



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE/GRADE 12

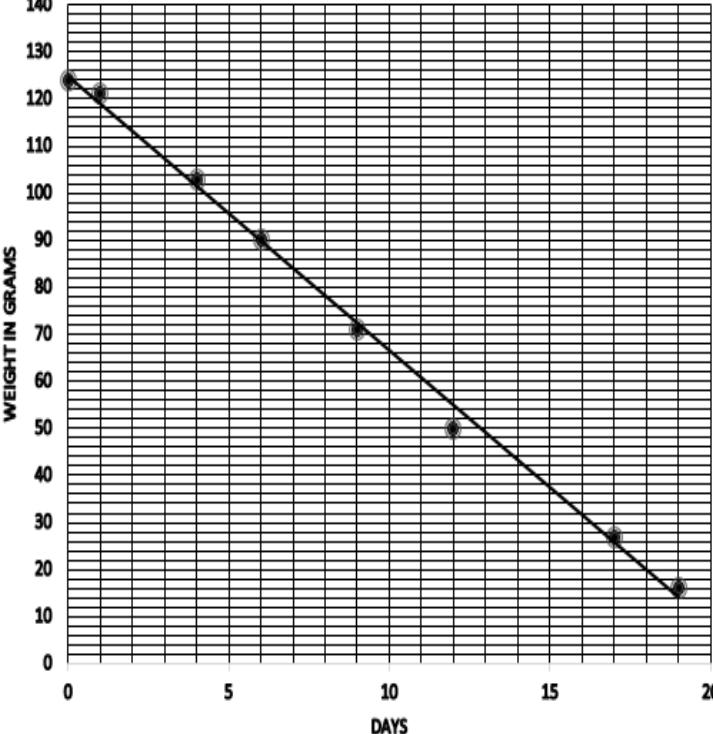
SEPTEMBER 2016

**MATHEMATICS P2 / WISKUNDE V2
MEMORANDUM**

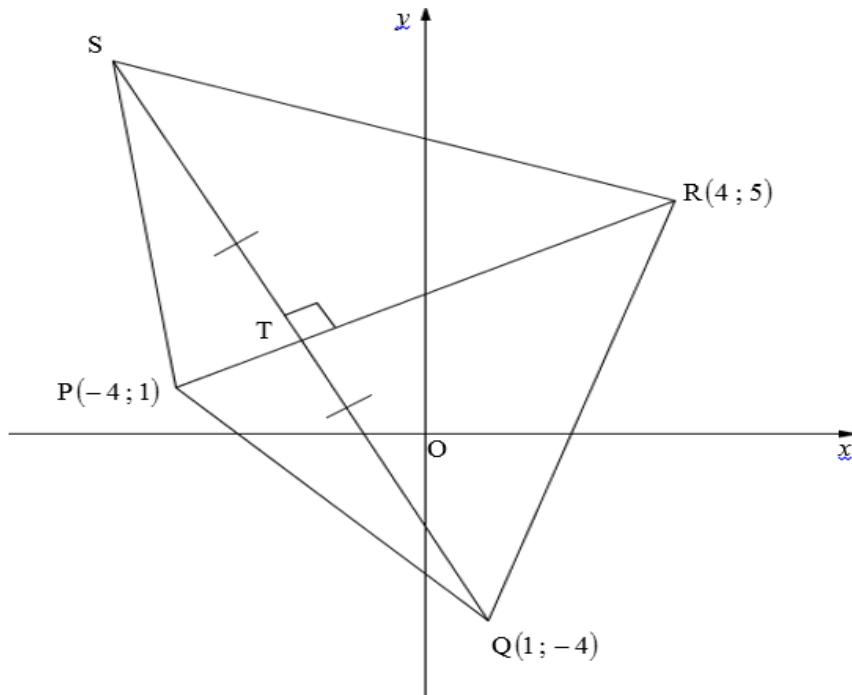
MARKS / PUNTE: **150**

This memorandum consists of 16 pages.
Hierdie memorandum bestaan uit 16 bladsye.

QUESTION 1 / VRAAG 1

	Day/Dag	0	1	4	6	9	12	17	19		
	Weight / Gewig	124	121	103	90	71	50	27	16		
1.1				<ul style="list-style-type: none"> ✓ 2-4 correct points ✓ 5-7 correct points ✓ plotting all points ✓ 2 – 4 punte korrek ✓ 5 – 7 punte korrek ✓ alle punte afgesteek 							
1.1											
1.2	$a = 124,84$ $b = -5,83$ $y = 124,84 - 5,83x$	<ul style="list-style-type: none"> ✓ A ✓ B ✓ equation / vergelyking 									
1.3	$(x; y) = (8; 5; 75,25)$ y-int 124,84	<ul style="list-style-type: none"> ✓ $(8; 5; 75,25)$ and/en ✓ y-int / y-afsnit 124,84 ✓ regression line / regressielijn 									
1.4	$124,84 - 5,83x = 80$ $5,83x = 44,84, 69$ $x = 7,69$ On the morning of the 8 th day the bar of soap will be less than 80 grams. <i>Op dieoggend van die 8^{ste} dag sal steen seep minder as 80 gram wees.</i>	<ul style="list-style-type: none"> ✓ substitution / vervanging ✓ answer / antwoord 									
1.5	$r = -0,998$	<ul style="list-style-type: none"> ✓ answer / antwoord 									
1.6	Very strong negative correlation. <i>Baie sterk negatiewe korrelasie.</i>	<ul style="list-style-type: none"> ✓ answer / antwoord 									
										[12]	

