Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE / *NASIONALE SENIOR SERTIFIKAAT*

GRADE 11 / *GRAAD 11*

NOVEMBER 2016

MATHEMATICS P2 / *WISKUNDE V2* MEMORANDUM

MARKS / *PUNTE*: 150

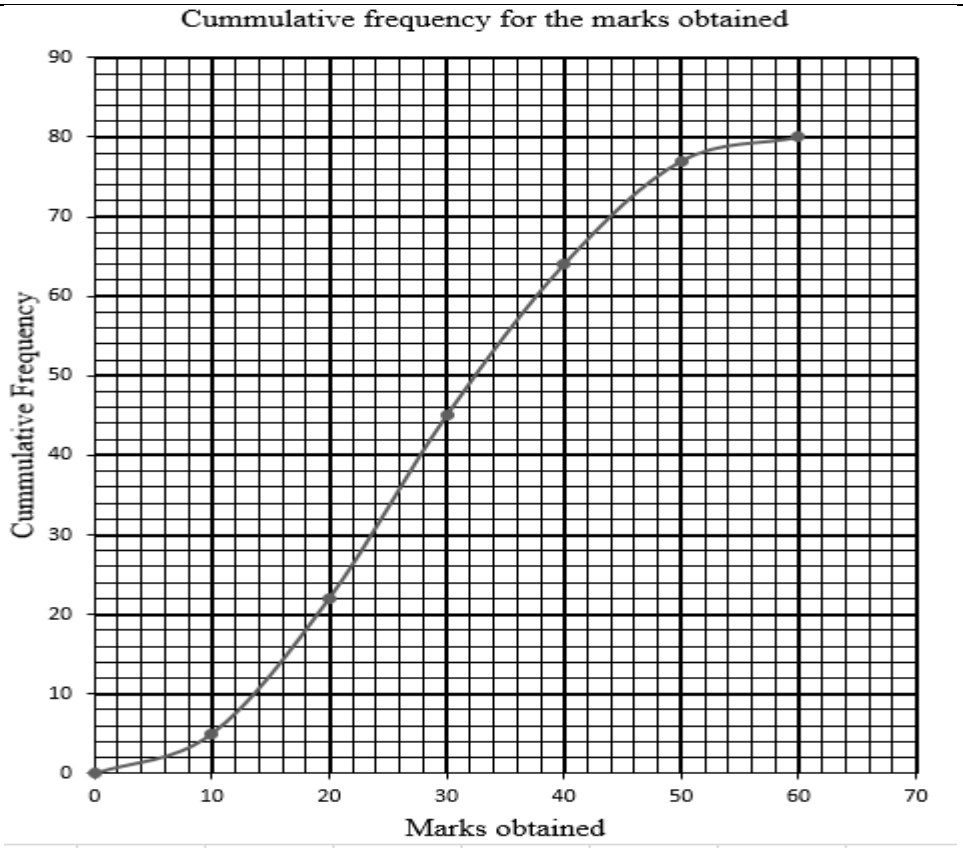
This memorandum consists of 16 pages./
Hierdie memorandum bestaan uit 16 bladsye.

NOTE:
- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is **NOT** acceptable.

LET WEL:
- Indien 'n kandidaat 'n vraag twee keer beantwoord, merk slegs die eerste poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, maar nie oorgedoen het nie, merk die doodgetrekte antwoord.
- Volgehoue akkuraatheid geld in ALLE aspekte van die memorandum.
- Aanname van antwoorde/waardes om 'n probleem op te los, is Onaanvaarbaar.

