Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2018

TECHNICAL MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

JTMATHE1

This question paper consists of 12 pages, including 2 diagram sheets and a formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining your answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. Where necessary, ALL answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams are NOT necessarily drawn to scale.

7. It is in your own interest to write legibly and to present your work neatly.

8. An information sheet with formulae is attached.

9. Diagram sheets are attached for QUESTION 3.1.2, QUESTION 6.2 and QUESTION 8.5. Write your name in the spaces provided and then hand in the diagram sheets with your ANSWER BOOK.

QUESTION 1

1.1 Evaluate:

$111010_2 - 10101_2$ (2)

1.2 Solve for x:

1.2.1 $x(x-3) = 0$ (2)

1.2.2 $x^2 + 3x + 1 = 0$ (correct to ONE decimal) (4)

1.2.3 $x^2 + 2 < -3x$ (4)

1.3 Solve for x and y simultaneously:

$y = x^2 - 1$ and $y = x + 1$ (5)

1.4 Given : $x^2 + bx + 4 = 0$

Determine the value(s) of b for which the roots of the equation will be equal. (4)

[21]

QUESTION 2

2.1 Simplify the following fully : $\dfrac{2^{x+1} - 2^{x-1}}{6^x}$ (4)

2.2 Prove that : $\dfrac{log_a 25 - log_a 125}{2[log_a 5^4 - log_a 5^6]} = \dfrac{1}{4}$ (5)

2.3 A rapidly growing population of rabbits on Robben Island consists of $1\,000 \times 2^{0,05t}$ rabbits after t days.

2.3.1 Approximately how many rabbits were there after 30 days? (2)

2.3.2 How long will it take for the rabbit population to reach 8 000? (3)

[14]

QUESTION 3

3.1 Given :

$$Z = -2 + i$$

3.1.1 Calculate the modulus of Z. (2)

3.1.2 Sketch Z in the argand plane on the DIAGRAM SHEET provided. (2)

3.1.3 Determine the argument of Z. (3)

3.1.4 Express Z in polar form. (2)

3.2 Solve for x and y if:

$$(x - yi)(1 + i) = -2 + i$$ (5)

[14]

QUESTION 4

4.1 An amount of R2 500 is deposited into a savings account at 14% interest per annum, compounded quarterly.

4.1.1 Calculate the nominal interest rate per quarter which the savings account will accumulate. (1)

4.1.2 Determine the effective interest rate per annum, correct to one decimal. (3)

4.1.3 Calculate the amount of money in the savings account at the end of 7 years. (3)

4.2 The Eastern Cape Construction Consortium opened a savings account into which a sum of R250 000 was deposited. The money in the account will be used to purchase a tipper truck with an estimated value of R800 000 after 5 years.

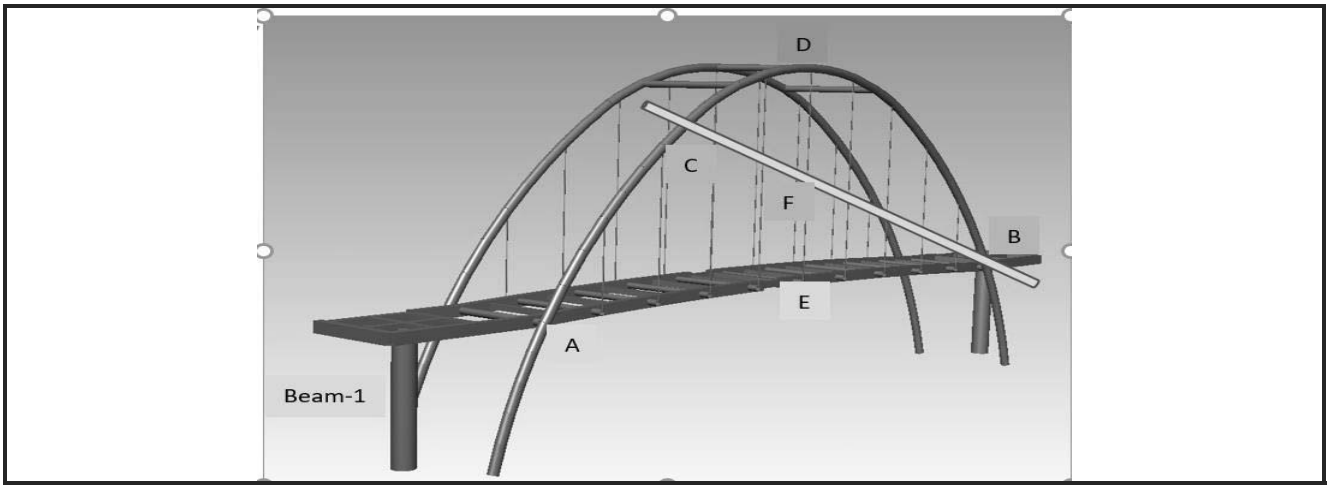
- The account accumulates an interest rate of 8% p.a. compounded monthly, for the first two years and 10% p.a. compounded quarterly for the remaining 3 years.
- A sum of R80 000 is deposited into the account at the end of the third year.

