N}$	✓substitution in correct formula *vervanging in korrekte formule* ✓simplification *vereenvoudiging* ✓standard form *standaard vorm* ✓factorization *faktorisering* ✓correct value of n *korrekte waarde van n.* (5)
3.2		$1 + (0,2) + (0,2)^2 + (0,2)^3$ a = 1 , r = $\frac{0,2}{1}$ $= 0,2$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_8 = \frac{1[1-(0,2)^8}{1-0,2}$ $= \frac{1-(0,2)^8}{0,8}$ $= 1,25$	✓ expansion *uitbreiding* ✓ ratio *verhouding* ✓ substitution in correct formula *vervanging in korrekte formule* ✓ answer *antwoord* (4)
3.3	3.3.1	$r = \frac{(x-1)^2}{x-1}$ $= x - 1$ $-1 < r < 1$ $-1 < x - 1 < 1$ $0 < x < 2$	✓ ratio *verhouding* ✓ substitution *vervanging* ✓ answer *antwoord* (3)

	3.3.2	$\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3$ $\therefore r = \frac{\frac{1}{3}^2}{\frac{1}{3}}$ $\therefore r = \frac{1}{3}$ $S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}}$ $= \frac{\frac{1}{3}}{\frac{2}{3}}$ $= \frac{1}{3} \times \frac{3}{2}$ $= \frac{1}{2}$	✓ ratio *verhouding* ✓ substitution in correct formula *vervanging in korrekte formule* ✓answer *antwoord* (3)
			[18]
QUESTION/*VRAAG* 4			
4.1	4.1.1	$T_n = ar^{n-1}$ $T_n = 400\ (1 + \frac{10}{100})^{n-1}$ $T_7 = 400(1,1)^{7-1}$ $= 400(1,1)^6$ $= 708,62$ $\therefore 7^{th}$ month/7^{de} *maand* R708,62	✓substitution in the correct formula *vervanging in korrekte formule* ✓simplification *vereenvoudiging* ✓answer *antwoord* (3)
	4.1.2	$S_n = \frac{a(r^n-1)}{r-1}$ $S_7 = \frac{a(r^n-1)}{r-1}$ $\therefore S_7 = \frac{400[(1,1)^7-1]}{1.1-1}$ $= 3794,87$ Saves in 7 months/*Spaar in 7 maande* R3 794,87	✓substitution into correct formula *vervanging in korrekete formule* ✓simplification *vereenvoudiging* ✓answer *antwoord* (3)
4.2	4.2.1	$a = 1900\ (1 + \frac{0,075}{12})$ $r = \frac{T_2}{T_1} = \frac{1900\ (1+\frac{0,075}{12})^2}{1900\ (1+\frac{0,075}{12})^1}$ $\therefore r = 1 + \frac{0,075}{12}$ $S_n = \frac{a(r^n-1)}{r-1}$ $= \frac{1900\ (1+\frac{0,075}{12})\ (1+\frac{0,075}{12})^{240}-1}{1+\frac{0,075}{12}-1}$ = R1 058 663,93 Alternative/*Alternatief* a = 1911,875 r = 1,00525 $= \frac{1911,875(1,00625^{240}-1)}{-1,00625}$ = R1 058 663,93	✓ the value of a *die waarde van a* ✓ the value of r *die waarde van r* ✓ substitution in correct formula *vervanging in korrekte formule* ✓ answer *antwoord* (4)
	4.2.2	7,5%	✓ answer antwoord (1)
	4.2.3	$1+i_e = (1 + \frac{7,5}{1200})^{12}$ i_e= 0,0776 Effective annual interest rate = 7,76% *Effektiewe jaarlikse rentekoers* = 7,76%	✓ substitution *substitusie* ✓ Simplification *vereenvoudiging* ✓ Answer *antwoord* (3)
			[14]