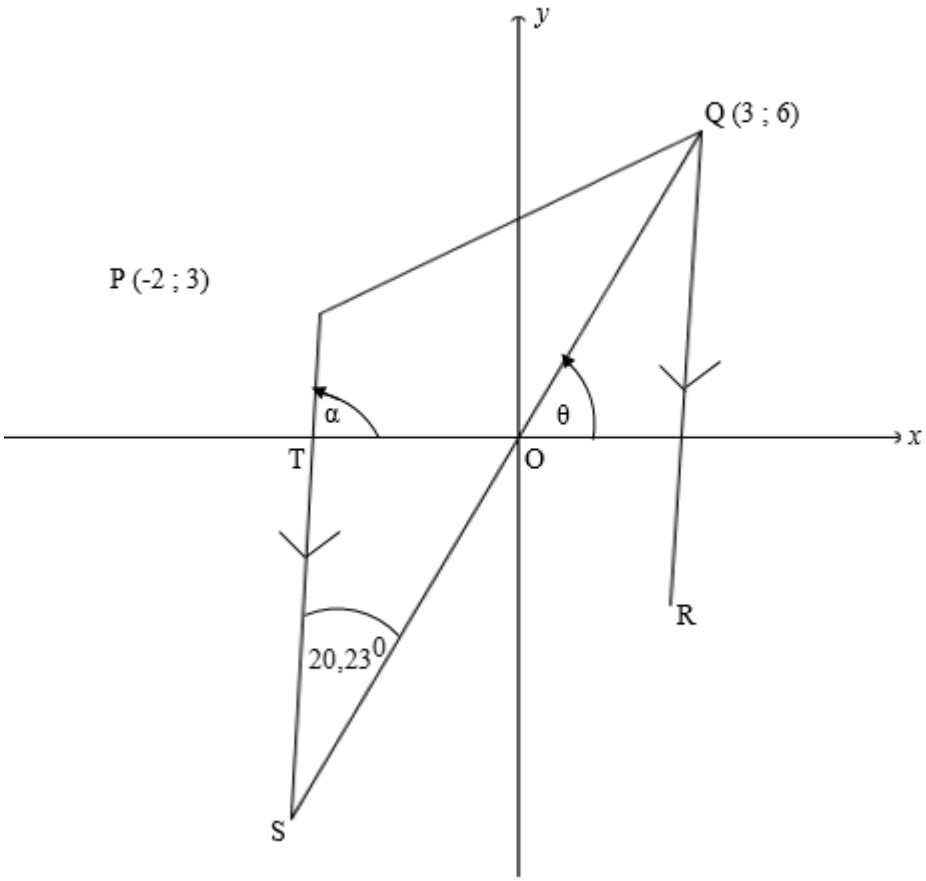
QUESTION 1 / *VRAAG* 1		
12 15 15 19 22 23 26 26 32 33 33 33 33 35 35		
1.1.1	26	✓ answer / *antwoord* (1)
1.1.2	IQR = 33-19 = 14 Answer only – full marks *Slegs antwoord – volpunte*	✓ Q_1 ✓Q_2 ✓ answer / *antwoord* (3)
1.1.3	Answer only – full marks *Slegs antwoord – volpunte* $\bar{x} = \frac{392}{15} = 26{,}13$	✓ $\frac{392}{15}$ ✓ answer / *antwoord* (2)
1.1.4	$\delta = 7.77$	✓ answer / *antwoord* ✓ (2)
1.2	$(\bar{x} - \delta; \bar{x} + \delta) = (18{,}36\ ;\ 33{,}90)$ 5 people that are outside one standard deviation of the mean 5 *mense is buite een standaardafwyking vanaf die gemiddelde* $\therefore$ It will be / *Dit sal*: $\frac{5}{15} \times 100 = 33{,}33\%$	✓interval / *interval* ✓ no. of people outside one SD / *aantal mense buite een SA* ✓ percentage / *persentasie* (3)
		[11]

QUESTION 2 / *VRAAG 2*



2.1	80	✓ answer / *antwoord* (1)
2.2	$80-22=58$	✓$80-22$ ✓answer / *antwoord* (2)
2.3	$Q_2 = 28$	✓✓answer / *antwoord* (2)
2.4	**Marks obtained** / ***Punte behaal*** | **Frequency** / ***Frekwensie*** $0 < x \leq 10$ | 5 $10 < x \leq 20$ | 17 $20 < x \leq 30$ | 23 $30 < x \leq 40$ | 19 $40 < x \leq 50$ | 13 $50 < x \leq 60$ | 3	✓ 17 ✓ 17 ✓ 23 ✓ 3 ✓ 19 ✓ 13 ✓ 3 (5) (2)
2.5	$20 < x \leq 30$	✓ (1)
		[11]

QUESTION 3 / *VRAAG 3*



3.1	$m_{QS} = \frac{6-0}{3-0} = 2$	✓ subst. / *vervang.* ✓ gradient / *gradiënt* (2)
3.2	$\tan\theta = 2$ $\therefore \theta = 63{,}43^0$	✓ $\tan\theta = m_{QS}$ ✓ $\theta = 63{,}43^0$ (2)
3.3		
3.3.1	$T\hat{O}S = \theta = 63{,}43^o$ [vert. opp. $\angle_s$ / *regoorst.*$\angle^{'e}$] $\alpha = 63{,}43^o + 20{,}23^o = 83{,}66^o$ [ext. ∠ of a Δ / ext. ∠ of a Δ / buite ∠ van Δ] $m_{ps} = \tan(83{,}66^o)$ $= 9$	✓ $T\hat{O}S = 63{,}43^0$ ✓ $\alpha = 83{,}66^0$ ✓ $m_{PS} = \tan(83{,}66^0)$ ✓ $m_{PS} = 9$ (4)

