Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2018

TECHNICAL MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

This question paper consists of 17 pages, 2 information sheets and a diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining the answers.

3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4. If necessary ALL answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams are NOT necessarily drawn to scale.

7. An information sheet with formulae is attached at the end of the question paper.

8. A diagram sheet is included for QUESTION 7.4 and QUESTION 9.4. Write your name in the space provided and hand in the diagram sheet with your ANSWER SHEET.

QUESTION 1

1.1 Solve for x if:

1.1.1 $x(x-3)=0$ (2)

1.1.2 $3x^2-2x-10=0$ (correct to ONE decimal place) (4)

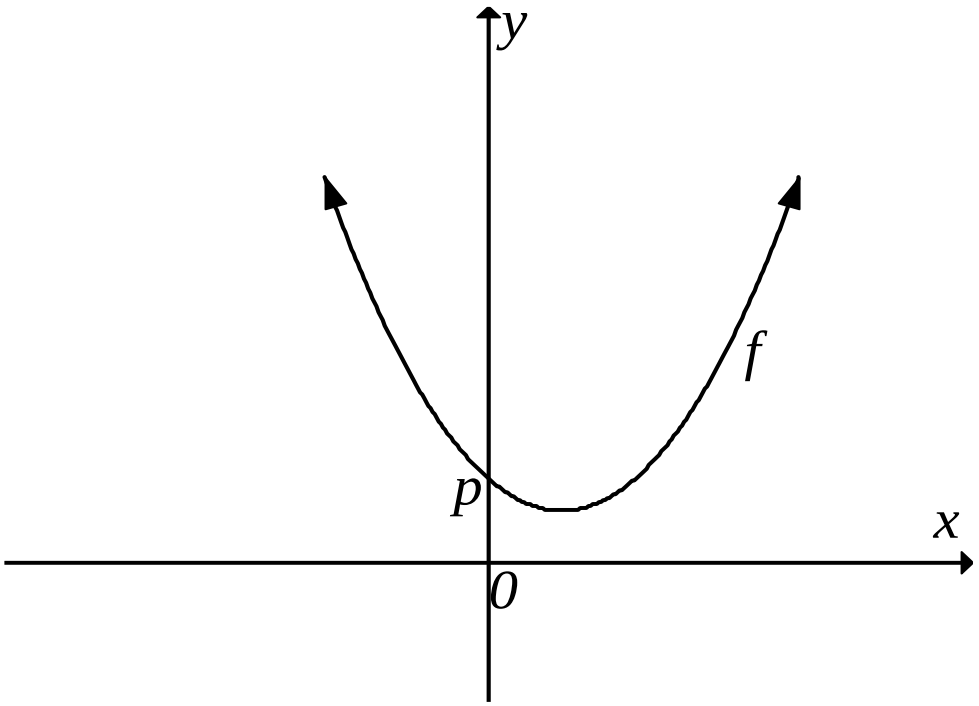
1.1.3 $2x^2-7x+3\geq 0$ (3)

1.2 Determine the value (**in scientific notation**) of $\frac{x^2-4}{x+2}$ if $x = 2\,000\,000\,000\,002$. (Show all your calculations) (3)

1.3 Solve for x and y simultaneously:

$2y+x=3$ and $y=x^2-x$ (6)

1.4 The graph defined by $f(x) = 2x^2 - 3x + p$ is drawn below, p is the y-intercept of f.



Determine the value of p for which the graph will always be above the x-axis. (3)

[21]

QUESTION 2

2.1 Simplify the following WITHOUT using a calculator:

2.1.1 $\dfrac{\sqrt{-18}\,.\,\sqrt{-12}}{\sqrt{-6}}$ (3)

2.1.2 $\log 6 + 2\log 20 - \log 3 - 3\log 2$ (5)

2.2

2.2.1 Show that $\sqrt{\dfrac{5^{x+1} - 5^{x}}{5^{x-1}} + 5} = 5$ (4)

2.2.2 Hence, solve for x if $\sqrt{\dfrac{5^{x+1} - 5^{x}}{5^{x-1}} + 5} = \left(\dfrac{1}{5}\right)^{x-2}$ (4)

2.2.3 Solve for x if $4\log_2 x - 1 = \log_2 8$ (4)

[20]