QUESTION/*VRAAG* 5		
5.1	$y \in \mathbb{R}\ ; y \neq 5$	✓ answer *antwoord* (1)
5.2	$y = -3x + 2$ (1) $y = \frac{-3}{x+1} + 5$ (2) $\therefore (-3x + 2) = \frac{-3}{x+1} + 5$ $\therefore -3x^2 - x + 2 = -3 + 5(x + 1)$ $-3x^2 - 6x = 0$ $-3x(x + 2) = 0$ $\therefore x = 0$ or/*of* $x = -2$ $\therefore y = -3(0) + 2$ or/*of* $y = -3(-2) + 2$ $= 2$ $= 8$ (0 ; 2) and/*en* (-2 ; 8)	✓ equating f(x) and g(x) *vergelyk f(x) en g(x)* ✓ simplification *vereenvoudiging* ✓ standard form *Standaard vorm* ✓ for both x values Vir beide x waardes ✓ for both y values *vir beide y waardes* ✓✓for each correct coordinates 1 mark *vir elke regte koördinaat 1 punt* (7)
5.3	$f(x) = \frac{-3}{x+1} + 5 \Rightarrow h\ x = \frac{3}{x+1} + 5$ reflection about asymptote: *refleksie in die asimptote*: $x = -1$ or/*of* reflection about asymptote: *refleksie in die asimptote*: $y = 5$	✓ for description *vir beskrywing* (1)
		[9]
QUESTION/*VRAAG* 6		
6.1	$y = a(x - p)^2 + q$ (3;-2): $y = a(x - 3)^2 - 2$ (0;0): $0 = a(0 - 3)^2 - 2$ $= 9a - 2$ $\therefore 9a = 2$ $\therefore a = \frac{2}{9}$ $g\ x = \frac{2}{9}(x - 3)^2 - 2$	✓ substituting (3;-2) ✓ *substitusie (0;0)* ✓ value of a *die waarde van* a (3)
6.2	B(6 ; 0)	✓ coordinate of B *koördinaat van B* (1)
6.3	g(x) is a one-too-many function; for every y-value there are two corresponding x-values. *g(x) is een-te-veel funksie; vir elke y-waarde is daar twee ooreenstemmende x-waardes*	✓ for answer/*vir antw.* ✓ substantiation/ *regverdiging* (2)

6.4		✓✓ both x intercepts *Beide x-afsnitte* ✓ turning point *draaipunt* (3)
6.5	Range/*Wydte* : $y \leq 3$ or/*of* $y \geq 3$	✓ range/*wydte* (1)
6.6	g: $y = \frac{2}{9}(x-3+2)^2 - 2$ $= \frac{2}{9}(x-1)^2 - 2$	✓✓ correct equation *korrekte vergelyking* (2)
		[14]
QUESTION/*VRAAG* 7		
7.1	For inverse/*Vir inverse* : x = $\left(\frac{1}{2}\right)^y$: y = $\log_{\frac{1}{2}} x$	✓ for base $\frac{1}{2}$ *vir basis* $\frac{1}{2}$ ✓ answer *antwoord* (2)
7.2		g: ✓ for curve *vir kurwe* ✓ y-intercept *y-afsnit* g^{-1}: ✓ for curve *vir kurwe* ✓ x-intercept *x-afsnit* (4)
7.3	see graph/*sien grafiek*	✓ dotted line *gebroke lyn* (1)
7.4	y = x	✓ y = x (1)
7.5	$\left(\frac{1}{2}\right)^x = \log_{\frac{1}{2}} x$ see graph/*sien grafiek*	(1)
		[9]

QUESTION/*VRAAG* 8

8.1

$P_v = 2\,000$; $i = \frac{i}{12}$ $n = 18$ $F_v = 2860$

$P_v(1 + i)^n = F_v \Rightarrow 2000\ \left(1 + \frac{i}{1200}\right)^{18} = 2860$

$\therefore \left(1 + \frac{i}{12}\right)^{18} = 1{,}43$

$\therefore 1 + \frac{i}{1200} = \sqrt[18]{1{,}43}$

$\therefore \frac{i}{1} = 0{,}020$

$\therefore i = 0{,}24083$

$\therefore i = 24{,}08\%$ or/*of* 24%

✓ correct value of *i* *korrekte waarde van i*
✓ substitution into correct formula *Vervanging in korrekte formule*
✓ answer *antwoord* (3)

8.2

$F_v = \frac{x[(1+i)^n - 1]}{i}$

$= \frac{100[(1+0{,}006)^{12} - 1}{0{,}006}$

$= \text{R}1\,244{,}99\ (< \text{R}1\,300)$

No, he will be R55,01 short
Nee, hy sal R55,01 kort

✓ value of *i* *waarde van i*
✓ substitution into correct formula *vervanging in korrekte formule*
✓ answer (correct conclusion) *antwoord (korrekte afleiding)* (3)