3.3.2	$y-3=9(x+2)$ $y=9x+21$	✓ $m_{PS}=9$ ✓ subst. *m* and (-2;3) / *vervang. m en* (-2 ; 3) ✓ equation of PS / *vergelyking van* PS (3)
3.4	$9x+21=2x$ $7x=-21$ $x=-3$ $y=2(-3)$ $=-6$ S(-3 ; -6)	✓ $9x+21=2x$ ✓ simplification / *vereenvoudiging* ✓ *x*-value / *x-waarde* ✓ *y*-value / *y-waarde* (4)
3.5	$QS=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(3+3)^2+(6+6)^2}$ $=6\sqrt{5}$	✓ correct formula / *korrekte formule* ✓ correct subst. / *korrekte vervanging* ✓ $QS=6\sqrt{5}$ (3)
3.6	$PS=\sqrt{(-2+3)^2+(3+6)^2}$ $=\sqrt{82}$ Area $\Delta PQS=\frac{1}{2}.QS.PS\sin 20{,}23$ $=\frac{1}{2}.6\sqrt{5}.\sqrt{82}\sin 20{,}23$ $=21{,}01$	✓ correct subst. / *korrekte vervanging* ✓ $PS=\sqrt{82}$ ✓ correct formula for Area of ΔPQS *korrekte formule vir Opp. van ΔPQS* ✓ correct subst. / *korrekte vervang.* ✓ 21,01 (5)
3.7	$\frac{-2+x}{2}=0$; $\frac{3+y}{2}=0$ [PO = OR, diag. of a parm / *hoeklyne van 'n ||m*] $x=2$ $y=-3$ R(2 ; -3) OF/OR Alternate proof $\frac{y-6}{x-3}=\frac{-6-3}{-3+2}$ $y-6=-9$ and $x-3=-1$ $y=-3$ and $x=-3$ R(2;-3)	✓ equation for mid point / *verg. van middelpunt.* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde* ✓ equation for gradient / *verg. vir gradiënt.* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde* (3)

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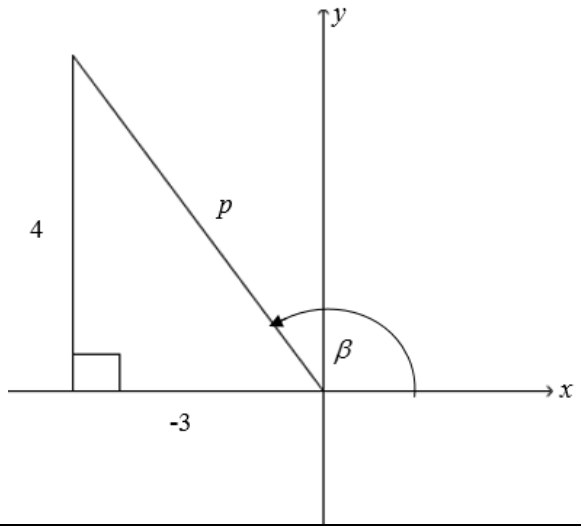
3.8	$m_{AB} = m_{BC}$ $\frac{4+1}{5-0} = \frac{2+1}{t-0}$ $1 = \frac{3}{t}$ $\therefore t = 3$	(collinear points have equal gradients) $(ko-linière\ punte\ het\ gelyke\ gradiënte)$	✓ $m_{AB} = m_{BC}$ ✓ substitution / *vervanging* ✓ $m_{AB} = 1$ ✓ $t = 3$ (4)
			[30]

QUESTION 4 / *VRAAG 4*			
4.1	4.1.1	$\sin\beta = \frac{4}{p}$ and/*en* $\cos\beta = -\frac{3}{p}$ β is in the 2nd quadrant where the ratio of $\sin\beta$ is positive and ratio of $\cos\beta$ is negative. / *β is in die 2de kwadrant waar die verhouding van sin β positief is en die verhouding van cos β negatief is.*	✓ $\sin\beta = \frac{4}{p}$ ✓ $\cos\beta = -\frac{3}{p}$ ✓ Any relevant explanation./ *Enige relevante verduideliking* (3)
	4.1.2	$\tan\beta = \frac{\sin\beta}{\cos\beta}$ $= \frac{4}{p} \times -\frac{p}{3}$ $= -\frac{4}{3}$ OR/*OF*  	✓ identity / *identiteit* ✓ $\frac{4}{p} \times -\frac{p}{3}$ (2) OR/*OF* ✓ diagram / *diagram* ✓ (2)
	4.1.3	$p = \sqrt{(-3)^2 + 4^2}$ $= 5$	✓ subst. in Pyth. *vervang. in Pyth.* ✓ value of *p / waarde van p* (2)

