



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2013

MATHEMATICS P3

MARKS: 100

TIME: 2 hours



This question paper consists of 14 pages, including a formula-sheet and 4 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, *et cetera*, which you have used in determining your answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number the answers correctly according to the numbering system used in this question paper.
6. Diagrams are NOT necessarily drawn to scale.
7. It is in your own interest to write legibly and to present work neatly.
8. FOUR diagram sheets for answering QUESTION 3.3, QUESTION 4.2.1, QUESTION 6.1 and QUESTIONS 7 to 10 are attached at the end of this question paper. Write your NAME/EXAMINATION NUMBER in the spaces provided and hand them in together with your ANSWER BOOK.
9. An information sheet with formulae is attached.

QUESTION 1

This is a very special sequence known as the *Fibonacci* sequence:

1; 1; 2; 3; 5; 8; 13; 21; ...

1.1 Write down the TENTH term of the sequence. (1)

1.2 Hence determine the *recursive formula* for the n^{th} term of the sequence in the form $T_n = \dots$ (4)
[5]

QUESTION 2

Inflation, according to the *Eastern Cape Tribune*, is expected to be consistent at 8,5% per year for the next 10 years. If the price of a T-shirt is priced at R100 today, answer the following questions by showing appropriate calculations.

2.1 What will the price of the T-shirt be in 1 YEAR from today? (1)

2.2 Hence, calculate the price of the T-shirt for the 2nd YEAR and the 3rd YEAR and write your answer as a sequence. (2)

2.3 Describe the sequence and motivate your answer. (2)

2.4 Use the appropriate formula and determine how long (in years) it will take to DOUBLE the price of the T-shirt as from today. (5)
[10]

QUESTION 3

The percentage marks of 50 learners for Mathematics Paper 3 of 2012 are given as follows:

37; 39; 44; 47; 50; 55; 58; 58; 58; 58; 60; 60; 62; 62; 64; 64; 66; 67; 68; 68; 68; 70; 70; 70; 70; 72; 74; 74; 75; 76; 77; 77; 77; 78; 80; 80; 82; 83; 85; 86; 86; 87; 87; 88; 88; 90; 91; 92; 94; 95

The claim is made that when a large sample is drawn from a normally distributed population the expected percentages for the standard deviation (SD) intervals of the mean are as follows:

1st SD intervals to both sides of the mean: approximately 68%

2nd SD intervals to both sides of the mean: approximately 95%

3rd SD intervals to both sides of the mean: approximately 100%

3.1 Calculate the mean (\bar{x}) to ONE decimal place. (1)

3.2 Calculate the standard deviation (σ) to ONE decimal place. (2)

3.3 Use the data and complete the following table in DIAGRAM SHEET 1.

Interval	Interval range	Observed no. of values	Expected %	Observed %
$\bar{x} - \sigma$ to $\bar{x} + \sigma$	54,6 to 85,0		Approx. 68%	
$\bar{x} - 2\sigma$ to $\bar{x} + 2\sigma$		48	Approx. 95%	
$\bar{x} - 3\sigma$ to $\bar{x} + 3\sigma$			Approx. 100%	

(7)

3.4 Based on your observation from the values you obtain in your table, would your values support or dispel the claim made on a normally distributed population?

(1)

[11]**QUESTION 4**

4.1 Given: $P(A) = \frac{2}{3}$

$$P(B) = \frac{1}{4}$$

$$P(A \cup B) = \frac{11}{12}$$

4.1.1 Are events A and B mutually exclusive? Use the appropriate probability rules and calculations to substantiate your answer.

(3)

4.1.2 Represent the data by using a Venn diagram.

(3)

4.2 Event A and event B are illustrated in a two-way contingency table below.

	B	not B	Total
A	30	b	d
not A	a	396	e
Total	34	c	700

4.2.1 Complete the table in DIAGRAM SHEET 1 by filling in the missing values (a, b, c, d and e).

(5)

4.2.2 Are events A and B dependent or independent events? Support your answer with appropriate probability rules and calculations.

(4)

4.3 Event A and Event B are independent events with $P(A) = 0,5$ and $P(B) = 0,4$. Find $P(A \cup B)$.

(4)

[19]**QUESTION 5**

5.1 FIVE new learners arrive at their new school where there are FIVE sports houses. In how many different ways can the learners be assigned to a sports house so that they are all in different houses?

(2)

5.2 There are THREE vacant teaching posts at a school and FIVE applicants. In how many ways can the applicants be chosen to fill the vacancies?

(3)

[5]

QUESTION 6

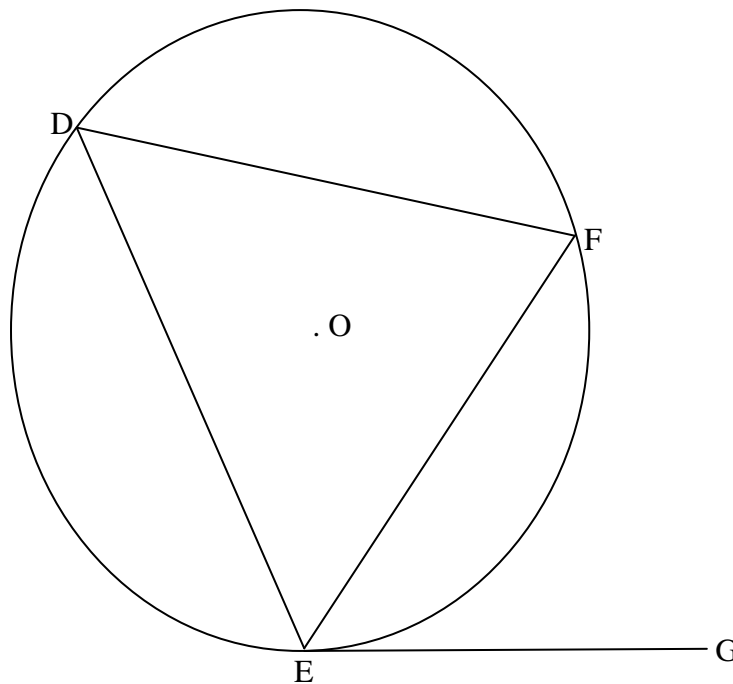
The human resources department of a well-known university wants to create a model to be used in determining the monthly income for its lecturers. TWELVE sources were consulted and the information is displayed in the following table:

Teaching experience (x)	26	1	3	5	6	6	10	14	12	13	20	8
Income in R1000/mth (y)	20	9	10,5	11	10	12	16	15	12	14	18	9

- 6.1 Use DIAGRAM SHEET 2 and draw a scatter plot of the data. (1)
 - 6.2 Calculate the equation of the regression line of best fit for the data. (2)
 - 6.3 Validate whether the point $(\bar{x}; \bar{y})$ lies on the line of best fit. (3)
 - 6.4 If a lecturer has been teaching for 35 years, what would his/her salary have been? (2)
 - 6.5 Is the amount determined in QUESTION 6.4 reasonable? Motivate your answer. (2)
 - 6.6 Dr. Fresh has taught for 12 years. Determine his monthly salary and state any ONE other factor you would consider in determining his monthly salary. (2)
- [12]**

QUESTION 7

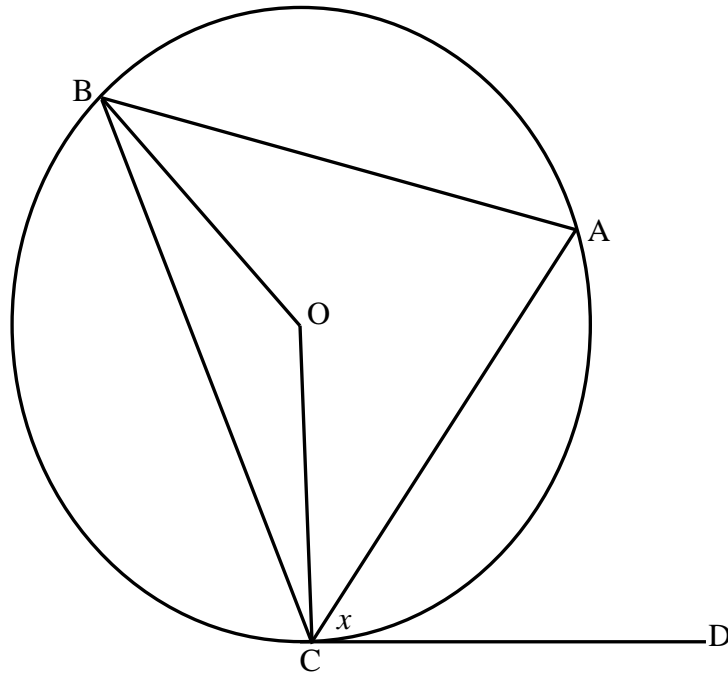
In the figure below O is the centre of the circle and EG is a tangent to the circle at E. Prove the THEOREM that states that the angle between a chord and a tangent is equal to the angle in the opposite segment. ($\widehat{FEG} = \widehat{D}$).