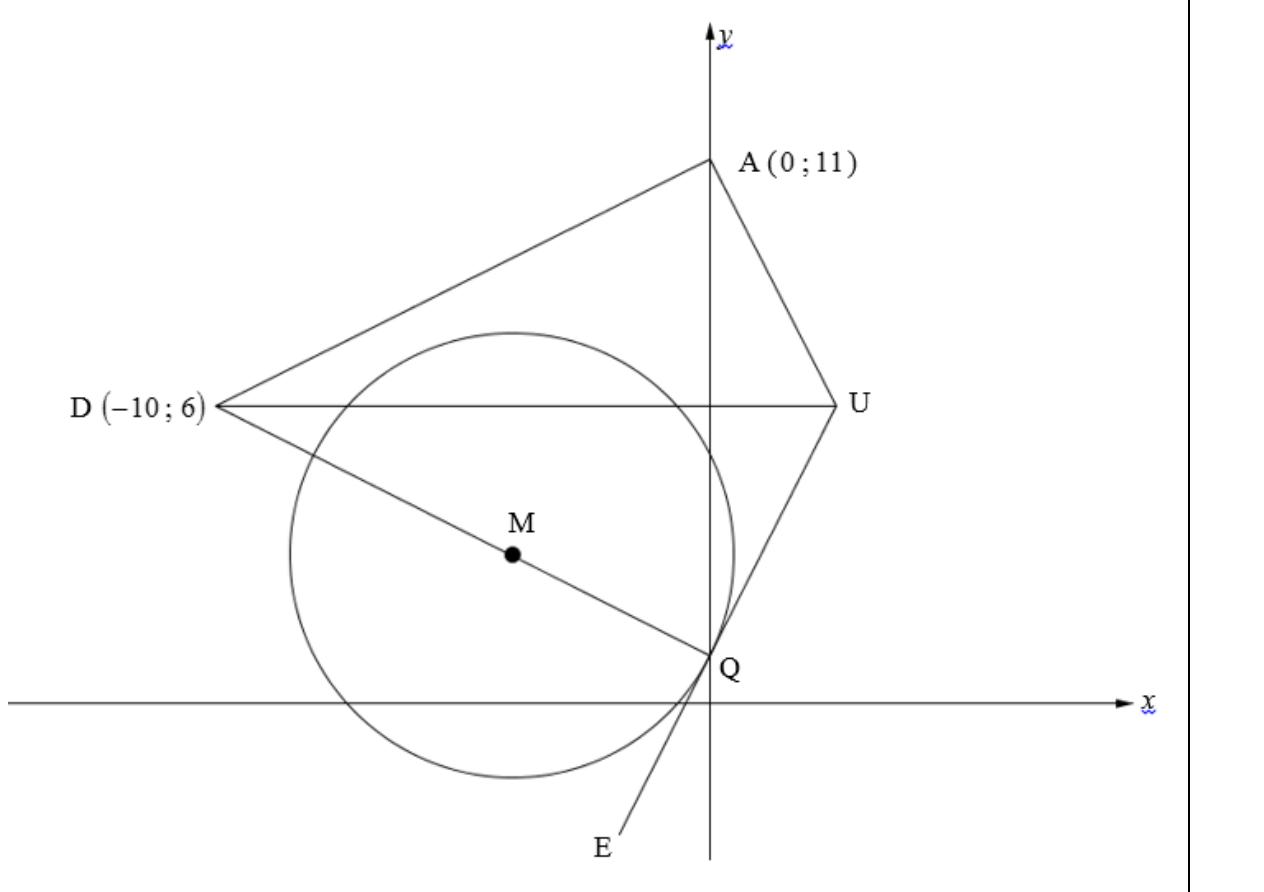
QUESTION 2 / VRAAG 2		
5	7	
4	3 4 4	
3	1 1 1 4 6 8	
2	2 4 8 9 9	
1	5 5 6 7	
0	3 5	
2.1	21 learners / leerders	✓ answer / antwoord (1)
2.2	3 pages / bladsye	✓ answer / antwoord (1)
2.3	$\bar{x} = 28,19$	✓✓ answer / antwoord (2)
2.4	$\sigma = 13,12$	✓ answer / antwoord (1)
2.5	$(28,19 - 13,12; 28,19 + 13,12)$ $(15,07; 41,31)$ $\therefore 8$ learners are outside one standard deviation 8 leerders is buite een standaardafwyking $\therefore \frac{8}{21} \times 100 = 38,10\%$	✓ interval / interval ✓ 8 learners / leerders ✓ 28,57% (3)
		[8]