Determine how much will be in the savings account after 5 years. (8)

[15]

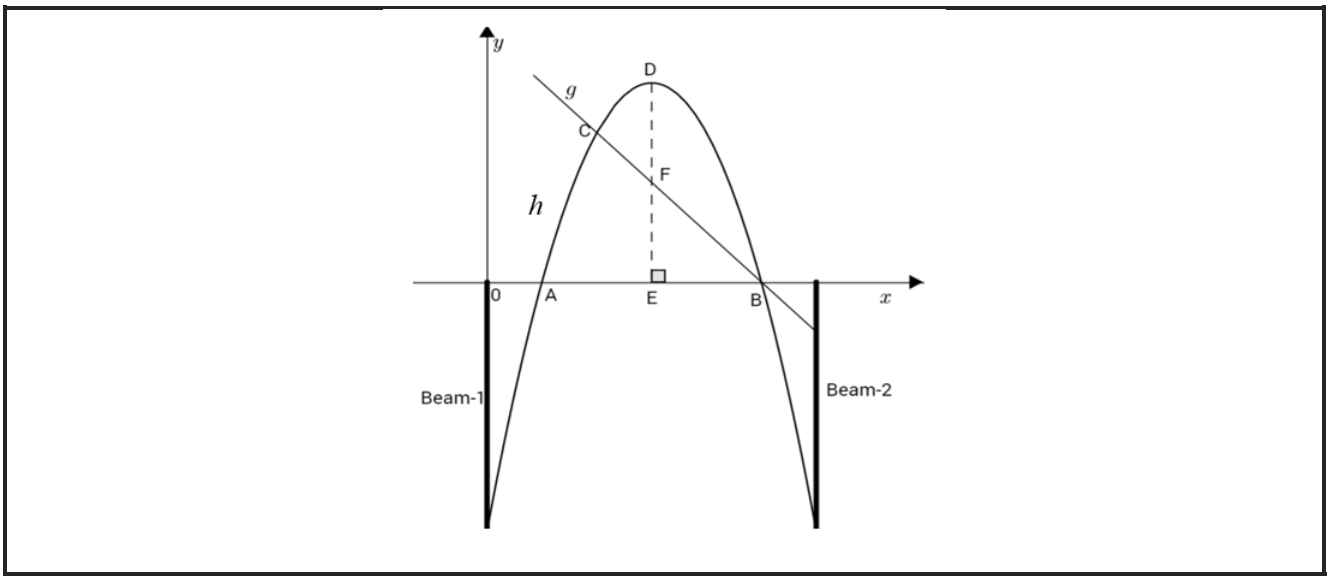
QUESTION 5

The external structural design of a newly erected bridge represents a parabolic function defined by $h(x) = -(x-3)^2 + 4$ and a linear function defined by $g(x) = -x + 5$ that represents a steel cable supporting one side of the parabolic structure.
The parabolic structure touches the horizontal roadside at A and B.
Point B and C are points of contact of h and g.
The vertical steel cable DE touches the horizontal roadside at E and cuts CB at F.
D is the maximum point of the bridge.
Two supportive vertical beams are at each edge of the bridge which are each 1 unit away from the point of contact between the bridge structure and the road surface.