[7]

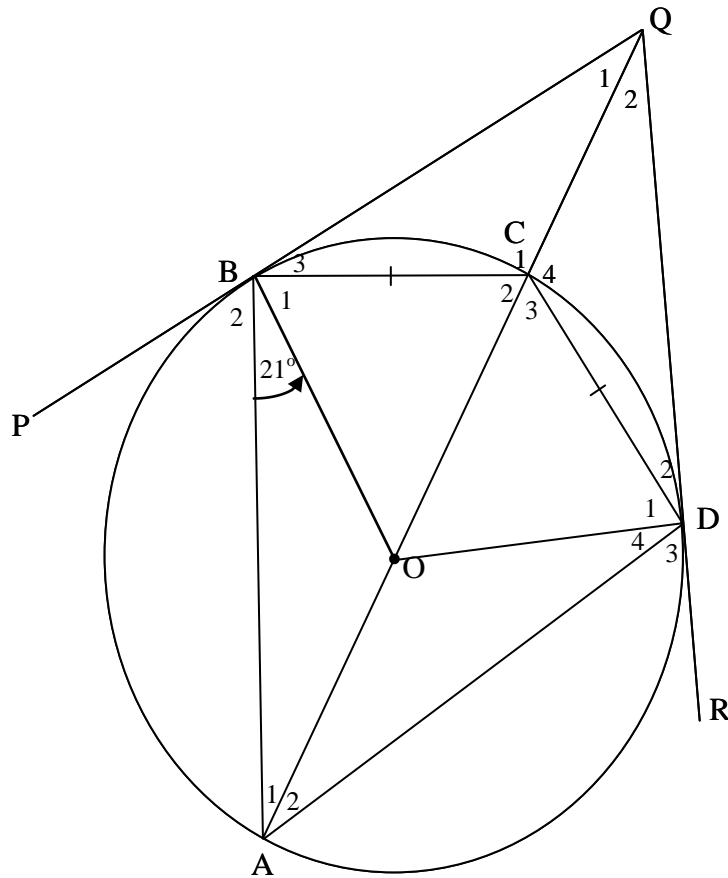
QUESTION 8

- 8.1 In the figure below O is the centre of the circle. DC is a tangent to the circle at C .
 $AB = AC$ and $\widehat{ACD} = x$.



- 8.1.1 Prove that $\widehat{BCA} = x$. (2)
- 8.1.2 Determine, with reasons, \widehat{OBC} in terms of x . (4)

8.2 In the figure below O is the centre of the circle with tangents PQ and QR joining at Q and $BC = CD$ and $\widehat{ABO} = 21^\circ$.

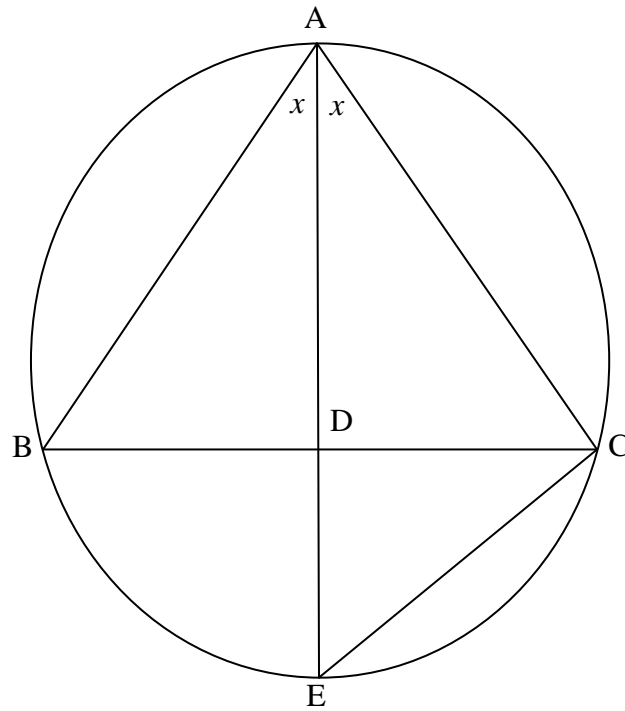


- 8.2.1 Name, with reasons, THREE other angles in the given figure each equal to 21° . (3)
- 8.2.2 Prove that BODQ is a cyclic quadrilateral. (2)
- 8.2.3 Determine with reasons the size of \widehat{Q}_2 . (2)
- 8.2.4 Hence, or otherwise, prove that AQ bisects \widehat{PQR} . (2)

[15]

QUESTION 9

In the figure A, B, C and D are points on a circle. AE bisects \hat{BAC} and BC and AE intersect in D.



Prove, stating reasons, that:

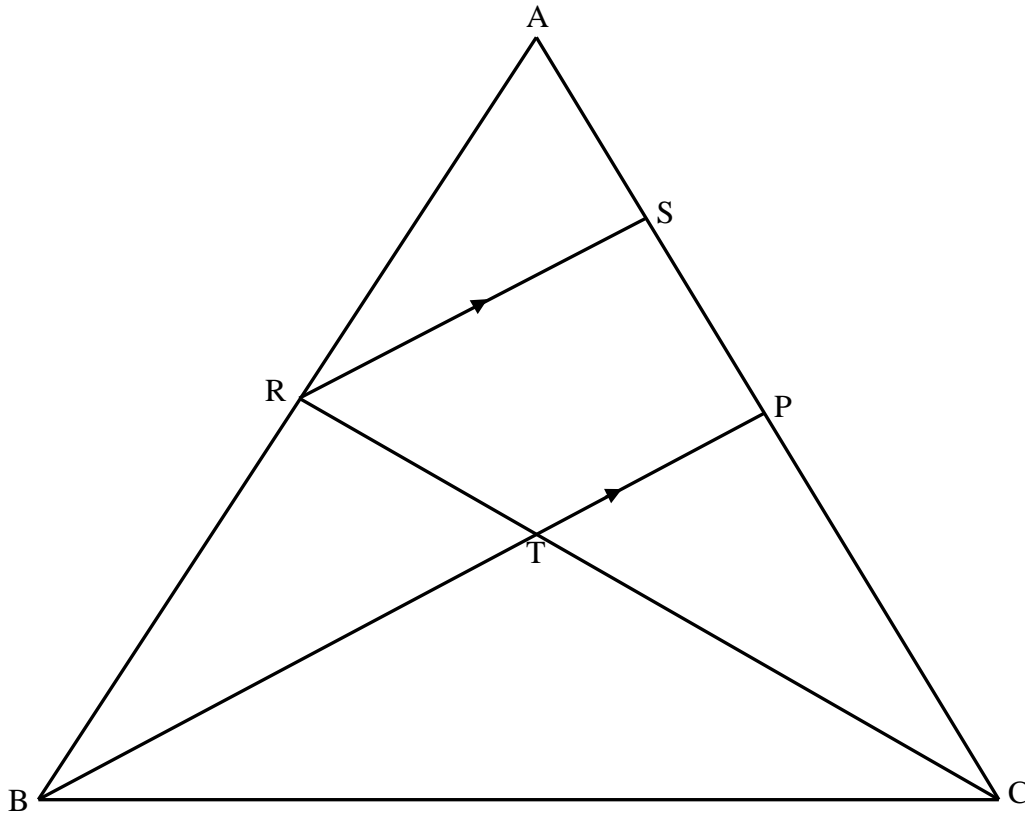
9.1 $\triangle ABD \sim \triangle CED$ (3)