QUESTION 3 / VRAAG 3

3.1	$m_{PR} = \frac{5-1}{4+4}$ $= \frac{1}{2}$	<ul style="list-style-type: none"> ✓ subst. P and R into correct formula <i>verv. van P en R in korrekte formule</i> ✓ $m_{PR} = \frac{1}{2}$ 	(2)
3.2	$m_{SQ} = -2$ $y + 4 = -2(x - 1)$ $y = -2x - 2$	<ul style="list-style-type: none"> ✓ $m_{SQ} = -2$ ✓ subst. m and Q into correct formula <i>verv. van m en Q in korrekte formule</i> ✓ $y = -2x - 2$ 	(3)
3.3	Equation of PR / <i>Vergelyking van PR</i> $y - 1 = \frac{1}{2}(x + 4)$ $y = \frac{1}{2}x + 3$ $-2x - 2 = \frac{1}{2}x + 3$ $-4x - 4 = x + 6$ $-5x = 10$ $\therefore x = -2$ $y = -2(-2) - 2 = 2$	<ul style="list-style-type: none"> ✓ substituting m and P into equation of a str. line / <i>vervanging van m en P in die vergelyking van reguitlyn</i> ✓ equation of PR / <i>vergelyking van PR</i> ✓ equating PR and SQ / <i>gelykstel van PR en SQ</i> ✓ x-value / <i>x-waarde</i> ✓ y-value / <i>y-waarde</i> 	
OR/OF			<ul style="list-style-type: none"> ✓ substituting m and R into equation of a

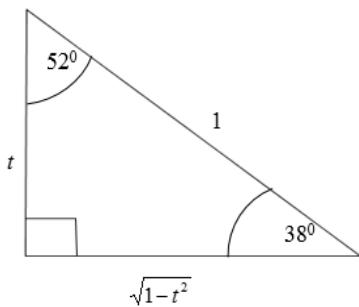
	$y - 5 = \frac{1}{2}(x - 4)$ $2y - 10 = x - 4$ $2y = x + 6$ $y = \frac{1}{2}x + 3$ $-2x - 2 = \frac{1}{2}x + 3$ $-4x - 4 = x + 6$ $-5x = 10$ $\therefore x = -2$ $y = -2(-2) - 2 = 2$	str. line / vervanging van m en R in die vergelyking van reguitlyn ✓ equation of PR / vergelyking van PR ✓ equating PR and SQ / gelykstel van PR en SQ ✓ x-value / x-waarde ✓ y-value / y-waarde	(5)
3.4	$-2 = \frac{x+1}{2} \quad 2 = \frac{y-4}{2}$ $\therefore x = -5 \quad \text{and/en} \quad y = 8$ $\text{S } (-5 ; 8)$	✓ substituting into correct formula vervanging in korrekte formule ✓ x-value / x-waarde ✓ y-value / y-waarde	(3)
3.5	$\text{SQ} = \sqrt{(-5-1)^2 + (8+4)^2}$ $= 6\sqrt{5}$ $\text{PT} = \sqrt{(-2+4)^2 + (2-1)^2}$ $= \sqrt{5}$ $\text{Area } \Delta \text{PQS} = \frac{1}{2} \times 6\sqrt{5} \times \sqrt{5}$ $= 15 \text{ unit}^2 / \text{eenhede}^2$ <p style="text-align: center;">OR/OF</p> $\therefore \text{area of } \Delta \text{ PQS} = ST \times PT$ $= 3\sqrt{5} \times \sqrt{5}$ $= 15 \text{ unit}^2 / \text{eenhede}^2$	✓ subt. into correct form / verv. in korrekte formule ✓ $\text{SQ} = 6\sqrt{5}$ ✓ $\text{PT} = \sqrt{5}$ ✓ Subt into correct form. / verv. in korrekte formule ✓ $15 \text{ units}^2 / \text{eenhede}^2$ ✓ $\text{SQ} = 6\sqrt{5}$ ✓ $\text{ST} = 3\sqrt{5}$ ✓ $\text{PT} = \sqrt{5}$ ✓ subst into form / verv. in formule ✓ $15 \text{ units}^2 / \text{eenhede}^2$	(5)
			[18]

QUESTION 4 / VRAAG 4



4.1	$\begin{aligned}x^2 + 8x + 16 + y^2 - 6y + 9 = 20 \\(x+4)^2 + (y-3)^2 = 20 \\\therefore M(-4; 3)\end{aligned}$	<ul style="list-style-type: none"> ✓ completing square <i>kwadraatsvoltooiing</i> ✓ $(x+4)^2 + (y-3)^2 = 20$ ✓ x-coordinate / x-koördinaat ✓ y-coordinate / y-koördinaat 	(4)
4.2	$\begin{aligned}(0+4)^2 + (y-3)^2 = 20 \\(y-3)^2 = 4 \\y = 3 \pm 2 \\\therefore y = 1 \\Q(0; 1)\end{aligned}$	<ul style="list-style-type: none"> ✓ subst. $x = 0$ into circle equation <i>verv. $x = 0$ in sirkel vergelyking</i> ✓ $(y-3)^2 = 4$ ✓ $y = 1$ 	(3)
4.3	$\begin{aligned}m_{radius} &= \frac{3-1}{-4-0} = -\frac{1}{2} \\m_{tan/rkl} &= 2 \\y-1 &= 2(x-0) \quad [\text{tangent } \perp \text{ radius}] \\&\quad [\text{raaklyn } \perp] \\\therefore y &= 2x+1\end{aligned}$	<ul style="list-style-type: none"> ✓ $m_{rad} = -\frac{1}{2}$ ✓ $m_{tan} = 2$ / $m_{raaklyn} = 2$ ✓ subst. $m_{tan/rkl} = 2$ and Q into correct form. / verv. $m_{tan/rkl} = 2$ en Q in korrekte formule. ✓ equation / vergelyking 	(4)
4.4	$y = 6$	✓ answer / antwoord	(1)