[Source: www.fbridgeworkoutpics.blogspot.com]

The following cartesian diagram models the above design:



5.1 Determine the:

5.1.1 Coordinates of A and B, the x-intercepts of h (4)

5.1.2 Coordinates of D, the turning point h (2)

5.1.3 Domain of h (3)

5.1.4 Maximum height of the bridge above the road (1)

5.1.5 Height of the beams, if the heights of the beams are the same (2)

5.1.6 Range of h (2)

5.1.7 the value(s) of x for which $h'(x).g(x) \leq 0$ (3)

5.2 A truck driver driving a truck of 4,5 metres high wishes to use the bridge as it shortens the distance he must travel.

Will the truck be able to go over the bridge?
Justify your answer by means of correct mathematical reasoning. (3)

5.3 Calculate the length of FD. (3)
[23]

QUESTION 6

Given the functions defined by: $f(x) = 2^x$ and $g(x) = \frac{-2}{x} + 1$

6.1 Determine the:

6.1.1 Coordinates of the x- intercept of g (2)

6.1.2 y- intercept of f (1)

6.1.3 Equations of asymptotes of f and g (2)

6.2 On the same set of axes, sketch the graphs of f and g on the DIAGRAM SHEET provided. Indicate all the intercepts, asymptotes and directions of the two graphs. (7)

6.3 Determine the:

6.3.1 Domain of g (2)

6.3.2 Values of x for which $f(x) > g(x)$ (1)
[15]

QUESTION 7

7.1 Use **first principles** to determine the derivative of $f(x) = -2x^2$ (5)

7.2 Find $\frac{dy}{dx}$ if $y = 2\sqrt{x} - \frac{1}{x}$ (leave your answer with positive exponents) (4)

7.3 Determine the equation of a tangent to the graph of a function defined by $g(x) = x^2 - 2x$ at $x = 2$ (5)

[14]

QUESTION 8

Given : $f(x) = x^3 + 4x^2 + x - 6$

8.1 Show that $x + 1$ is not a factor of $f(x) = x^3 + 4x^2 + x - 6$ (1)

8.2 Determine all coordinates of the x-intercepts of f. (5)

8.3 Determine the y-intercept of f. (1)

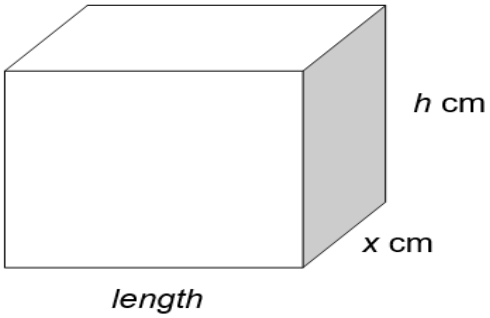
8.4 Find the coordinates of the turning points of f. (4)

8.5 Sketch the graph of f indicating all the intercepts and the stationary points on the DIAGRAM SHEET provided at the back of the question paper. (5)

[16]

QUESTION 9

9.1 A manufacturer has designed a closed box of which the base side has the width = x cm as indicated below. The length of the box is twice the width. The surface area of the box is exactly 120 cm^2.