9.2 $AB \cdot AC = AD^2 + AD \cdot DE$ (4)

[7]

QUESTION 10

In $\triangle ABC$, P is the midpoint of AC. $RS \parallel BP$ and $\frac{AR}{AB} = \frac{3}{5}$. CR and BP intersect at T.
(Hint: Let $AR = 3k$ and $AB = 5k$)



Determine with reasons:

10.1 $\frac{AS}{SP}$ (2)

10.2 $\frac{AS}{SC}$ (2)

10.3 $\frac{RT}{TC}$ (1)

10.4 $\frac{\text{Area } \triangle TPC}{\text{Area } \triangle RSC}$ (4)

[9]

TOTAL: 100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta ; y \cos \theta - x \sin \theta) \quad (x; y) \rightarrow (x \cos \theta - y \sin \theta ; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

NAME AND SURNAME:

DIAGRAM SHEET 1

QUESTION 3.3

Interval	Interval range	Observed no. of values	Expected %	Observed %
$\bar{x} - \sigma$ to $\bar{x} + \sigma$	54,6 to 85,0		Approx. 68%	
$\bar{x} - 2\sigma$ to $\bar{x} + 2\sigma$		48	Approx. 95%	
$\bar{x} - 3\sigma$ to $\bar{x} + 3\sigma$			Approx. 100%	

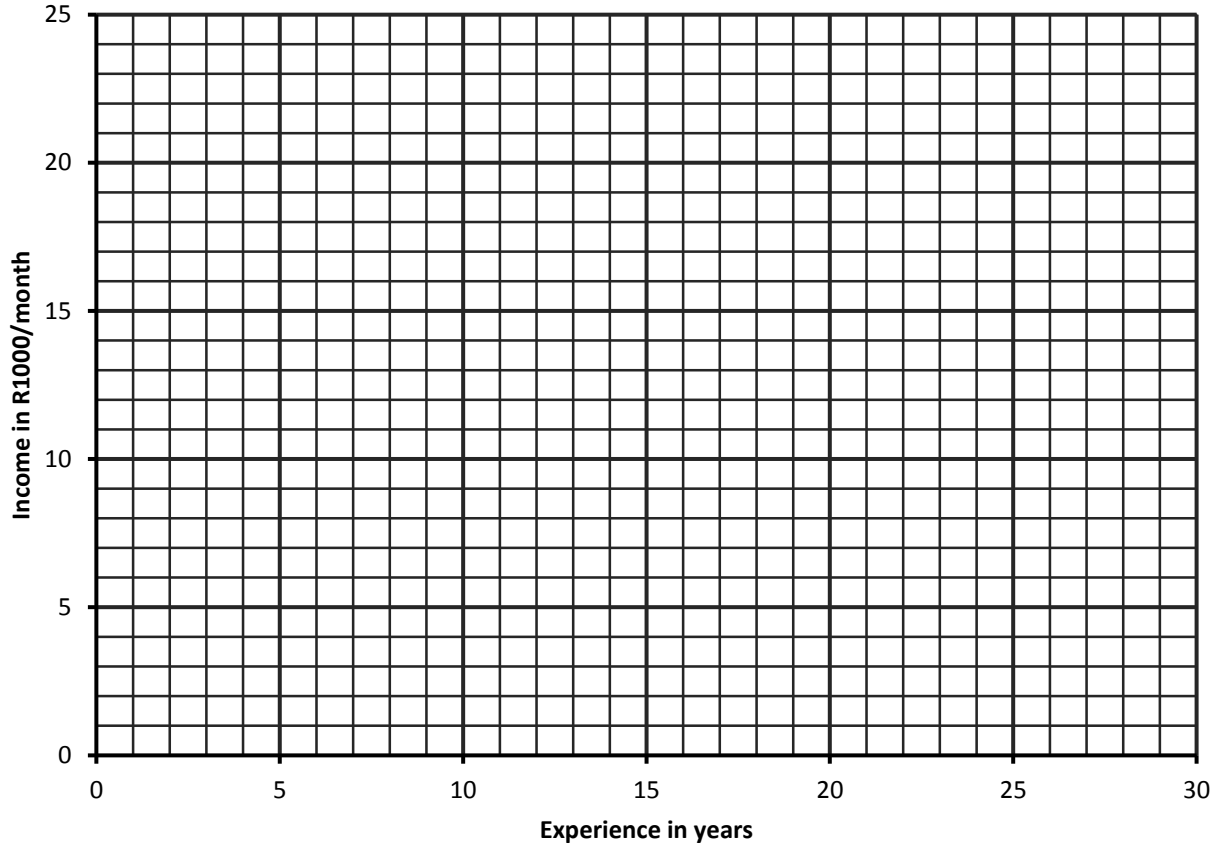
QUESTION 4.2.1

	B	not B	Total
A	30	b _____	d _____
not A	a _____	396	e _____
Total	34	c _____	700

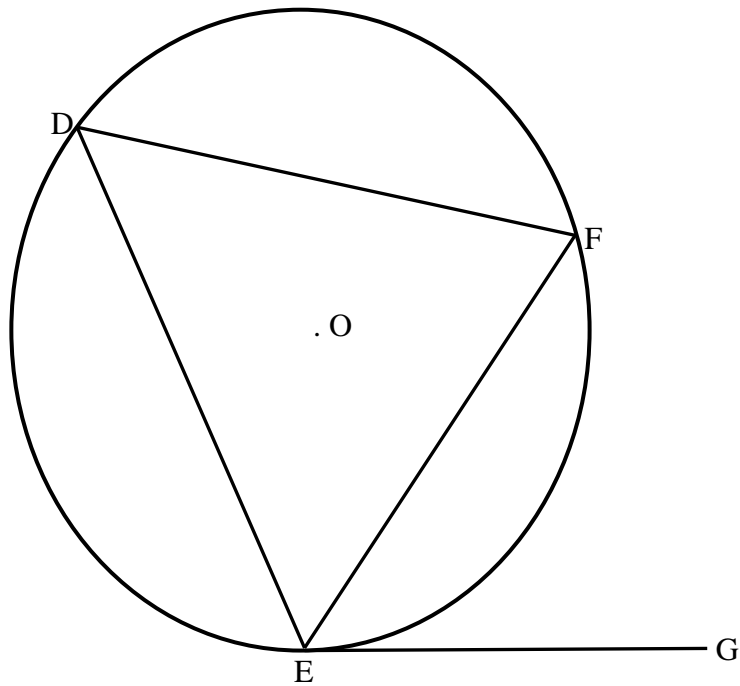
NAME AND SURNAME:

DIAGRAM SHEET 2

QUESTION 6.1



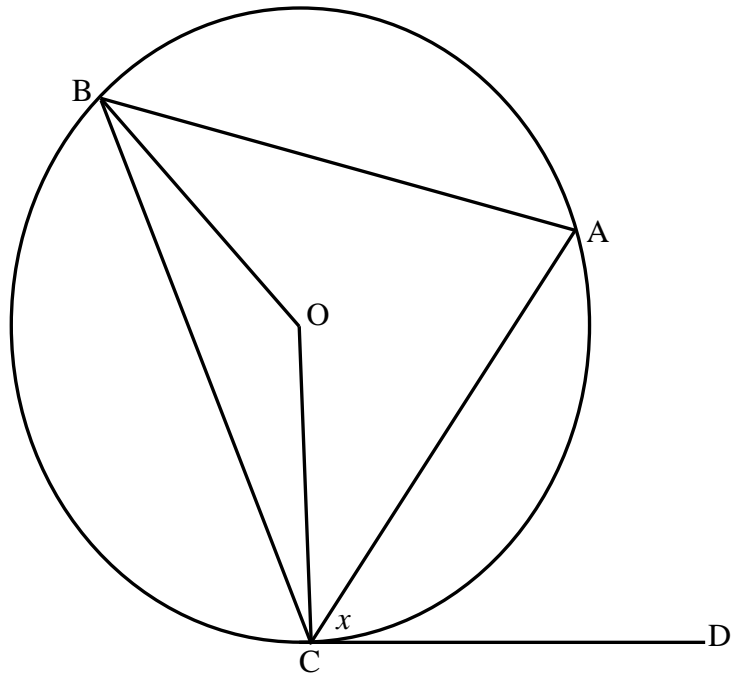
QUESTION 7



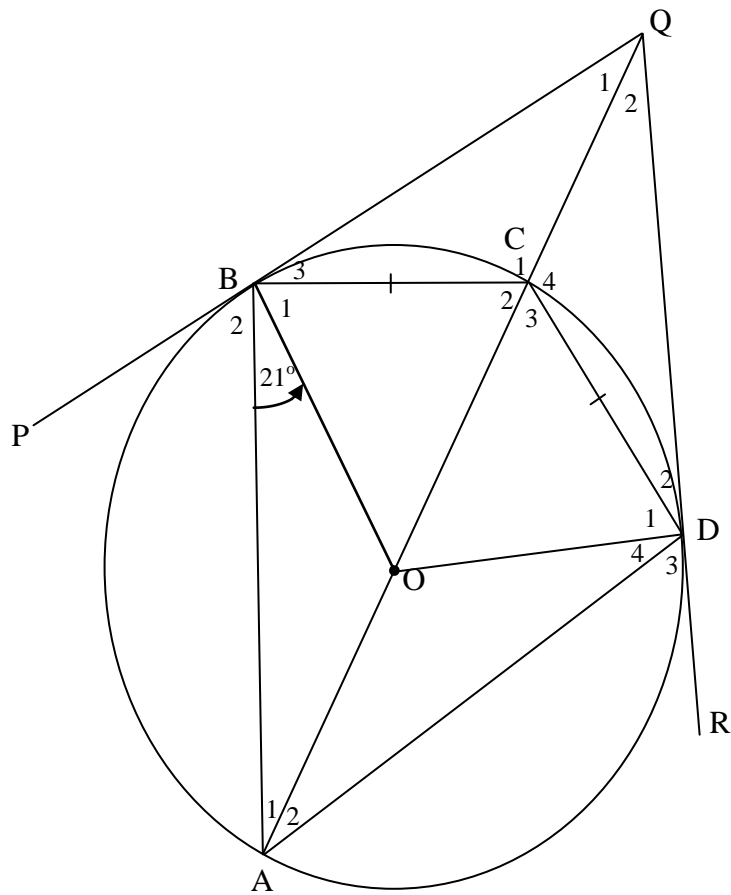
NAME AND SURNAME:

DIAGRAM SHEET 3

QUESTION 8.1



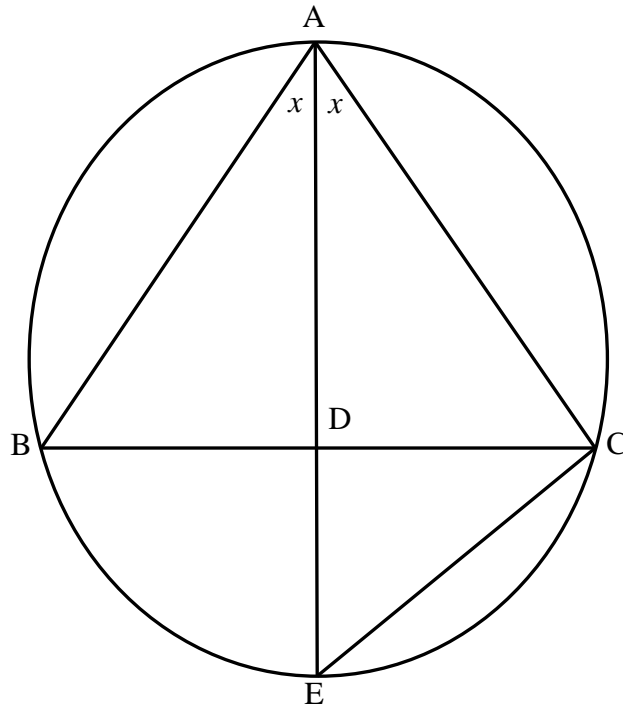
QUESTION 8.2



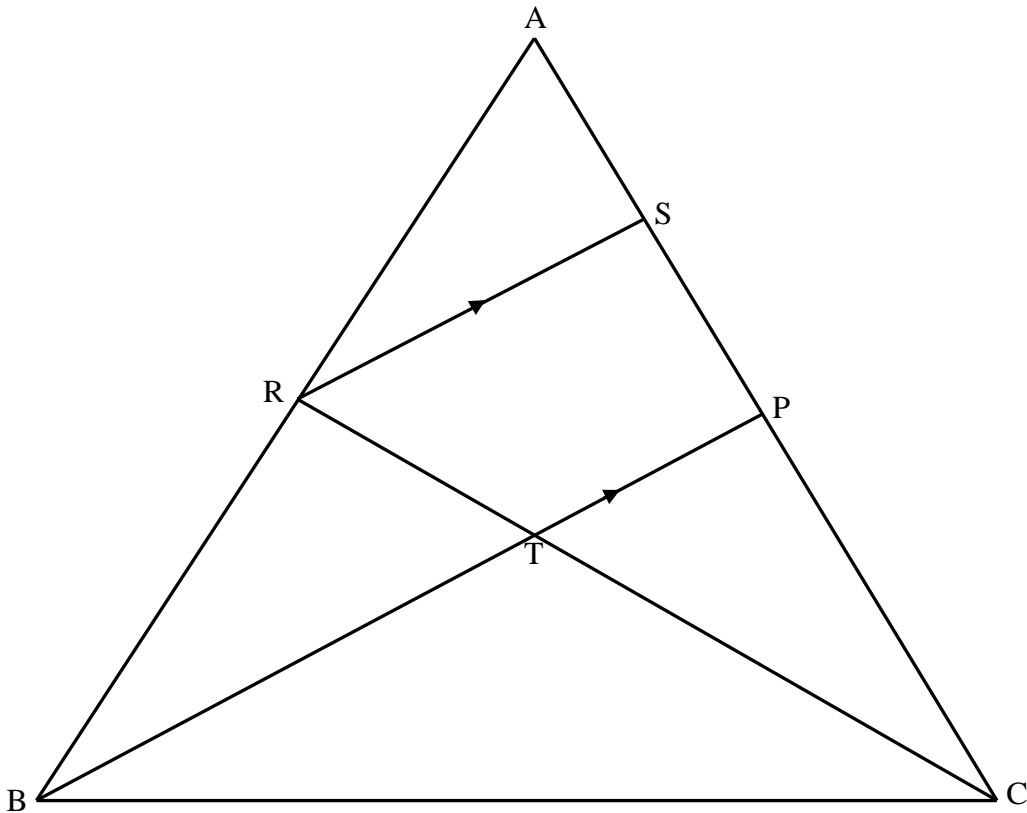
NAME AND SURNAME:

DIAGRAM SHEET 4

QUESTION 9



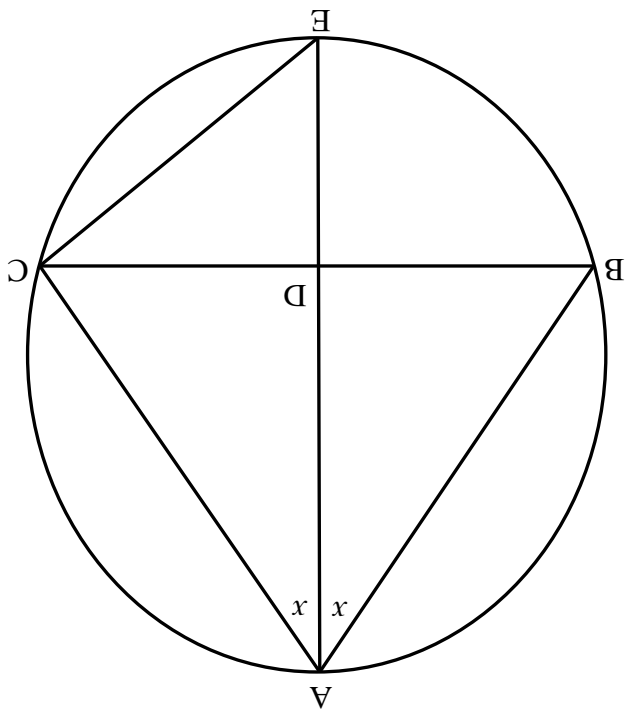
QUESTION 10



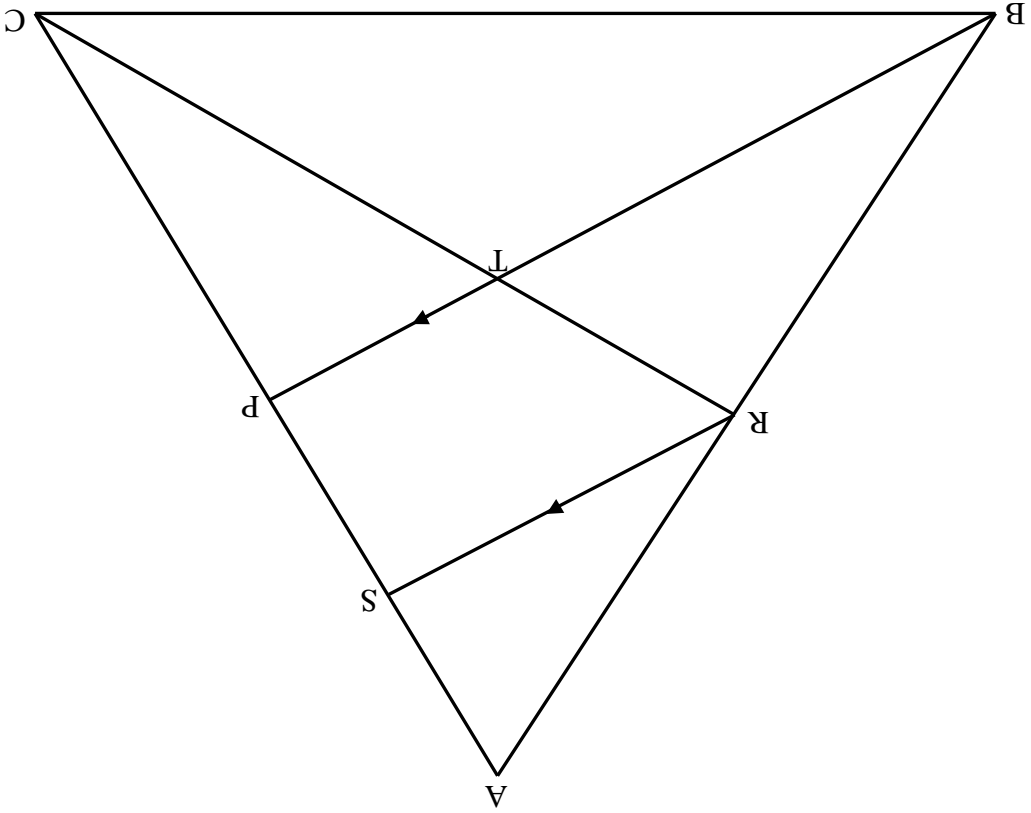
NAAM EN VAN:

DIAGRAMMEL 4

VRAAG 9



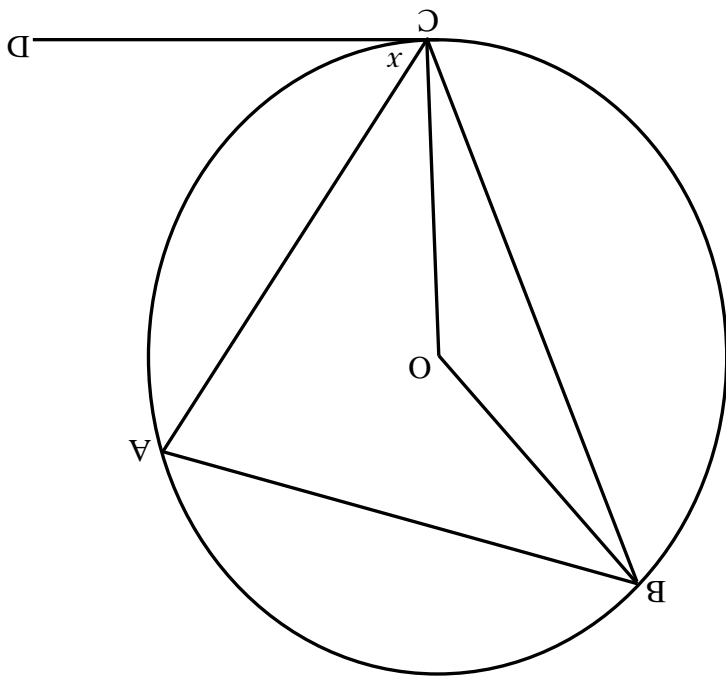
VRAAG 10



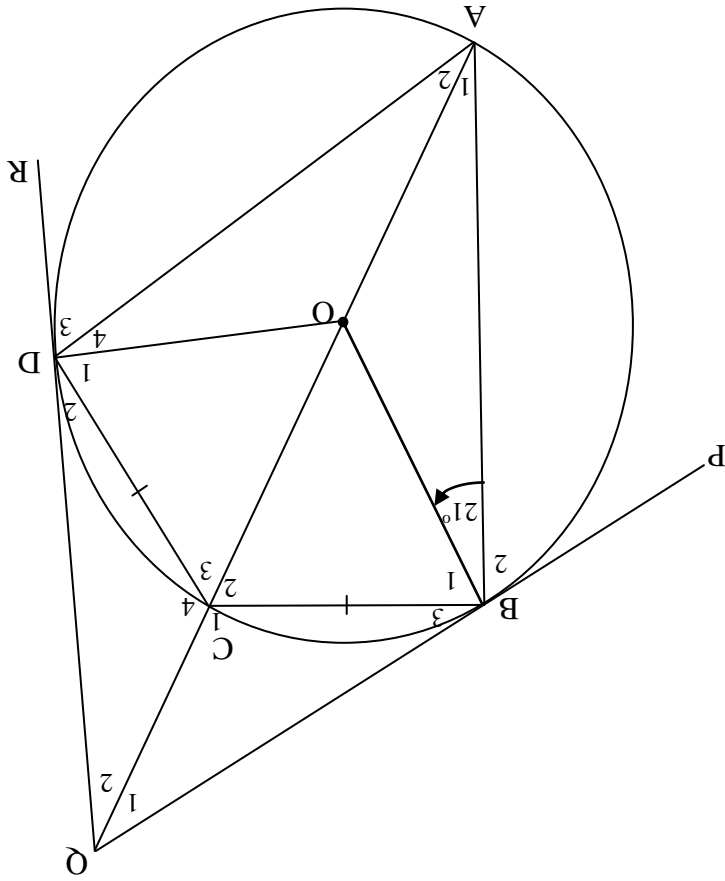
NAAM EN VÁN:

DIAGRAMVEL 3

VRAAG 8.1



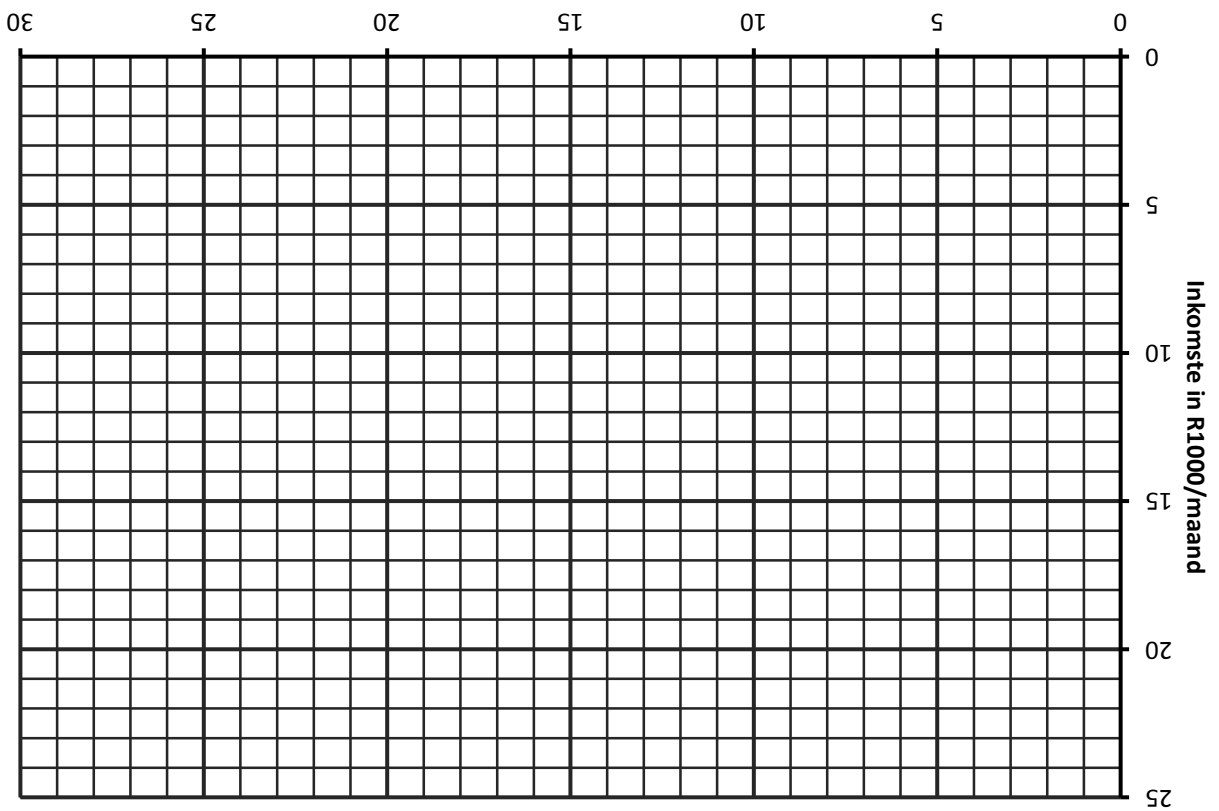
VRAAG 8.2



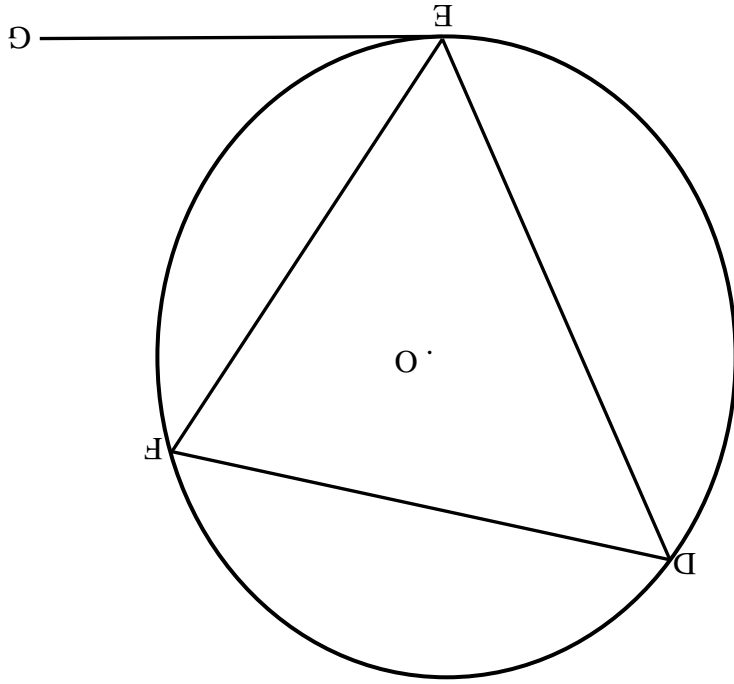
NAAM EN VAN:

DIAGRAMVEL 2

VRAAG 6.1



VRAAG 7



VRAAG 4.2.1

Totaal	34	c	700
nie A	a	396	e
A	30	b	d
B	nie B	Totaal	

VRAAG 3.3

DIAGRAMMEL 1

NAAM EN VAN:

Interval	Intervalwydte	Waargenome nr. van waardes	Verwagte %	Waargenome %
$\bar{x} - \sigma$ tot $\bar{x} + \sigma$	54,6 tot 85,0		Naast. 68%	
$\bar{x} - 2\sigma$ tot $\bar{x} + 2\sigma$		48	Naast. 95%	
$\bar{x} - 3\sigma$ tot $\bar{x} + 3\sigma$			Naast. 100%	

INLICHTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n \quad \sum_{i=1}^n \frac{1}{n(i+1)} = \frac{1}{n} \quad T_n = a + (n-1)d \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \quad S_n = a \frac{r^n - 1}{r - 1} \quad ; \quad r \neq 1 \quad S_\infty = \frac{a}{1-r} \quad ; \quad -1 < r < 1$$

$$F = \frac{x^n(1+i)^n - 1}{i}$$

$$P = \frac{x^n(1+i)^n - 1}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{Area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \quad \sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \quad \cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\left. \begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha &= 2\cos^2 \alpha - 1 \\ 2\cos^2 \alpha - 1 &= 1 - 2\sin^2 \alpha \end{aligned} \right\}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x;y) \leftarrow (x \cos \theta + y \sin \theta; y \sin \theta - x \cos \theta) \quad (x;y) \leftarrow (x \cos \theta - y \sin \theta; y \sin \theta + x \cos \theta)$$

$$\sum_{i=1}^n x^i = \frac{x^{n+1} - x}{x - 1}$$

$$\sum_{i=1}^n x^i = \frac{x^{n+1} - x}{x - 1}$$

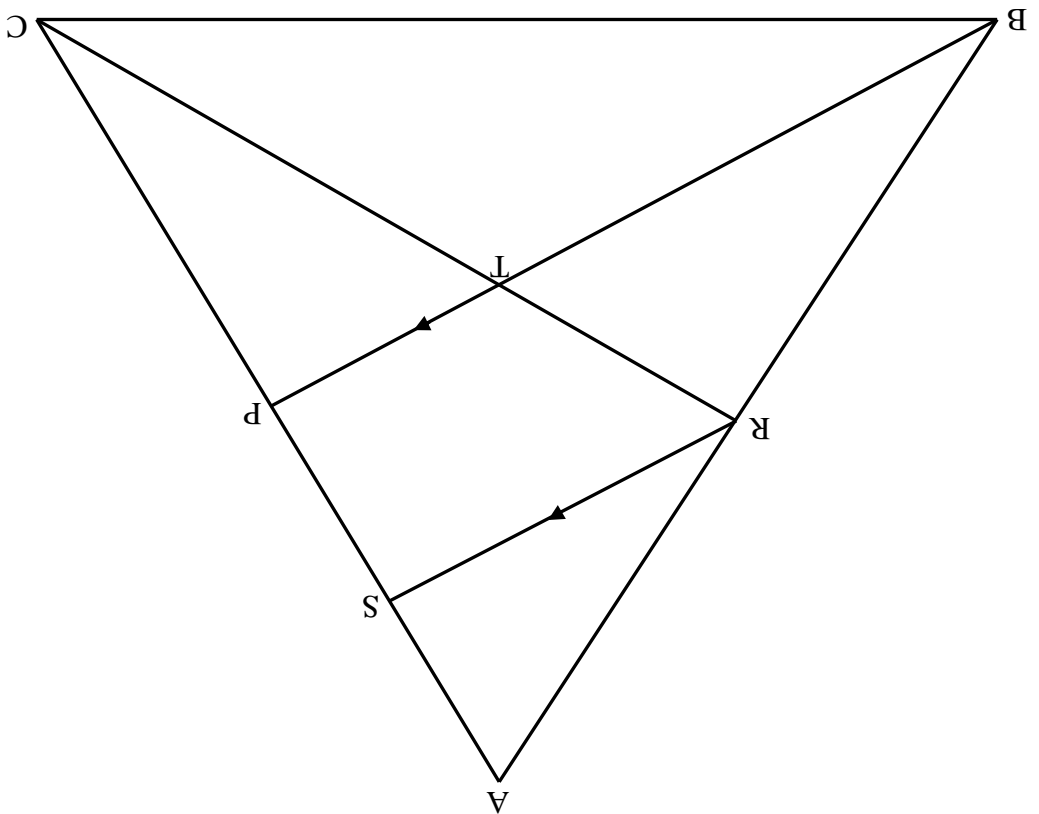
$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \cap B)$$