4.5	$6 = 2x + 1$ $x = \frac{5}{2}$ $U\left(\frac{5}{2}; 6\right)$	$\checkmark \quad 6 = 2x + 1$ $\checkmark \quad x = \frac{5}{2}$	(2)
4.6	$m_{AU} = \frac{11-6}{0-\frac{5}{2}}$ $= -2$ $m_{AD} = \frac{6-11}{-10-0}$ $= \frac{1}{2}$ $m_{AU} \times m_{DA} = -2 \times \frac{1}{2}$ $= -1$ $\therefore AU \perp DA$ $\therefore \hat{A} = 90^\circ$ $D\hat{Q}U = 90^\circ$ [tangent \perp radius] / [raaklyn \perp radius] $\therefore QUAD$ is a cyclic quad. [opp. \angle^s add up to 180°] $QUAD$ is 'n koordevierhoek [teenoorst. \angle^e se som is 180°]	$\checkmark \quad m_{AU} = -2$ $\checkmark \quad m_{AD} = \frac{1}{2}$ $\checkmark \quad m_{AU} \times m_{DA} = -1$ $\checkmark \quad \hat{A} = 90^\circ$ $\checkmark \quad D\hat{Q}U = 90^\circ$ $\checkmark \quad R$	(6)
			[20]