4.2	$\sin 143^0.\cos 127^0 - \sin 53^0.\cos(37^0)$ $= \sin(180^0 - 37^0).\cos(90^0 + 37^0) - \sin(90^0 - 37^0).\cos 37^0$ $= \sin 37^0.(-\sin 37^0)(-\cos 37^0).\cos 37^0$ $= -\sin^2 37^0 - \cos^2 37^0$ $= -(\sin^2 37^0 + \cos^2 37^0)$ $= -1$	✓ $\sin 37^0$ ✓ $-\sin 37^0$ ✓ $-\cos 37^0$ ✓ $-(\sin^2 37^0 + \cos^2 37^0)$ ✓ -1 (5)
4.3	L.H.S/LK. $= \left(\tan y + \frac{1}{\tan y}\right)(1 - \cos^2 y)$ $= \left(\frac{\sin y}{\cos y} + \frac{\cos y}{\sin y}\right)(\sin^2 y)$ $= \left(\frac{\sin^2 y + \cos^2 y}{\cos y.\sin y}\right)\sin^2 y$ $= \frac{1}{\cos y.\sin y}.\sin^2 y$ $= \frac{\sin y}{\cos y}$ $= \tan y$	✓ $\frac{\sin y}{\cos y}$ ✓ $\frac{\cos y}{\sin y}$ ✓ $\sin^2 y$ ✓ $\frac{\sin^2 y + \cos^2 y}{\cos y.\sin y}$ ✓ 1 ✓ $\frac{\sin y}{\cos y}$ (6)
4.4	$\cos\theta - \frac{1}{\cos\theta} = \frac{5}{6}$ $6\cos^2\theta - 5\cos\theta - 6 = 0$ $(3\cos\theta + 2)(2\cos\theta - 3) = 0$ $\cos\theta = -\frac{2}{3}$ or/*of* $\cos\theta = \frac{3}{2}$ no solution/*geen oplossing* $\theta = \pm 131{,}81^0 + 360^0.k;\ k \in Z$	✓ standard form *standaardvorm* ✓ factors / *faktore* ✓ both equations *beide oplossings* ✓ no solution / *geen oplossing* ✓ $\theta = \pm 131{,}81^0$ ✓ $360^0.k$ and/*en* $k \in Z$ (6)
		[24]