$$\sum_{i=1}^n (x-x)^2 = q \quad \sum_{i=1}^n (x-x)^2 = q$$

VRAAG 10

In $\triangle ABC$, is P die middelpunt van AC, RS \parallel BP en $\frac{AR}{AB} = \frac{3}{5}$. CR en BP sny by T.
 (Wenk: Stel $AR = 3k$ en $AB = 5k$)



Bepaal met redes:

10.1 $\frac{AS}{SP}$

10.2 $\frac{AS}{SC}$

10.3 $\frac{TC}{RT}$

10.4 $\frac{\text{Oppervlakte } \triangle TPC}{\text{Oppervlakte } \triangle RSC}$

(4) [9]

(1)

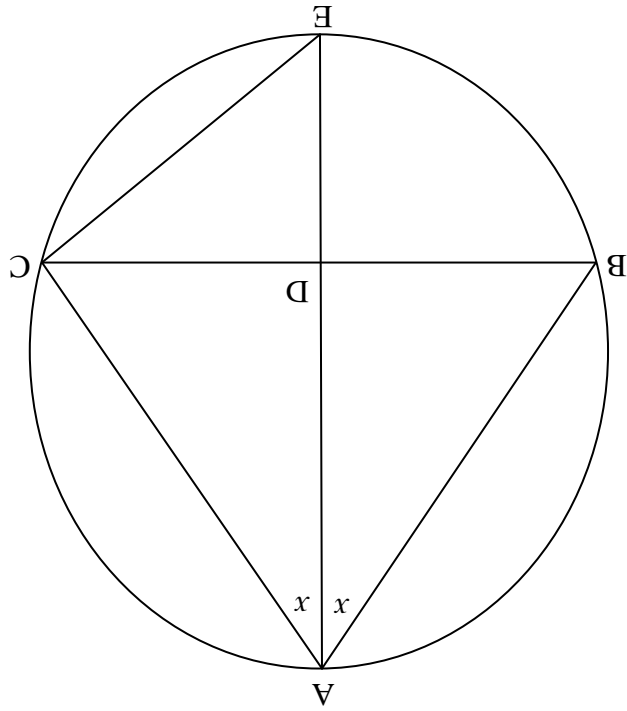
(2)

(2)

TOTAAL: 100

VRAAG 9

In die figuur is A, B, C en D punte op 'n sirkel. AE halveer \widehat{BAC} en BC en AE sny mekaar in D.



Bewys, deur redes te verskaf, dat:

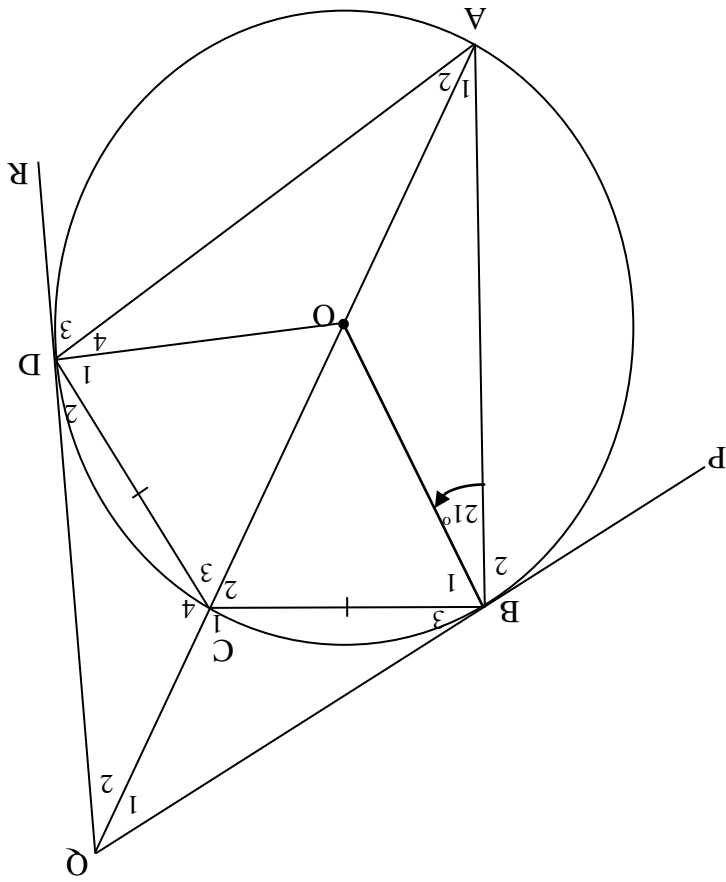
9.1 $\triangle ABD \parallel \triangle CED$

9.2 $AB \cdot AC = AD^2 + AD \cdot DE$

(4) [7]

(3)

8.2 In die figuur hieronder is O die middelpunt van die sirkel met raaklyne PQ en QR wat by Q verbind en $BC = CD$ en $\angle ABO = 21^\circ$.



8.2.1 Noem, met redes, DRIE ander hoeke in die gegee figuur wat elk gelyk is aan 21° .

8.2.2 Bewys dat $BODQ$ 'n koordevierhoek is.

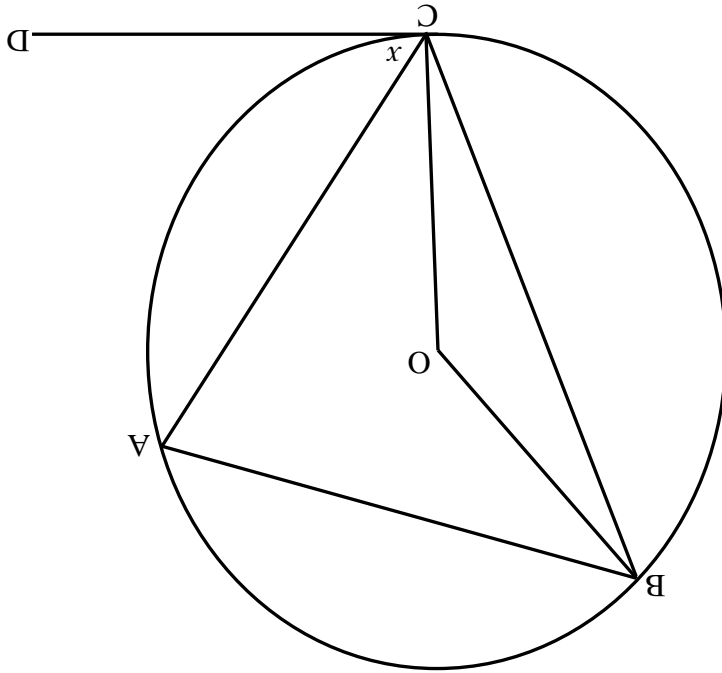
8.2.3 Bepaal, met redes, die grootte van $\angle z$.