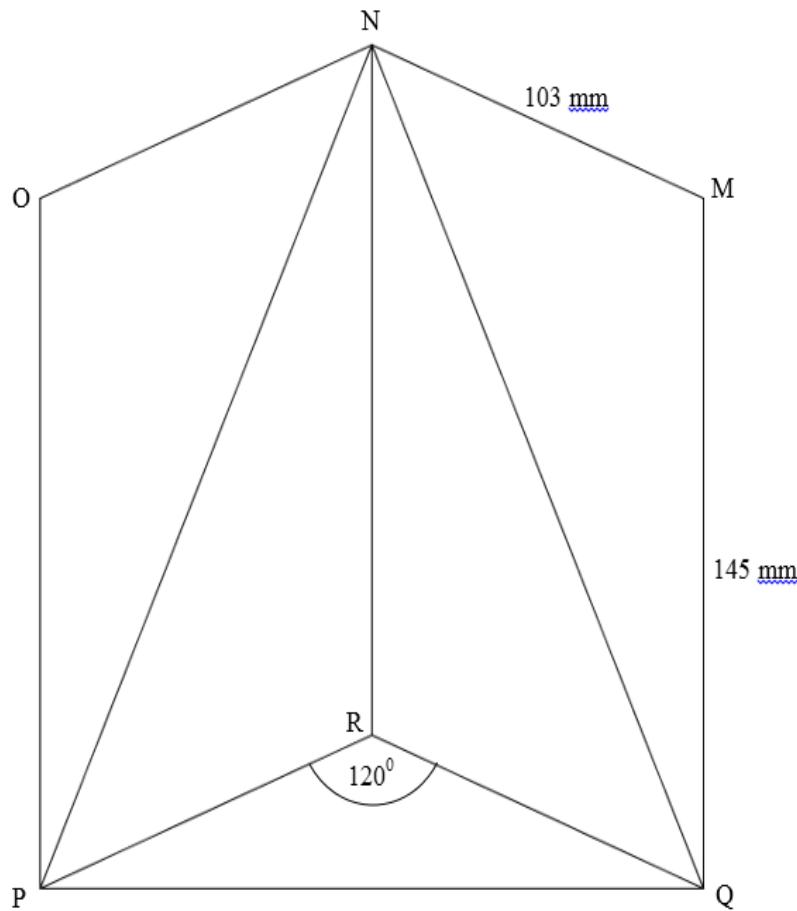
QUESTION 5 / VRAAG 5

5.1.1	$\sin(-52^\circ) = -\sin 52^\circ$ $= \sqrt{1-t^2}$	✓ - sin 52° ✓ ✓ answer (3)
5.1.2	$\cos(2.19^\circ) = \cos 38^\circ$ $2\cos^2 19^\circ - 1 = \cos 38^\circ$ $\cos 19^\circ = \sqrt{\frac{\cos 38^\circ + 1}{2}}$ $\therefore \cos 19^\circ = \sqrt{\frac{\sqrt{1-t^2} + 1}{2}}$	✓ $\cos(2.19^\circ) = \cos 38^\circ$ ✓ $\cos(2.19^\circ) = 2\cos^2 19^\circ - 1$ ✓ simplification / vereenvoudiging ✓ answer / antwoord (4)
5.2	$\frac{2\cos(180^\circ + x).\sin(180^\circ - x).\sin 74^\circ}{\sin(x + 360^\circ).\sin 37^\circ.\sin 53^\circ.\sin(x - 90^\circ)}$ $= \frac{2(-\cos x).\sin x.\sin 74^\circ}{\sin x.\sin 37^\circ.\cos 37^\circ.(-\cos x)}$ $= \frac{2\sin 74^\circ}{\frac{1}{2}\sin 74^\circ}$ $= 4$ <p>OR/OF</p> $\frac{2\cos(180^\circ + x).\sin(180^\circ - x).\sin 74^\circ}{\sin(x + 360^\circ).\sin 37^\circ.\sin 53^\circ.\sin(x - 90^\circ)}$ $= \frac{2(-\cos x).\sin x.\sin(2 \times 37^\circ)}{\sin x.\sin 37^\circ.\cos 37^\circ.(-\cos x)}$ $= \frac{2\sin(2 \times 37^\circ)}{\sin 37^\circ.\cos 37^\circ}$ $= \frac{4\sin 37^\circ.\cos 37^\circ}{\sin 37^\circ.\cos 37^\circ}$ $= 4$	✓ - cos x ✓ sin x ✓ sin x ✓ cos 37° ✓ - cos x ✓ $\frac{1}{2}\sin 74^\circ$ ✓ answer / antwoord ✓ - cos x ✓ sin x ✓ sin x ✓ cos 37° ✓ - cos x ✓ $2\sin(2 \times 37^\circ)$ ✓ answer / antwoord (7)

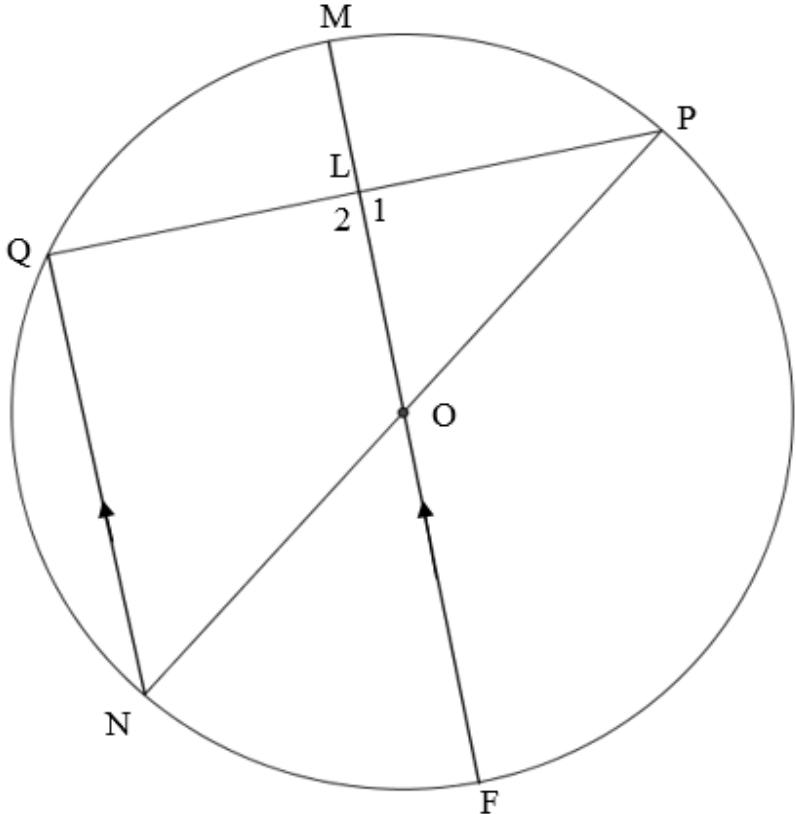
5.3.1	$\begin{aligned} 1 - \cos 2x &= 0 \\ \cos 2x &= 1 \\ \therefore 2x &= 0^\circ + k \cdot 360^\circ \\ x &= 0^\circ + k \cdot 180^\circ; k \in \mathbb{Z} \end{aligned}$	$\begin{aligned} \checkmark \quad 1 - \cos 2x &= 0 \\ \checkmark \quad 2x &= 0^\circ + 360^\circ \cdot k \\ \checkmark \quad x &= 0^\circ + 180^\circ \cdot k; \\ \checkmark \quad k \in \mathbb{Z} & \end{aligned}$	(4)
5.3.2	$\begin{aligned} \text{L.H.S/LK} &= \frac{2\sin x}{2(1-\cos 2x)} \\ &= \frac{2\sin x}{2-2\cos 2x} \\ &= \frac{2\sin x}{2-2(1-2\sin^2 x)} \\ &= \frac{2\sin x}{2-2+2\sin^2 x} \\ &= \frac{2\sin x}{2\sin^2 x} \\ &= \frac{1}{\sin x} \end{aligned}$	$\begin{aligned} \checkmark \quad 1-2\sin^2 x & \\ \checkmark \quad \text{removing brackets} & \\ \checkmark \quad \text{verwyder hakies} & \\ \checkmark \quad 2\sin^2 x & \end{aligned}$	(3)