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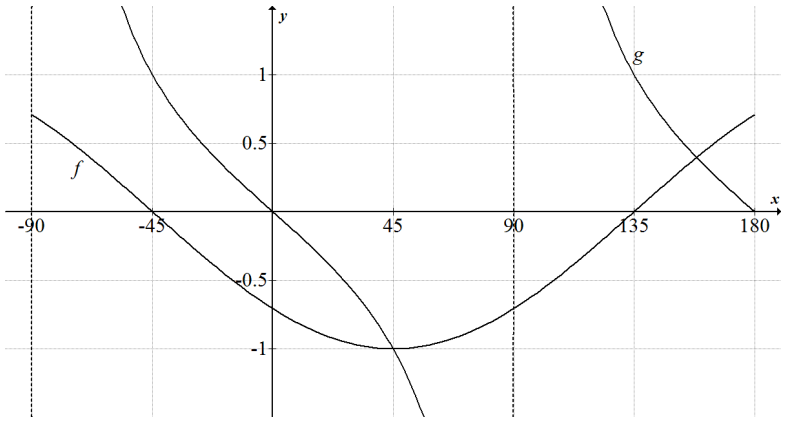
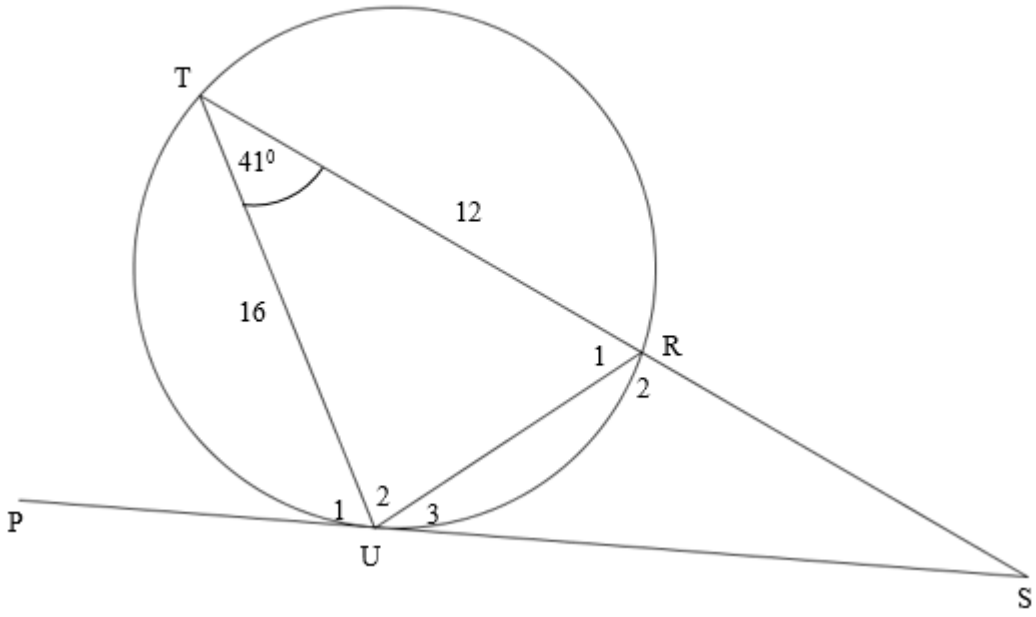
QUESTION 5 / *VRAAG 5*		
5.1		*f*: ✓ *x*-intercepts *x-afsnit* ✓ min. and max. *min. en maks.* ✓ shape / *vorm* *g:* ✓ intercepts / *afsnitte* ✓ asymptotes *asimptote* ✓ shape / *vorm* (6)
5.2	$-90^0 < x \leq 45^0$	✓ correct critical value / *korrekte kritiese waarde* ✓ notation / *notasie* (2)
5.3	$h(x) = -\{-\cos(45^0 - x + 45^0)\}$ $= -\{-\cos(90^0 - x)\}$ $= \sin x$	✓ subst. / *vervang.* ✓ sin *x* (2)
		[10]

QUESTION 6 / *VRAAG 6*

6.1 $c^2 = a^2 + b^2 - 2ab\cos\hat{C}$ ✓ answer / *antwoord* (1)

6.2



6.2.1

$UR^2 = TU^2 + TR^2 - 2TR.TU.\cos\hat{T}$
$= 16^2 + 12^2 - 2(16)(12)\cos 41^0$
$= 110{,}191$
$\therefore UR = 10{,}50$

✓ cosine rule / *cosinusreël*
✓ correct subst./*korrekte vervang.*
✓ UR = 10,50 (3)

6.2.2

$\frac{\sin\hat{U}_2}{12} = \frac{\sin\hat{T}}{10{,}50}$
$\sin\hat{U}_2 = \frac{12.\sin 41^0}{10{,}5}$
$\hat{U}_2 = 48{,}57^0$

OR/OF

$TR^2 = UR^2 + TU^2 - 2.UR.TU.\cos\hat{U}_2$
$(12)^2 = (10{,}50)^2 + (16)^2 - 2(10{,}50)(16)\cos\hat{U}_2$
$\cos\hat{U}_2 = \frac{(10{,}5)^2 + (16)^2 - (12)^2}{2(10{,}50)(16)}$
$\hat{U}_2 = 48{,}57^0$

✓ use of sine rule / *gebruik van sinusreël*
✓ correct subst./*korrekte vervang.*
✓ $\hat{U}_2$ (3)

✓ use of cosine rule *gebruik van cosinusreël*
✓ correct substitution *korrekte vervanging*
✓ $\hat{U}_2$ (3)

	6.2.3	$\hat{U}_3 = 41^0$ $[\text{tan chord theorem}]$ $[\textit{raaklyn koord stelling}]$ $\therefore \hat{U}_{2+3} = 89{,}57^0$ $\hat{S} = 49{,}43^0$ $[\angle_s \text{in } \Delta \text{URS}]$ $\frac{RS}{\sin 41^0} = \frac{10{,}5}{\sin 49{,}43^0}$ $RS = 9{,}07$ $\therefore T\hat{R}S = 21{,}07$	✓ S ✓ S ✓ S ✓ correct subst. in sine rule / *korrekte vervang. in sinusreël* ✓ $T\hat{R}S$ (5)
			[12]