8.2.4 Bewys vervolgens of andersins dat AQ vir PQR halveer.

[15]

VRAAG 8

8.1 In die figuur hieronder is O die middelpunt van die sirkel. DC is 'n raaklyn aan die sirkel by C . $AB = AC$ en $\angle ACD = x$.



8.1.1 Bewys dat $\angle BCA = x$.

(2)

8.1.2 Bepaal, met redes, $\angle OBC$ in terme van x .

(4)

VRAAG 6

Die menslike hulpbronne departement van 'n welbekende universiteit wil 'n model ontwerp wat gebruik moet word om die maandelikse inkomste van sy lektore te bepaal. TWAALF promone was nageslaan en die inligting word in die volgende tabel ten toon gestel:

Onderwys ervaring (x)	26	1	3	5	6	6	10	10	11	10,5	9	20	Inkomste in R1000/mnd (y)
	8	20	13	12	14	10	14	14	12	15	16	12	14
	9	18	14	12	15	16	12	10	11	10,5	9	20	14

6.1 Gebruik DIAGRAMVEL 2 en teken 'n spreidingsdiagram van die gegewens. (1)

6.2 Bereken die vergelyking van die regressielyn van bestepas vir die gegewens. (2)

6.3 Bekragtig of die punt $(\bar{x}; \bar{y})$ op die lyn van bestepas lê. (3)

6.4 As 'n lektor vir 35 jaar onderwys gee, wat sou sy/haar salaris wees? (2)

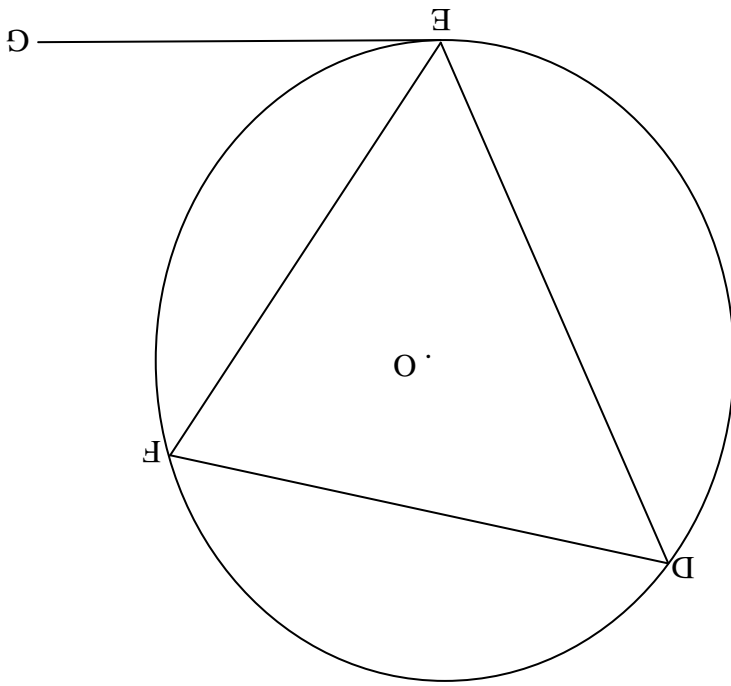
6.5 Is die bedrag in VRAAG 6.4 redelik? Motiveer jou antwoord. (2)

6.6 Dr. Fresh het vir 12 jaar onderwys gegee. Bepaal sy maandelikse salaris en verskat enige EBN faktor wat jy sal in ag neem om sy salaris te bepaal. (2)

[12]

VRAAG 7

In die figuur hieronder is O die middelpunt van die sirkel en EG is 'n raaklyn by E. Bewys die STELLING wat beweer dat die hoek tussen 'n koord en raaklyn gelyk is aan die hoek in die teenoorstaande segment. ($\angle FEG = \angle D$).



[7]

3.3 Gebruik die gegewens en voltooi die volgende tabel in DIAGRAMVEL 1.

Interval	Interval-wyde	Waargetal nr van waardes	Verwagte %	Waargetal %
$\bar{x} - \sigma$ tot $\bar{x} + \sigma$	54,6 tot 85,0		Nast. 68%	
$\bar{x} - 2\sigma$ tot $\bar{x} + 2\sigma$		48	Nast. 95%	
$\bar{x} - 3\sigma$ tot $\bar{x} + 3\sigma$			Nast. 100%	

(7)

3.4 Gebaseer op jou waarneming van die waardes in jou tabel, sal jou waardes die bewering oor 'n normaal verspreide bevolking ondersteun of verwerp?

(1)

VRAAG 4

4.1 Gegee: $P(A) = \frac{3}{2}$

$$P(B) = \frac{1}{4}$$

$$P(A \cup B) = \frac{11}{12}$$

4.1.1 Is gebeurtenisse A en B onderling uitsluitlik? Gebruik die toepaslike waarskynlikheidsreëls en berekeninge om jou antwoord te staaf.

(3)

4.1.2 Gebruik 'n Venn-diagram om die gegewens voor te stel.

(3)

4.2 Geburtenis A en gebeurtenis B word geïllustreer in 'n tweerigting tabel hieronder.

	B	nie B	Totaal
A	30	b	d
nie A	a	396	e
Totaal	34	c	700

4.2.1 Voltooi die tabel in DIAGRAMVEL 1 deur die ontbrekende waardes in te vul (a, b, c, d en e).

(5)

4.2.2 Is gebeurtenisse A en B afhanklik of onafhanklike gebeurtenisse? Ondersteun jou antwoord met toepaslike waarskynlikheidsreëls en berekeninge.

(4)

4.3 Geburtenis A en gebeurtenis B is onafhanklike gebeurtenisse met $P(A) = 0,5$ en $P(B) = 0,4$.

Bepaal $P(A \cup B)$.

(4)

VRAAG 5

5.1 VYF nuwe leerlinge dag by hulle nuwe skool op waar daar VYF sportshuise is. In hoeveel verskillende maniere kan die leerlinge aan 'n sportshuis toegeken word sodat hulle almal in verskillende huise kan wees?

(2)

5.2 Daar is DRIE vakante onderwysposte by 'n skool en VYF aansoekers. In hoeveel verskillende maniere kan die aansoekers gekies word om die vakante poste te vul?

(3)

[5]

[19]

Lees die volgende instruksies sorgvuldig deur voordat die vrae beantwoord word.

1. Hierdie vraestel bestaan uit 10 vrae. Beantwoord AL die vrae.
2. Dui ALLE berekeninge, diagramme, grafieke ensovoorts duidelik aan wat jy gebruik het in die bepaling van jou antwoorde.
3. n Goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) mag gebruik word, tensy anders vermeld.
4. Waar nodig moet antwoorde tot TWEE desimale plekke afgerond word, tensy anders vermeld.
5. Nummer jou antwoorde korrek, volgens die nommeringstelsel wat op die vraestel gebruik word.
6. Diagramme is NIE noodwendig volgens skaal geteken NIE.
7. Dit is in jou eie belang om leesbaar te skryf en jou werk netjies aan te bied.
8. VIER diagramvulle word aan die einde van die vraestel aangebied om VRAAG 3.3, VRAAG 4.2.1, VRAAG 6.1 en VRAE 7 tot 10 te beantwoord. Skryf jou NAAM/EKSAAMENNUMMER in die spasies wat voorsien word en handig dit saam met jou ANTWOORDEBOEK in.
9. n Inligtingsblad met formules is aangebied.

Hierdie vraestel bestaan uit 14 bladsye, insluitende 7 formuleblad en 4 diagramme.



TYD: 2uur

PUNTE: 100

WISKUNDE V3

SEPTEMBER 2013

GRAAD 12

**NASIONALE
SENIOR SERTIFIKAT**

Province of the
EASTERN CAPE
EDUCATION