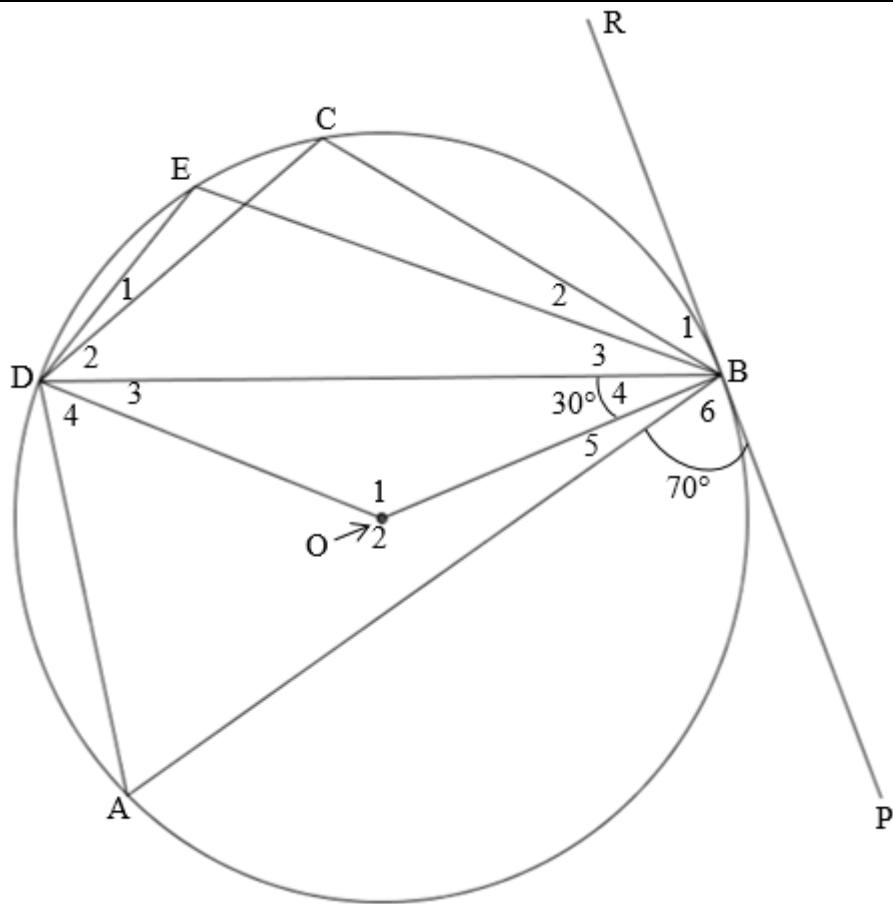
QUESTION 6 / VRAAG 6

6.1	$\sin(x + 60^\circ) = \sin(90^\circ - 2x)$ $\therefore x + 60^\circ = 90^\circ - 2x + 360^\circ \cdot k$ or/of $x + 60^\circ = 180^\circ - (90^\circ - 2x) + 360^\circ \cdot k$ $3x = 30^\circ + 360^\circ \cdot k$ or/of $x = -30^\circ + 360^\circ \cdot k$ $x = 10^\circ + 120^\circ \cdot k$; $k \in \mathbb{Z}$ $\therefore x = -30^\circ; 10^\circ; 130^\circ$ <p style="text-align: center;">OR/OF</p> $\cos 2x = \cos(30^\circ - x)$ $2x = 30^\circ - x + 360^\circ \cdot k$ or $2x = -(30^\circ - x) + 360^\circ \cdot k$; $k \in \mathbb{Z}$ $3x = 30^\circ + 360^\circ \cdot k$ or $x = -30^\circ + 360^\circ \cdot k$ $\therefore x = -30^\circ; 10^\circ; 130^\circ$	✓ co-ratio / ko-funksie ✓ both gen. solns beide algemene oplossings ✓ -30° ✓ 10° ✓ 130° ✓ co-ratio / ko-funksie ✓ both gen. solns beide algemene oplossings ✓ -30° ✓ 10° ✓ 130°	(5)
6.2		g: ✓ x-intercept. x-afsnit ✓ y-intercept. y-afsnit ✓ shape / vorm f: ✓ x-intercept. x-afsnit ✓ y-intercept. y-afsnit ✓ shape / vorm	(6)
6.3	240°	✓ answer / antwoord	(1)
6.4	$h(x) = \cos(2x - 90^\circ) - 1 = \sin 2x - 1$	✓ substitution / vervanging ✓ $\sin 2x - 1$	(2)
			[14]

QUESTION 7 / VRAAG 7

7.1	$\begin{aligned} NP^2 &= ON^2 + OP^2 \\ &= (103)^2 + (145)^2 \\ \therefore NP &= 177,86 \end{aligned}$	<ul style="list-style-type: none"> ✓ using Pyth theorem correctly <i>korrekte gebruik van stelling van Pythagoras</i> ✓ answer / antwoord 	(2)
7.2	$\begin{aligned} PQ^2 &= (103)^2 + (103)^2 - 2(103)(103)\cos(120^\circ) \\ PQ &= 178,40 \end{aligned}$	<ul style="list-style-type: none"> ✓ subst. into cosine rule <i>verv. in cosinus formule</i> ✓ answer / antwoord 	(2)
7.3	$\begin{aligned} \cos \hat{N} &= \frac{(177,86)^2 + (177,86)^2 - (178,40)^2}{2(177,86)(177,86)} \\ \therefore \hat{N} &= 60,20^\circ \end{aligned}$	<ul style="list-style-type: none"> ✓ substitution / vervanging ✓ \hat{N} 	(2)
			[6]