QUESTION 3

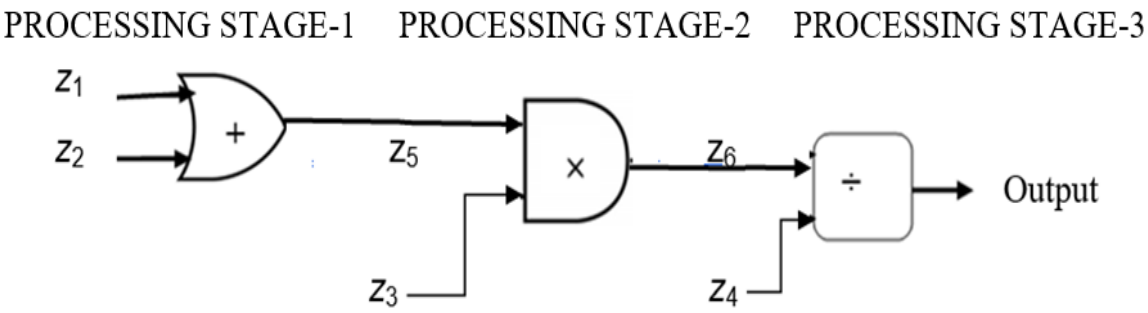
3.1 The picture below shows a CAD machine which is used to cut out different shapes from any material depending on the data input.

http://www.marchantdice.com/

The CAD machine performs 3 processing stages:

- Processing stage 1 performs an addition of two complex numbers entered.
- Processing stage 2 performs multiplication of two complex numbers entered.
- Processing stage 3 performs division of two complex numbers entered.

The diagram below models the order of the 3 processing stages to yield the required output:



Given:

$z_1 = 2 + 3i$ $z_2 = -3 - 2i$ $z_3 = -4 + i$ $z_4 = 2 + i$

Determine (in the form $a + ib$):

3.1.1 z_5 (2)

3.1.2 z_6 (2)

3.1.3 Output = $\frac{z_6}{z_4}$ (4)

3.2 To cut out a circular shape using the CAD machine, the following **OUTPUT** must be obtained:

- The modulus of the output must be greater than 5.
- The argument of the output must be in the first or fourth quadrant.

3.2.1 Calculate the modulus of the output. (2)

3.2.2 Sketch an Argand diagram of the output. (2)

3.2.3 Use the results obtained in QUESTION 3.2.1 and 3.2.2 to determine whether a circular shape was cut by the machine. (2)

[14]

QUESTION 4

4.1 A group of learners are tasked to clean a fish pond that has been neglected wherein a certain type of bacteria has been developing. At the beginning of the cleaning process, a 2-litre sample is tested and 12 000 bacteria are found. After 30 minutes, another 2-litre sample is tested and the bacteria count has decreased to 4 000. It is found that the bacteria decrease rate follows the compound decrease formula.

4.1.1 Calculate the estimated decrease rate of the bacteria per minute. (4)

4.1.2 Use the results obtained in QUESTION 4.1.1 to determine how many bacteria will be present at the end of 1 hour. (3)

4.2 Mabeka Construction Company bought a second hand TLB machine as shown below.

http://blog.truckandtrailer.co.za

- The TLB machine cost R600 000 from a dealership.
- The company paid 18% deposit and took out a bank loan to pay the remaining amount.
- The bank charges 15 % interest per annum, compounded monthly.