QUESTION 7 / *VRAAG 7*

7.1	Surface area of a hemisphere: *Buite oppervlak van hemisfeer:* $= 2\pi r^2$ $= 2 \times \pi \times 10^2 = 628{,}32\,m^2$ or/of $= 2\pi rh$ $= 2 \times \pi \times 10 \times 10$ $= 628{,}32\,m^2$ Surface area of a cylinder: *Buite oppervlak van silinder:* $= 2 \times \pi \times 10 \times 75 + \pi \times 10^2 = 5026{,}55m^2$ $\therefore$ TSA of the tank / *TBO van die tenk* $= 5654{,}87\,m^2$	✓ correct subst./*korrekte vervang*. ✓ Area of hemisphere / *Oppervlakte van hemisfeer* Using π =3,14 SA/BO = 628 m^2 ✓ correct subst./*korrekte vervang.* ✓ Area of a cylinder / *Oppervlakte van silinder* Using π =3,14 SA/BO = 5024 m^2 ✓ TSA / TBO Using π =3,14 TSA/TBO = 5652 m^2 (5)
7.2	V. hemisphere/*V. van hemisfeer* $= \frac{2}{3} \times \pi \times 10^3 = 2094{,}40\,m^3$ V. cylinder/*V. van silinder* $= \pi \times 10^2 \times 75 = 23561{,}94\,m^3$ V. of the tank/*V. van die tenk* $= 25656{,}34\,m^3$	✓ correct subst./*korrekte vervang.* ✓ V. of hemi. / *V. van hemisfeer* Using π = 3,14 Volume = 2 093 m^3 ✓ Correct subst./*korrekte vervang.* ✓ V. of cylinder / *V. van silinder* Using π = 3,14 $V_{cyl/sil}$ = 23 550 m^3 ✓ V. of the tank / *V. van tenk* Using π = 3,14 $Vol_{tank/tenk}$ = 25 643 m^3 (5)
		[10]

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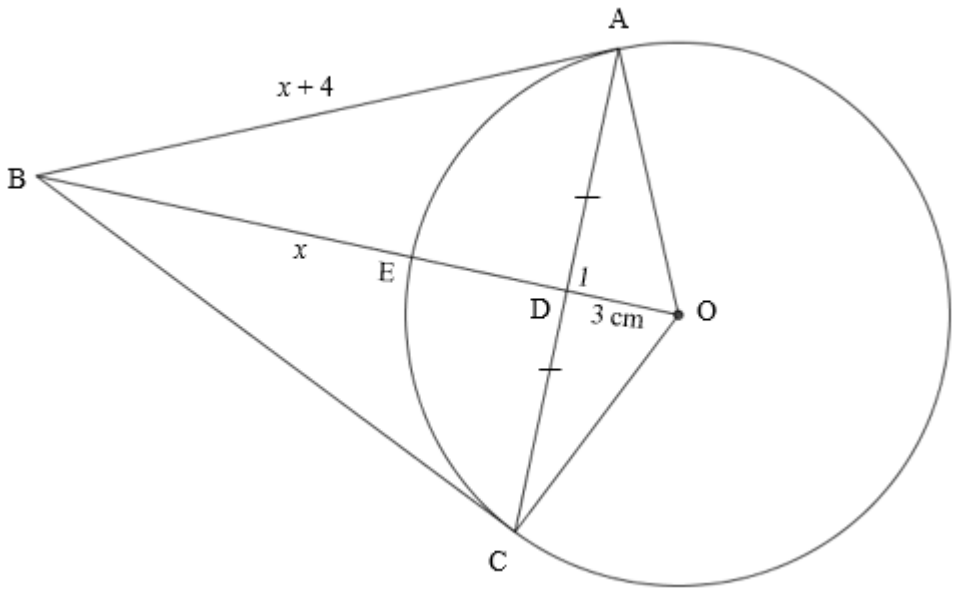
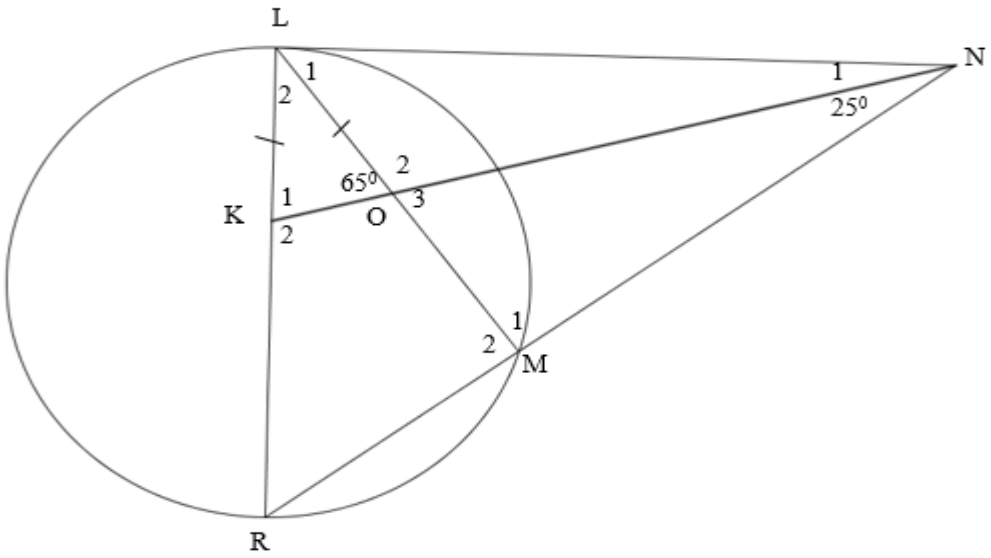
QUESTON 8 / *VRAAG 8*