9.1.1 Write down an expression of the height of the box in terms of x. (3)

9.1.2 Show that the volume of the box is given by:

$$\text{Volume} = 40x - \frac{4}{3}x^3 \quad (2)$$

9.1.3 Determine the value(s) of x for which the volume of the box will be maximum. (3)

9.2 The temperature increase inside an engine in degrees Celsius after t seconds is given by:

$$T = t^3 - 9t^2 + 50t - 66$$

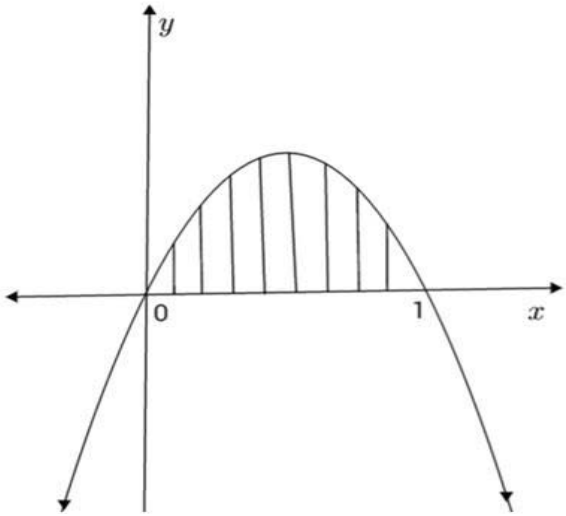
Determine the rate of temperature increase after 5 seconds. (3)

[11]

QUESTION 10

10.1 Simplify $\int(3x^2 - x)\,dx$ (3)

10.2 Determine the striped area bounded by the graph of $f(x) = -x^2 + x$ and the x axis, as shown below: (4)



[7]