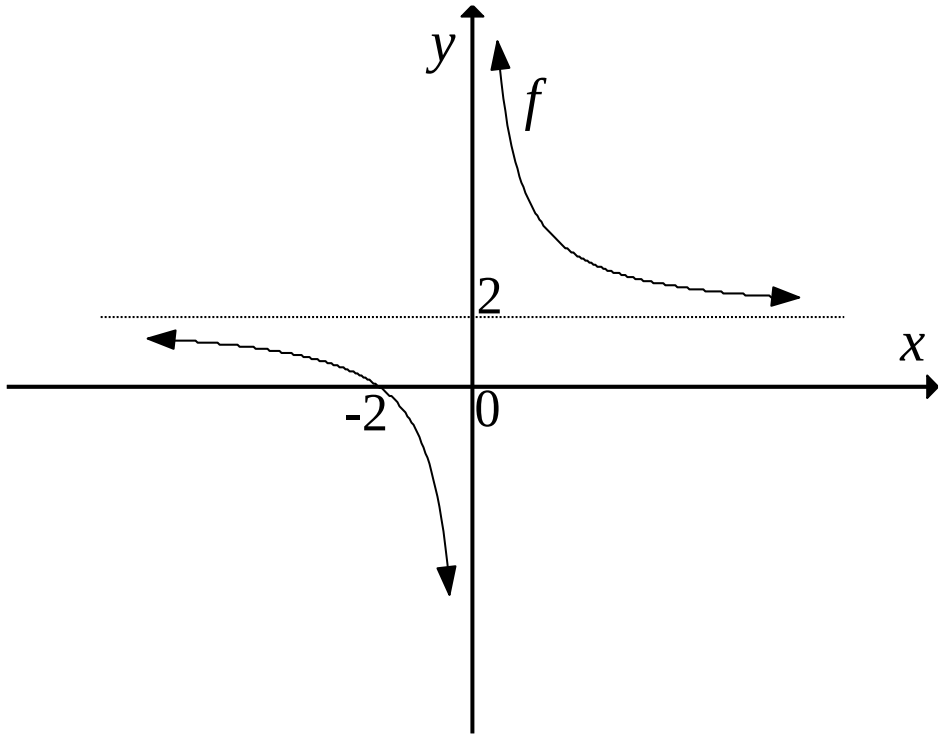
4.2.1 Calculate the amount the company loaned from the bank. (2)

4.2.2 Determine the number of years it took Mabeka Construction Company to pay off the loan if the company paid R1 204 860, 32 in total. (4)

[13]

QUESTION 5

The graph defined by $f(x) = \frac{a}{x} + q$ is drawn below. The graph cuts the x- axis at – 2



Determine:

5.1 The numerical value of q (1)

5.2 The numerical value of a (2)

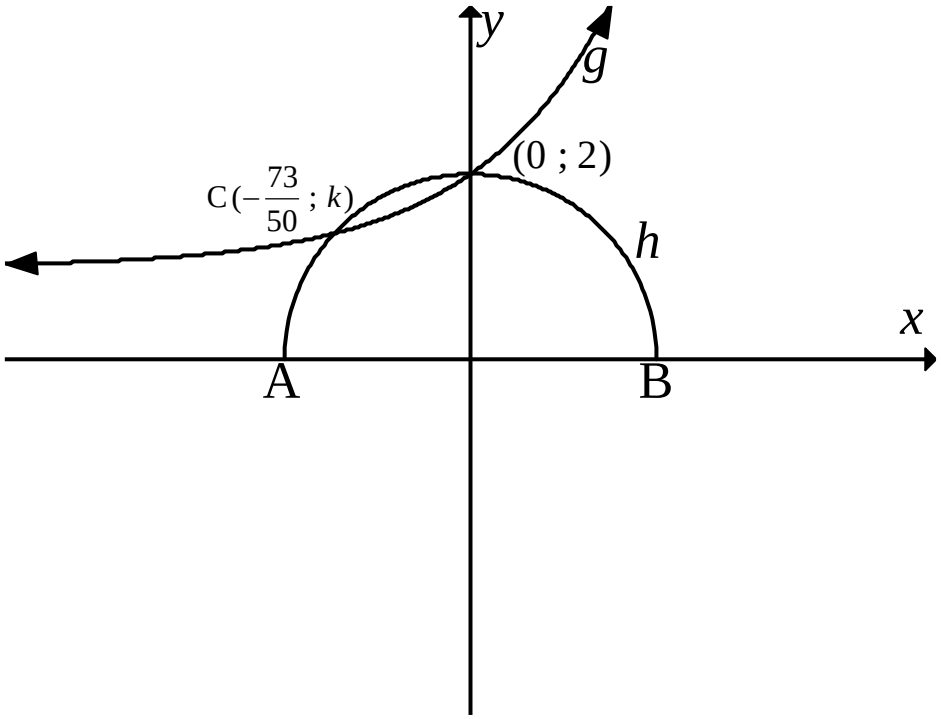
5.3 The equations of asymptotes of f (2)