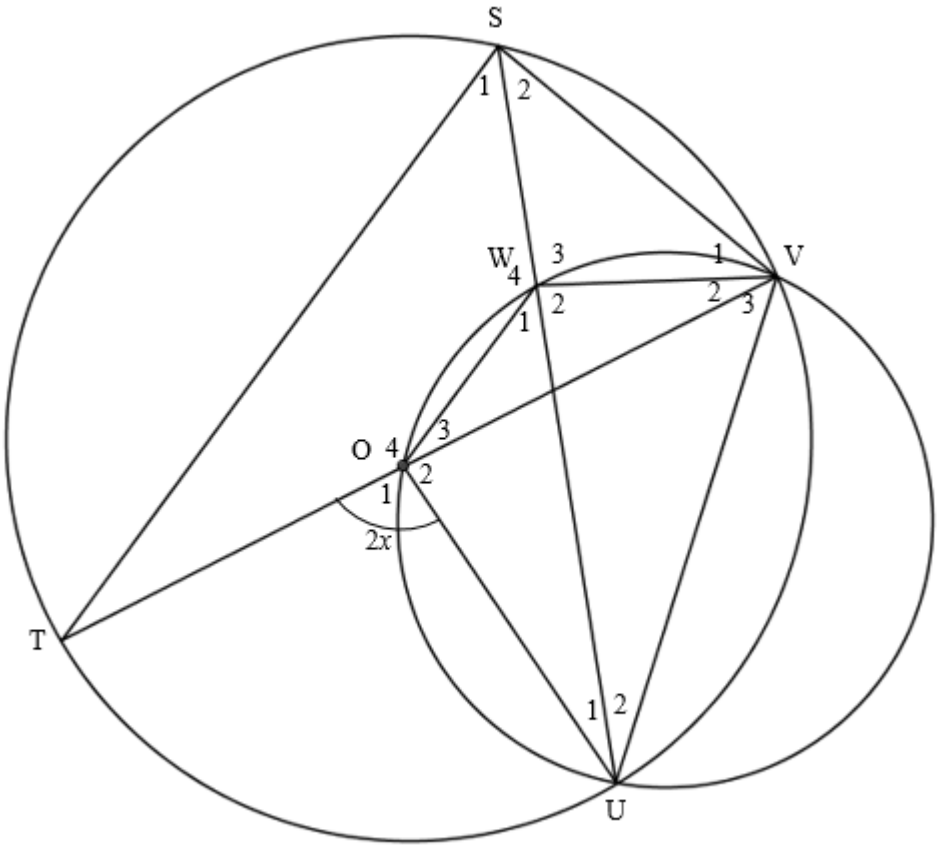
8.1	$\hat{D}_1 = 90^0$ [line from centre to midpoint of chord] [*lyn vanaf middelpt van sirkel na middelpt van koord*]	✓ S ✓R (2)
8.2	$OA^2 = OD^2 + AD^2$ $= (3\text{ cm})^2 + (4\text{ cm})^2$ $\therefore OA = 5\text{ cm}$	✓ subst. in Pyth. *vervang. in Pyth* ✓ answer/ *antwoord* (2)
8.3	$\hat{A} = 90^0$ [tan ⊥ rad] / [*raaklyn ⊥ radius*]	✓S✓R (2)
8.4	$(x+5)^2 = (x+4)^2 + 5^2$ $x^2 + 10x + 25 = x^2 + 8x + 16 + 25$ $2x = 16$ $x = 8$	✓ subst. into Pyth / *vervang. in Pyth* ✓ simplification / *vereenvoudiging* ✓ standard form / *standaardvorm* ✓ *x*-value / *x-waarde* (4)
		[10]