8.3

8.3.1

The deposit/*Die deposito* = 15% of R125 000,00
= R18 750
∴ The loan/*Die lening* = 125 000 – 18 750
= R106 250
or/*of*
The loan/*Die lening* = 85% of R125 000
= R106 250

✓ value of loan *waarde van* (1)

8.3.2

$P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$106\,250 = \frac{x[1-(1+0{,}010\,416)^{-72}]}{0{,}010\,416}$

$\therefore 50{,}476x = 106\,250$

$\therefore x = \frac{106\,250}{50{,}476}$

$= \text{R}2\,104{,}96$

✓ value of *i* *waarde van i*
✓ substitution into correct formula *Vervanging in korrekte formule*
✓ answer *antwoord* (3)

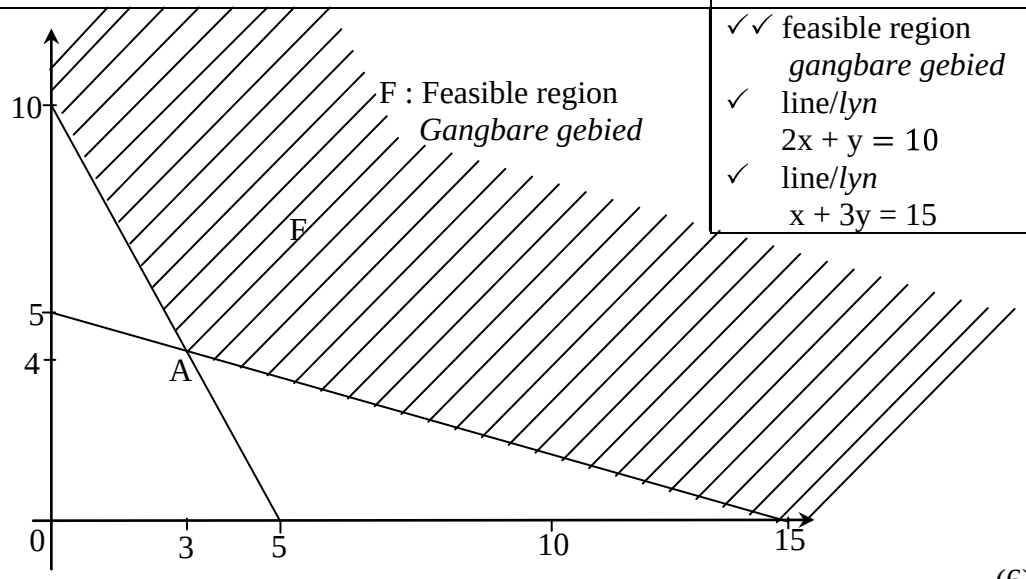
[10]

QUESTION/*VRAAG* 9		
9.1	$\lim_{h\to 0}\frac{2h}{h} = \lim_{h\to 0} 2$ $= 2$	✓ answer *antwoord* (1)
9.2	$V = \frac{1}{2}Ar - \pi r^3$ $\frac{dv}{dr} = \frac{1}{2}A - 3\pi r^2$	✓✓ one mark for each correct term *een punt vir elke korrekte term* (2)
9.3	$D_x \ a^2x^2 + x^{\frac{1}{2}} = 2a^2x + \frac{1}{2}x^{-\frac{1}{2}}$ $= 2a^2x + \frac{1}{2\sqrt{x}}$	✓ 1 mark for each derivative 1 *punt vir elke afgeleide* (2)
9.4	$f(x) = x^3$ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{f(x+h)^3-x^3}{h}$ $= \lim_{h\to 0}\frac{x^3+3x^2+3xh^2+h^3-x^3}{h}$ $= \lim_{h\to 0}(3x^2+3xh+h^2)$ $= 3x^2$	✓ substitution into correct formula *vervanging in korrekte formu*le ✓ simplification *vereenvoudiging* ✓ simplification v*ereenvoudigin*g ✓ answer *antwoord* (4)
		[9]
QUESTION/*VRAAG* 10		
10.1	$y = x(x^2 - 27)$ $= x^3 - 27x$ At turning points/*By draaipunt*: $\frac{dy}{dx} = 0$ $3x^2 - 27 = 0$ $x^2 = 9$ $\therefore x = 3$ or -3 $\therefore y = 3^3 - 27(3)$ or/*of* $y = (-3)^3 - 27(-3)$ $= -54$ $= 54$ Turning points/*Draaipunte* (3 ; -54) or/*of* (-3 ; 54)	✓ $\frac{dy}{dx} = 0$ ✓ both values of x *Beide waardes van x* ✓ both values of y *Beide waardes van y* ✓ 1 mark for both coordinates 1 *punt vir koördinate vir beide draaipunte*. (4)