QUESTION 8 / VRAAG 8			
8.1	Bisects the chord / <i>Halveer die koord</i>	✓ answer / <i>antwoord</i>	(1)
8.2			
8.2.1	$\hat{Q} = 90^\circ$ [\angle in semi circle.] / [\angle in halwe sirkel] $\hat{L}_1 = \hat{Q} = 90^\circ$ [corresp. \angle^s , $QN \parallel LO$] [oorseenk. \angle^e , $QN \parallel LO$] $QL = LP$ [line from centre perp. to chord] [lynstuk vanaf middelpunt is loodreg op koord]	✓ S ✓R ✓ S/R ✓ R	(4)
8.2.2	$MF = 8ML$	✓	(1)
8.2.3	$OP^2 = OL^2 + LP^2$ $(4ML)^2 = (3ML)^2 + 7^2$ $16ML^2 = 9ML^2 + 49$ $7ML^2 = 49$ $\therefore ML = \sqrt{7}$	✓ using Pyth correctly korrekte gebruik van Pythagoras ✓ simplification / vereenvoudiging ✓ ML	(3)
			[9]

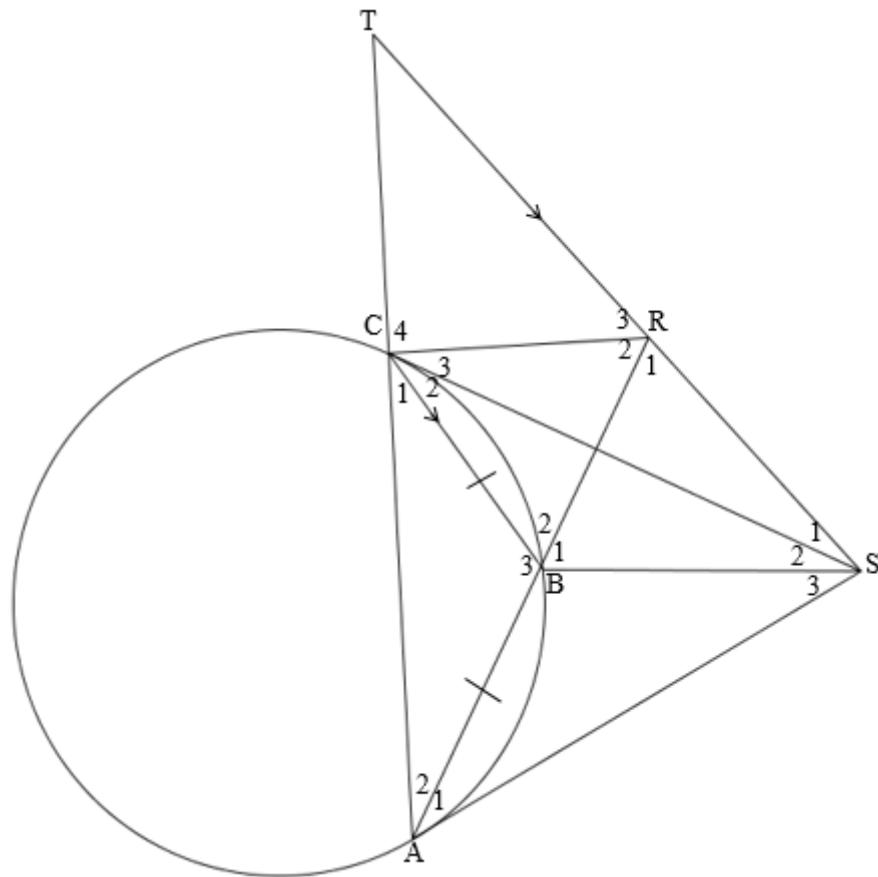
QUESTION 9 / VRAAG 9

9.1	$\hat{D}_3 = 30^\circ$ [∠s opp equal sides] / [\angle^e teenoor gelyke sye] $\therefore \hat{O}_1 = 120^\circ$ [sum of ∠s of a Δ.] / [som v/d ∠e van 'n Δ]	✓ S ✓R ✓ S/R	(3)
9.2	$\hat{A} = 60^\circ$ [∠ at centre = 2 ∠ at circumf.] / [middelpunts ∠ = 2 × omtrekshoek]	✓ S ✓R	(2)
9.3	$\hat{C} = 120^\circ$ [opp. ∠s of cyclic quad.] / [teenoorst. ∠e van k.v]	✓ S ✓R	(2)
9.4	$A\hat{D}B = 70^\circ$ [tan chord theorem.] / [raaklyn koord stelling]	✓ S ✓R	(2)
			[9]