QUESTION 9 / *VRAAG 9*



9.1	$\hat{O}_3 = 65^0$ [vert.opp ∠s]/[*regoorst.∠e*] $\hat{M}_1 = 90^0$ [sum of ∠s of a Δ]/[*som van ∠e van* Δ] ∴ LN is a diameter to circle LMN [chord subtends 90^0 **or** converse ∠ in a semi-circle] ∴ LN *is 'n middellyn van sirkel* LMN [*koord onderspan* 90^o **of** *omgekeerde van ∠ in semi-sirkel*]	✓ S ✓R ✓ S ✓R ✓R (5)
9.2	$\hat{N}_1 = 25^0$ [ON bisects $\hat{N}$]/[*ON halveer* $\hat{N}$] $\hat{L}_1 = 40^0$ [ext ∠ of a Δ LON = sum of int opp ∠s]/[*buite ∠ van* Δ LON = *som van teenoorst.binne ∠e*] $\hat{K}_1 = 65^0$ [∠s opp = sides]/[*∠e teenoor gelyke sye*] $\hat{L}_2 = 50^0$ [sum of ∠s of a Δ]/[*som van ∠e van 'n* Δ] $\therefore \hat{L} = 90^0$	✓S/R ✓S/R ✓S ✓R ✓S/R (5)
9.3	$\hat{R} = 40^0$ [sum of ∠s of a Δ]/[*som van ∠e van 'n* Δ] $\therefore \hat{R} = \hat{L}_1$ ∴ LN is a tangent [∠ between a line and a chord]/[*∠ tussen lyn en koord*]	✓S ✓R ✓R (3)
9.4	Yes. LR subtends 90^0 **OR** $\hat{M}_2 = 90^0$ [LR is a diameter] *JA.* LR *onderspan* 90^o **OF** $\hat{M}_2 = 90^0$ [LR *is 'n middellyn*]	✓Yes / *Ja* ✓Reason / *Rede* (2)
		[15]

QUESTION 10 / *VRAAG 10*



10.1	$\hat{S}_1 = x$	$[\angle \text{at centre} = 2 \times \angle \text{at circumf.}]$ [*Middelpts* $\angle = 2 \times$ *Omtreks* $\angle$]	✓S ✓R
	$\hat{V}_3 = x$	$[\angle \text{s in same segment}]/[\angle e \text{ in dieselfde segment}]$	✓S ✓R
	$\hat{U} = x$	$[\text{ext.} \angle \text{ of a } \Delta \text{OVU} = \text{sum of opp int} \angle \text{s}]$ [*buite* $\angle$ *van* ΔOVU = *som van teenoorst.binne* $\angle e$]	✓S ✓R
	$\hat{W}_1 = \hat{V}_3 = x$	$[\angle \text{s in same segmet}]/[\angle e \text{ in dieselfde segment}]$	✓ S ✓R (8)
10.2	$\hat{O}_2 = 180^0 - 2x$	$[\angle \text{s on a straight line}]/[\angle e \text{ op 'n reguitlyn}]$	✓S ✓R
	$\hat{W}_2 = \hat{O}_2 = 180^0 - 2x$	$[\angle \text{s in same segment}]/[\angle e \text{ in dieselfde segment}]$	✓S ✓R (4)

10.3	$\hat{S}_{1+2} = 90^0$ $\left[\angle \text{in a semi} - \text{circle}\right]/[\angle \textit{in semi} - \textit{sirkel}]$	✓S	✓R
	$\therefore \hat{S}_2 = 90^0 - x$		
	$\hat{V}_1 = \hat{W}_2 - \hat{S}_2$ $\left[\text{ext.}\angle \text{of a}\Delta\right]/[\textit{buite}\angle \textit{van'n}\Delta]$	✓S	✓R
	$= 180^0 - 2x - \left(90^0 - x\right)$		
	$= 90^0 - x$		
	$\therefore \text{SW} = \text{WV}$ $\left[\text{sides opp.} = \angle s\right]/[\textit{sye teenoor gelyke hoeke}]$		✓R
			(5)
			[17]
		TOTAL/TOTAAL:	**150**

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