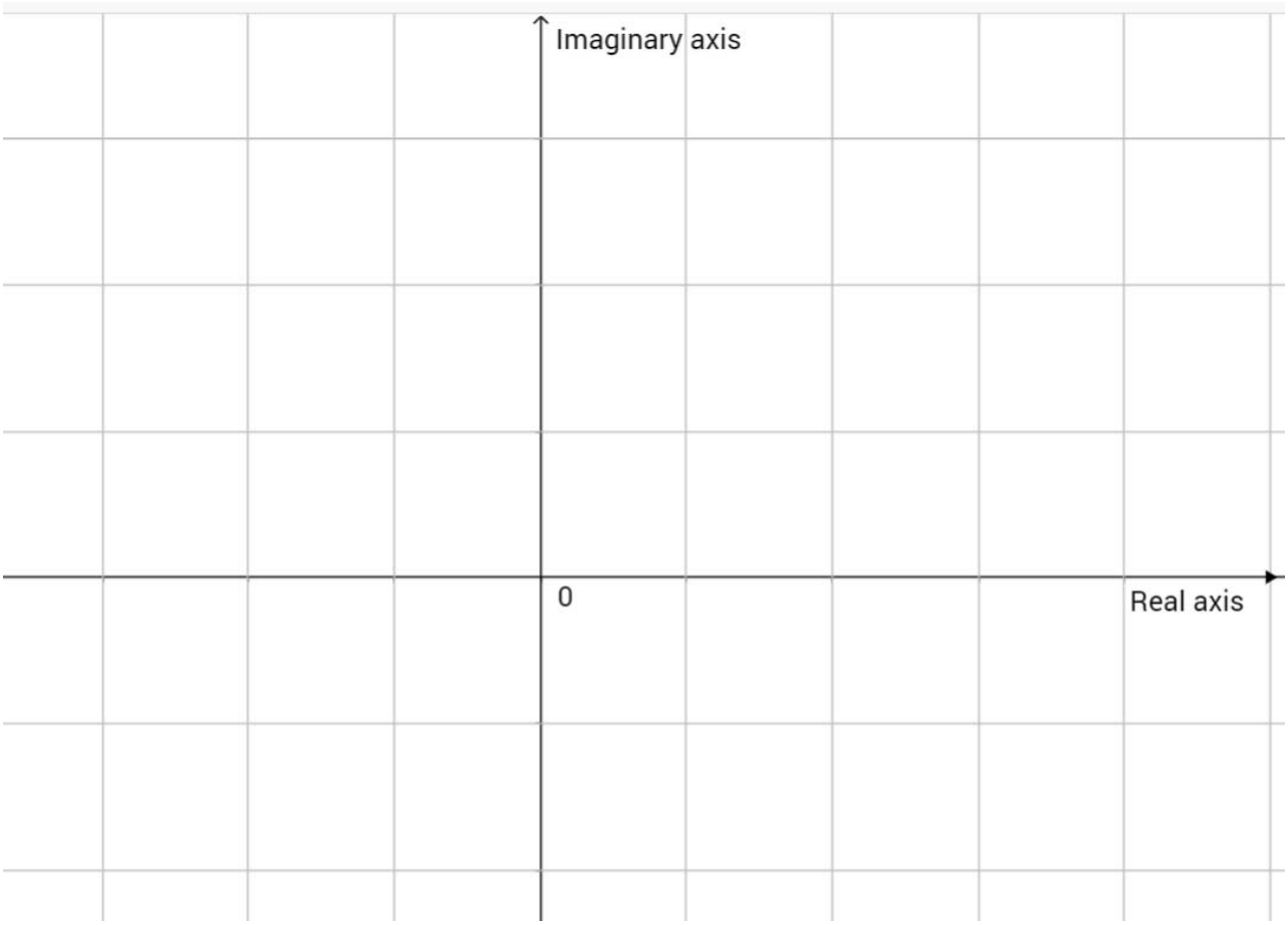
TOTAL: 150

DIAGRAM SHEETS

Learner Name: …….. Class: ……..