10.2	$f(x) = 4 + 12x - 3x^2 - 2x^3$ $\therefore f'(x) = 12 - 6x - 6x^2$ $\therefore f''(x) = -6 - 12x$ At point of inflection/*By punt van infleksie*: $\therefore f''(x) = 0$ $\therefore -6 - 12x = 0$ $-12x = 6$ $\therefore x = \frac{-1}{2}$ $\therefore y = 4 + 12(-\frac{1}{2}) - 3(-\frac{1}{2})^2 - 2(-\frac{1}{2})^3$ $= 4 - 6 - \frac{3}{4} + \frac{1}{4}$ $= -2\frac{1}{2}$ $\therefore$ coordinate are/*koördinate is* $-\frac{1}{2}$; $-2\frac{1}{2}$	✓ 1st derivative *1ste afgeleide* ✓ 2nd derivative *2de afgeleide* ✓ value for x *waarde van x* ✓ value for y *waarde van y* ✓ coordinate *koördinate* (5)
10.3	$\frac{dy}{dx} = 3mx^2 - 6x - 12$ $3mx^2 - 6x - 12 = 0$ $x = 2 \Rightarrow \therefore 3m(2)^2 - 6(2) - 12 = 0$ $\therefore 12m - 12 - 12 = 0$ $\therefore 12m = 24$ $\therefore m = 2$ $\therefore y = 2x^3 - 3x^2 - 12x + n$ At x = 2 is y = -3 $\therefore -3 = 2(2)^3 - 3(2)^2 - 12(2) + n$ $\therefore -3 = 16 - 12 - 24 + n$ $\therefore -3 = -20 + n$ $\therefore n = 17$	✓ differentiation *differensiasie* ✓ substitution *vervanging* ✓ value of m *waarde* van m ✓ substitution of x and y *vervanging van x en y* ✓ value of n *waarde* van n (5)
		[14]
QUESTION/*VRAAG* 11		
11.1	Volume = $\pi r^2 h$ $10\,000 = \pi r^2 h$ $\therefore h = \frac{10\,000}{\pi r^2}$	✓ formula *formule* ✓ substitution *vervanging* ✓ *h* (3)
11.2	Exterior area/*Buite oppervlakte* $A = 2(\pi r^2) + 2\pi rh$ $= 2\pi r^2 + \frac{2\pi r\ 10\,000}{\pi r^2}$ $= 2\pi r^2 + \frac{20\,000}{r}$	✓ formula *formule* ✓ substitution of *h* *vervanging van h* (2)

11.3	For minimum/*Vir minimum*: $\frac{dA}{dr} = 0$ $\therefore 4\pi r - \frac{20\,000}{r^2} = 0$ $\therefore 4\pi r^3 - 20\,000 = 0$ $\therefore 4\pi r^3 = 20\,000$ $\therefore r = 11{,}68$ and/*en* $h = \frac{10\,000}{\pi r^2}$ $= 23{,}35$ Allow/*Laat toe*: 23,33 or/*of* 23,34	✓ differentiation of A *differensiasie van A* ✓ simplification *vereenvoudiging* ✓ value of r *waarde van r* ✓ value of *h*. *waarde van h* (4)
		[9]

QUESTION/*VRAAG* 12

12.1	$2x + y \geq 10$✓✓ $x + 3y \geq 15$✓✓ $x > 0$ ✓ $y > 0$✓	✓✓ for 1st inequality *vir 1ste ongelykheid* ✓✓ 2nd inequality *2de ongelykheid* ✓ $x > 0$ ✓ $y > 0$ (4)
		✓✓ feasible region *gangbare gebied* ✓ line/*lyn* $2x + y = 10$ ✓ line/*lyn* $x + 3y = 15$ (6)
12.3	Minimum number of capsules and tablets: *Minimum aantal kapsule en tablette* 3 tablets/*tablette* Read at A./*Gelees by A.* 4 capsules/*kapsules*	✓✓ tablets *tablette* ✓✓ capsules *kapsules* (3)
		[13]
	TOTAL/*TOTAAL*:	**150**