5.4 The domain of f (2)
[7]

QUESTION 6

The graphs of a semi-circle defined by $h(x) = +\sqrt{4 - x^2}$, centred at the origin and an exponential funtion defined by $g(x) = 2^x + q$ are drawn below :

- A and B are the *x*-intercepts of *h*
- Points (0; 2) and $C\left(-\frac{73}{50}; k\right)$ are points of intersection of *g* and *h*
- The horizontal asymptote of *g*(*x*) cuts the *y*-axis at *q*



Determine:

6.1 The coordinates of A and B (2)

6.2 The numerical value of *q* (1)

6.3 The equation of a straight line passing through B and point (0;2) (2)

6.4 The values of *x* for which $g(x) < h(x)$ (2)
[7]

QUESTION 7

Given:

$$f(x) = -(x-2)^2 + 4$$

7.1 Determine the *x*-intercepts of *f*. (3)

7.2 Write down the *y*-intercept of *f*. (1)

7.3 Write down the coordinates of the turning point of *f*. (2)

7.4 On the DIAGRAM SHEET provided, sketch the graph of *f*. Clearly show all the intercepts with the axis and the turning point of the graph. (4)

7.5 Write down the range of *f*. (1)

7.6 Determine the coordinates of the turning point of *g* if $g(x)$ is the result of shifting $f(x)$, 1 unit to the right and 1 unit downwards. (2)

[13]

QUESTION 8

8.1 Calculate the average gradient of $f(x) = 2x^2 + x - 1$ between $x = 1$ and $x = 3$ (5)

8.2 Determine the derivative of $f(x) = 3x$ by using **FIRST PRINCIPLES**. (4)

8.3 Determine $\frac{dy}{dx}$:

8.3.1 $3x - 2y = \sqrt{x}$ (4)

8.3.2 $y = 6 - \frac{4}{\sqrt[3]{x}} + \frac{1}{x^4}$ (4)

8.4 Determine the point of contact of a tangent to the graph defined by $g(x) = x^2 + 2x - 3$ if the gradient of the tangent is – 2. (4)

[21]

QUESTION 9

Given:

$$f(x) = (x - 5)(x + 1)^2$$

9.1 Determine the co-ordinates of the *y*-intercept of f. (1)

9.2 Show that (5 ; 0) is one of the *x*-intercepts of f and hence, find other coordinates of the *x*-intercepts of f. (2)

9.3 Determine the co-ordinates of the turning points of f. (5)

9.4 Sketch the graph of f on the DIAGRAM SHEET provided. Clearly show all the intercepts with the axes and the stationary points. (5)

[13]

QUESTION 10

The image below shows a spilled paint blot. The area of an expanding paint blot (in cm^2), t seconds after it has been spilled, is given by the formula $A = 6 + 4t - t^2$.

Determine:

10.1 The initial area of the paint blot (2)

10.2 The rate of increase in area at t =1 second (3)

10.3 The time at which the paint blot stops spreading (2)

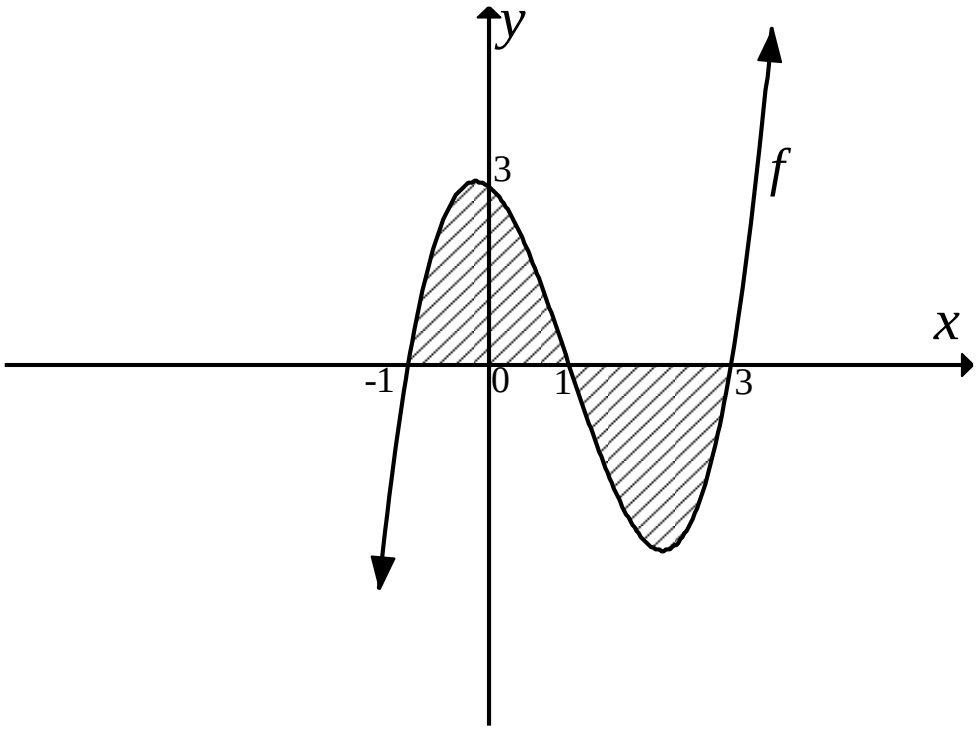
10.4 The area at the time the paint blot stops spreading (2)
[9]

QUESTION 11

11.1 Integrate:

$$\int (2x - 4)\,dx$$ (3)

11.2 The graph defined by $f(x) = x^3 - 3x^2 - x + 3$, is drawn below.
The x-intercepts of the graph are $x = -1$, $x = 1$ or $x = 3$



Determine the area of the stripped region of the graph of f bounded by the graph and the x-axis, between $x = -1$ and 3. (9)

[12]