QUESTION 10 / VRAAG 10

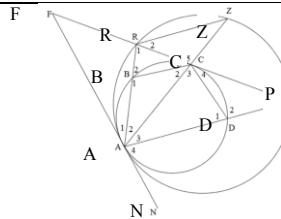
10.1		✓ constr. konstr.	
	$\hat{U}_2 = 90^\circ - \hat{U}_1$ [tan \perp radius.] / [raaklyn \perp radius] $\hat{Z} = 90^\circ$ [\angle in semi circle] / [\angle in halwe sirkel] $\hat{S} = 180^\circ - (90^\circ - \hat{U}_1) - 90^\circ$ [sum of \angle s of a Δ] / [som van die \angle e van 'n Δ] $\therefore \hat{S} = \hat{U}_1$ $\hat{S} = \hat{Y}$ [\angle s in same segment] / [\angle e in dieselfde segment] $\therefore \hat{U}_1 = \hat{Y}$	✓ S/R ✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)

10.2



10.2.1	$\hat{A}_2 = x$ [$\angle s$ opp. = sides] / [\angle^e teenoor gelyke sye]	✓ S/R	
	$\hat{C}_2 = \hat{A}_2 = x$ [tan chord theo.] / [raaklyn koord stelling]	✓ S/R	
	$\hat{A}_1 = \hat{C}_1$ [tan chord theo.] / [raaklyn koord stelling]	✓ S/R	
	$\hat{C}_2 = \hat{S}_1$ [alt. $\angle s$, $CB \parallel TS$] / [verw. \angle^e , $CB \parallel TS$]	✓ S/R	
	$\hat{C}_1 = \hat{T}$ [corresp $\angle s$, $CB \parallel TS$] / [ooreenk. \angle^e , $CB \parallel TS$]	✓ S/R	(5)
10.2.2	$\hat{S}_1 = \hat{T} = x$ [proven in 10.2.1] / [bewys in 10.2.1] $\therefore CS = CT$ [sides opp. = $\angle s$] / [sye teenoor gelyke hoeke]	✓ S ✓ R	(2)
10.2.3	$\frac{AR}{BR} = \frac{AT}{CT}$ [line \parallel to one side of a Δ] [lynstuk \parallel aan een sy van 'n Δ] $CS = CT$ [proved in 10.2.2] / [bewys in 10.2.2] $AT = \frac{3}{2} \times 4$ $AT = 6\text{cm}$	✓ S/R ✓ CS = CT ✓ substitution vervanging ✓ AT	(4)
			[16]

QUESTION 11 / VRAAG 11



11.1	$\hat{A}_1 = \hat{C}_2$ [tan chord theo.] / [raaklyn koord stelling] $= \hat{Z}$ [tan chord theo.] / [raaklyn koord stelling] $\therefore \hat{C}_2 = \hat{Z}$ [both = x] / [beide = x] $\therefore BC \parallel RZ$ [corresp. \angle s =] / [ooreenk. \angle e =]	✓ S ✓R ✓ S/R ✓ R	(4)
11.2	$\hat{Z} = \hat{P}$ [\angle s in same segment] / [\angle e in dieselfde segment] $= \hat{C}_2$ [corresp. \angle s; $BC \parallel RZ$] / [ooreenk. \angle e; $BC \parallel RZ$] $\therefore BC$ is a tangent to circle ACP [conv. of tan chord theorem] BC is 'n raaklyn aan sirkel ACP [omgekeerde van raaklyn koord stelling]	✓ S/R ✓ S	
11.3	$\hat{B}_1 = \hat{D}_2$ [ext. \angle of a cyclic quad.] [buite \angle van koordevierhoek] $\hat{R} = \hat{B}_1$ [corresp. \angle s, $BC \parallel RZ$] [ooreenk. \angle e, $BC \parallel RZ$] $\therefore \hat{R} = \hat{D}_2$ $\hat{Z} = \hat{P}$ [\angle s in same segment] [\angle e in dieselfde segment] $\hat{A}_2 = \hat{C}_4$ [3^{rd} \angle] / [3^{de} \angle] $\therefore \Delta RZA \sim \Delta DPC$ [equiangular or $\angle\angle\angle$] [gelykhoekig of $\angle\angle\angle$]	✓ S ✓R ✓ S/R ✓ S/R ✓ R	(5)
11.4	$\frac{ZA}{PC} = \frac{RA}{DC}$ [similar Δ s] / [gelykvormige Δ e] $\therefore RA = \frac{ZA \times DC}{PC}$... (1) $\frac{AR}{AB} = \frac{AZ}{AC}$ [line \parallel to one side of a Δ] [llynstuk \parallel aan een sy van 'n Δ] $\therefore AR = \frac{AZ \times AB}{AC}$... (2) $\frac{ZA \times DC}{PC} = \frac{AZ \times AB}{AC}$ $\therefore \frac{DC}{PC} \times \frac{AC}{AB} = 1$	✓ S/R ✓ RA = $\frac{ZA \times DC}{PC}$ ✓ S/R ✓ RA = $\frac{AZ \times AB}{AC}$ ✓ simplification	(5)
			[17]
		TOTAL/TOTAAL:	150