School Name: ……

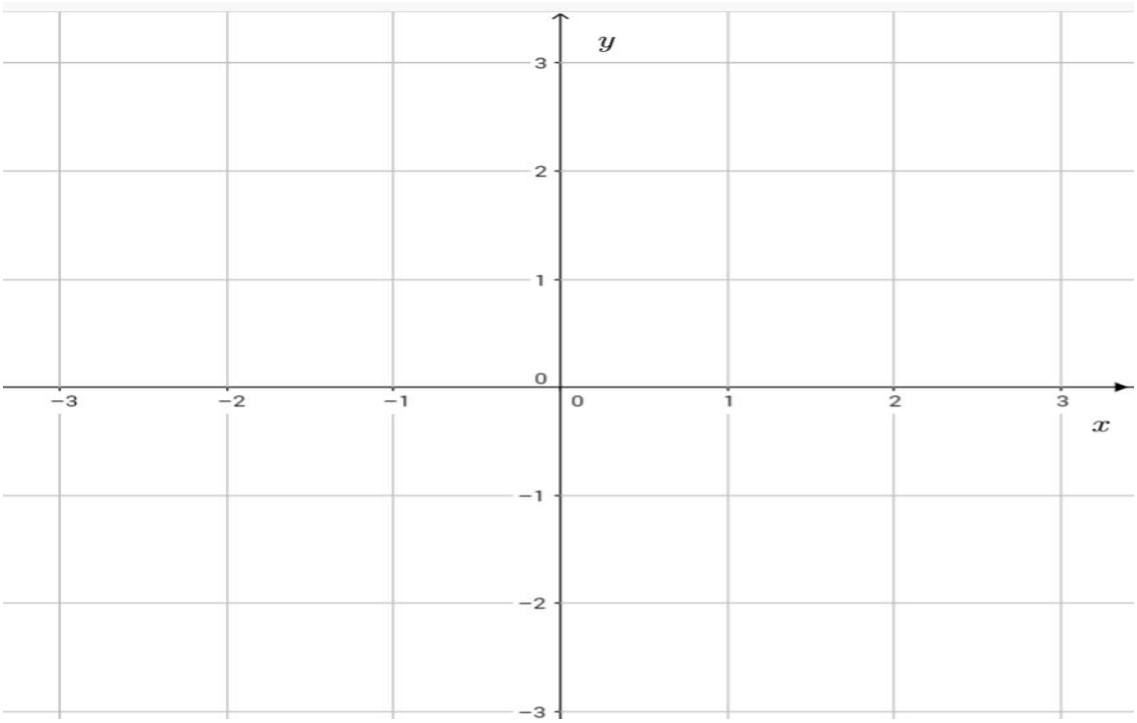
QUESTION 3.1.2



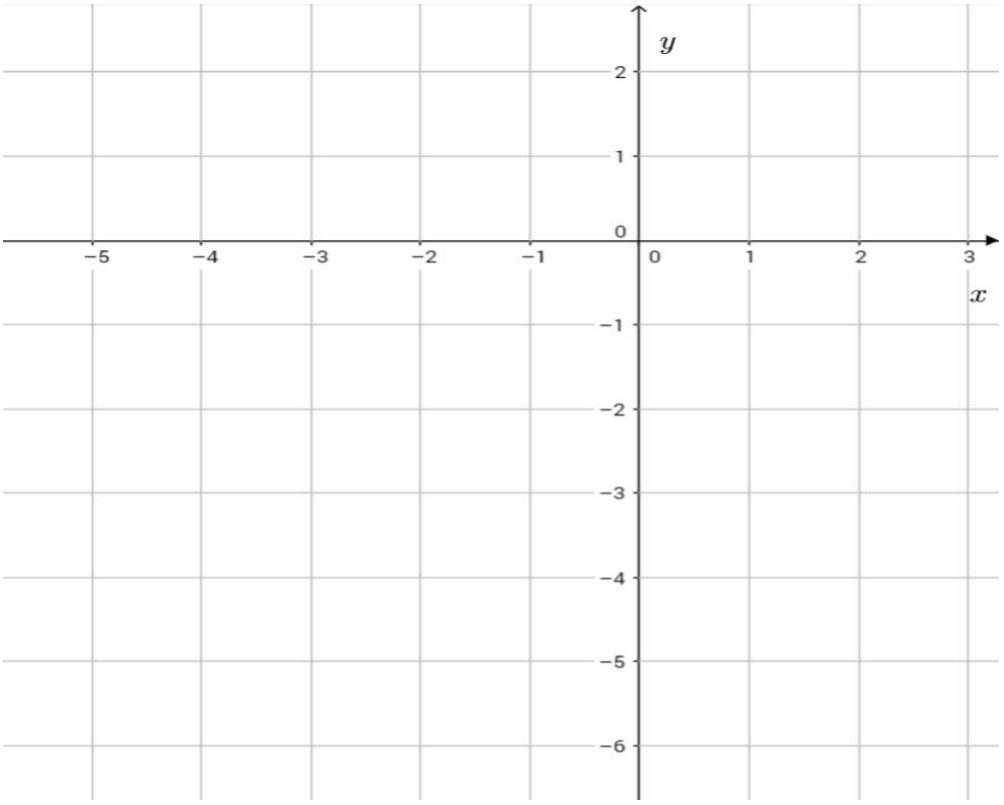
Learner Name: …….. Class: ……..

School Name: ……

QUESTION 6.2



QUESTION 8.5



INFORMATION SHEET FOR TECHNICAL MATHEMATICS: EC/2018
INLIGTINGSBLAD VIR TEGNIESE WISKUNDE: OK/2018

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad i_{eff} + 1 = \left(1 + \frac{i^m}{m}\right)^m$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \int ax^n dx = \frac{ax^{n+1}}{n+1} + C, n \neq -1$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$x^2 + y^2 = r^2 \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$s = r\theta \qquad \omega = 2\pi n \qquad v = \pi D n$$

$$4h^2 - 4dh + x^2 = 0 \qquad \omega = \frac{\theta}{t} \qquad v = \omega r$$

$$Area = \frac{r^2\theta}{2}$$

In ΔABC:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \qquad a^2 = b^2 + c^2 - 2bc.cosA \qquad Area = \frac{1}{2}ab.sinC$$

Right prism / Reghoekige prisma	$Area = 2lh + 2bh + 2bl$	$Volume = blh$
Cylinder/Silinder	$Area = 2\pi r^2 + 2\pi rh$	$Volume = \pi r^2 h$
Cone / Keël	$Area = \pi r^2 + \pi rl$ $= \pi r^2 + \pi r\sqrt{h^2 + r^2}$	$Volume = \frac{1}{3}\pi r^2 h$
Sphere / Sfeer	$Area = 4\pi r^2$	$Volume = \frac{4}{3}\pi r^3$