TOTAL: 150

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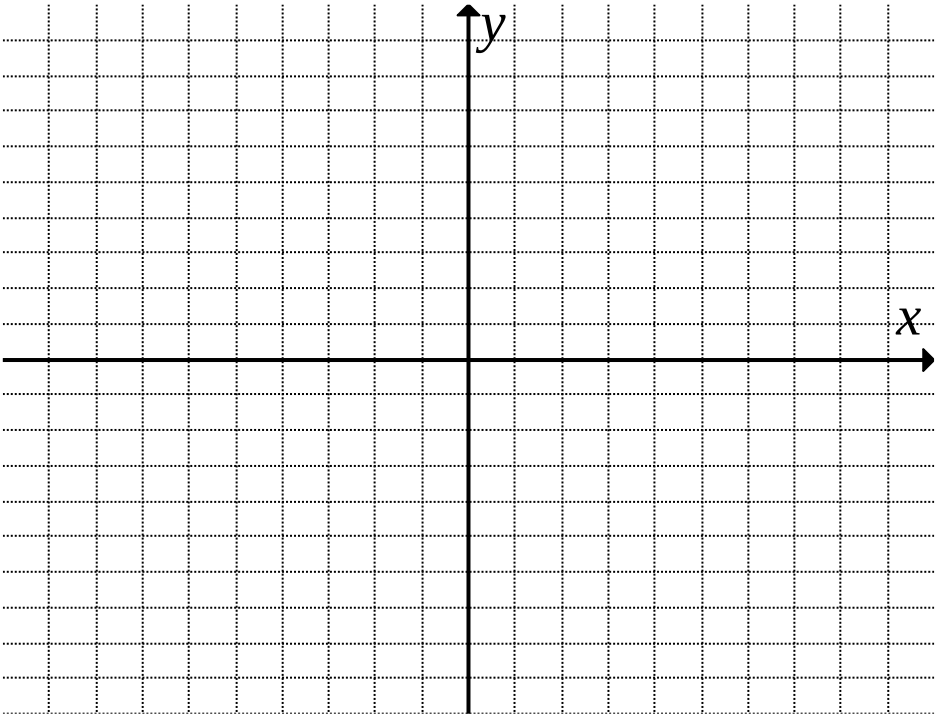
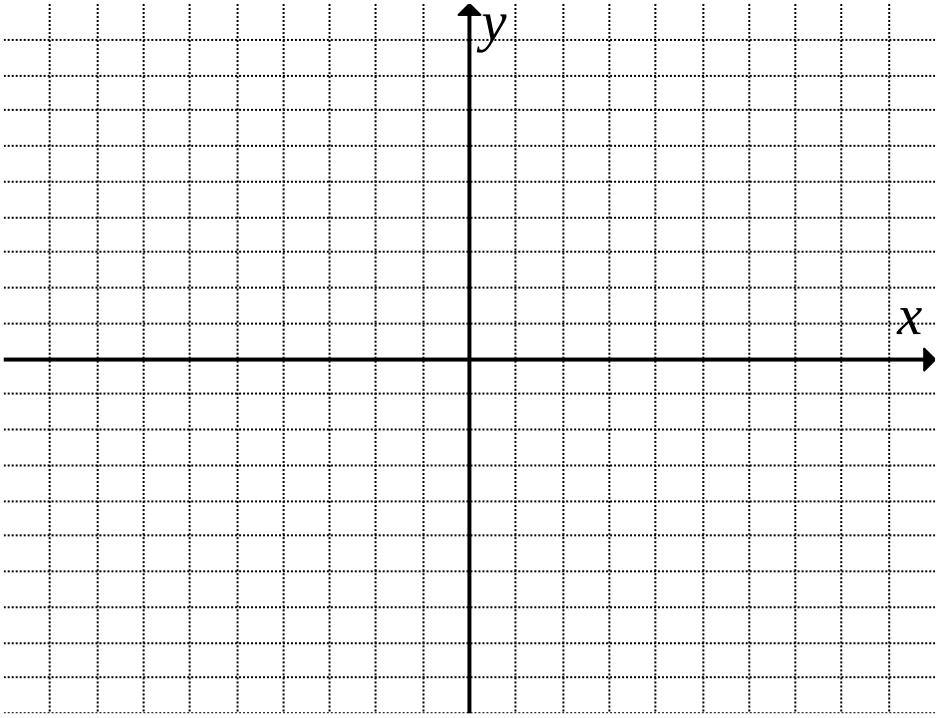
QUESTION 7.4



QUESTION 9.4



INFORMATION SHEET FOR TECHNICAL MATHEMATICS: EC/2018
INLIGTINGSBLAD VIR TEGNIESIE WISKUNDE: OK/2018

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b \ , \qquad a > 0 \ , \ a \neq 1 \ \text{ and } \ b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \qquad x > 0 \qquad\qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $a^2=b^2+c^2-2bc.\cos A$

$area\ of\ \Delta ABC=\frac{1}{2}ab\cdot\sin C$

$\sin^2\theta+\cos^2\theta=1$ $1+\tan^2\theta=\sec^2\theta$ $\cot^2\theta+1=\text{cosec}^2\theta$

$\pi rad=180°$

Angular velocity $=\omega=2\pi n=360°n$ where n = rotation frequency

Circumferencial velocity $=v=\pi Dn$ where D = diameter and n = rotation frequency

$s=r\theta$ where r = radius and θ = central angle in radians

Area of a sector $=\frac{rs}{2}=\frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2-4dh+x^2=0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T=a\left(m_1+m_2+m_3+\ldots+m_n\right)$ where a = equal parts, $m_1=\frac{o_1+o_2}{2}$ and n = number of ordinates

OR

$A_T=a\left(\frac{o_1+o_n}{2}+o_2+o_3+o_4+\ldots+o_{n-1}\right)$ where a = equal parts, $o_i=i^{th}$ ordinate and n = number of ordinates