Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

SEPTEMBER 2019

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

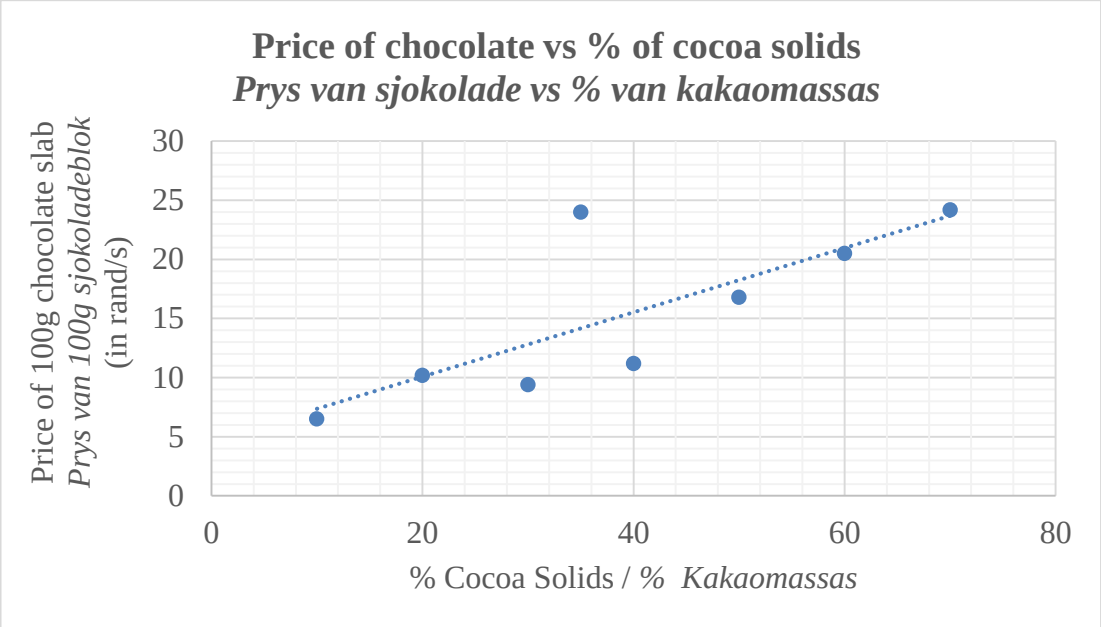
MARKS/*PUNTE*: 150

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

QUESTION 1/*VRAAG 1*			
1.1	$\bar{x} = \frac{420,8}{80}$ $= 5,26$ Answer only: Full marks *Slegs antwoord: Volpunte*	✓ $\frac{420,8}{80}$ ✓ answer / *antwoord*	(2)
1.2	total of heights / *totaal van hoogtes* $= 4,86 \times 7 = 34,02$ Error/*fout* = 4,98 – 4,89 = 0,09 new total of heights / *nuwe totaal van hoogtes* = 33,93 new mean / *nuwe gemiddelde* = $\frac{33,93}{7}$ = 4,85 m	✓ error / *fout* = 0,09 ✓ new total / *nuwe totaal* ✓ answer / *antwoord*	(3)
1.3	Standard deviation of individual heights of the 80 giraffes would be bigger than that of 7 males. There is a bigger spread of heights among population of juveniles and adults and babies. *Standaardafwyking van individuele hoogtes van 80 kameelperde sal groter wees as die van die 7 manlike kameelperde.* *Daar is 'n groter verspreiding van hoogtes onder die jeugdige en volwasse en baba bevolking.*	✓ bigger/larger standard deviation *groter standaardafwyking* ✓ bigger spread of heights *groter verspreiding van hoogtes*	(2)
			[7]

QUESTION 2/*VRAAG 2*

2.1



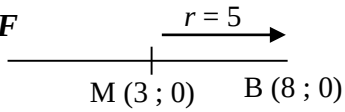
✓✓all points plotted correctly
alle punte korrek afgesteek (2)

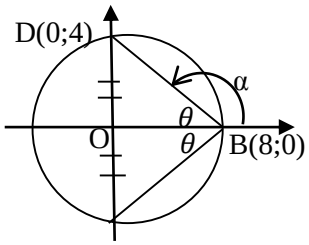
2.2	$\hat{y} = a + bx$ $a = 4{,}64 \quad b = 0{,}27$ $\hat{y} = 4{,}64 + 0{,}27x$		✓ value of/*waarde van a* ✓ value of/*waarde van b* ✓ equation / *vergelyking* (3)
2.3	See line in 2.1 / *Sien lyn in* 2.1		✓ correct gradient / *korrekte gradiënt* ✓ correct line / *korrekte lyn* (2)
2.4	$r = 0{,}78$		✓ $r = 0{,}78$ (1)
2.5	fairly strong positive correlation in the % of cocoa and the price. *redelike sterk positiewe korrelasie tussen die % van kakao en die prys.*		✓ correct description of *r* *korrek beskrywing van r* (1)
	2.6.1	Brand D / *Handelsmerk* D **OR / *OF*** (35 ; 24)	✓ answer / *antwoord* (1)
	2.6.2	$\hat{y} = 4{,}64 + 0{,}27(35)$ $= R14{,}09$ Overpriced by/*Te duur*: R24,00 – R14,09 = R9,91	✓ substitution of 35 *vervanging van 35* ✓ R14,09 ✓ R9,91 (3)
			[13]

QUESTION 3 / *VRAAG 3*			
3.1	D (4 ; 0)	✓ answer / *antwoord*	(1)
3.2	$m_{DE} = \dfrac{-\frac{4}{3}-0}{0-4}$ $= -\dfrac{4}{3}\times\left(-\dfrac{1}{4}\right)$ $= \dfrac{1}{3}$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.3	Angle of inclination of DE / *Inklinasiehoek van DE* $= \tan^{-1}\dfrac{1}{3}$ $= 18,43°$ $\hat{O}DE = 18,43°$ (vert. opp. ∠s)/(*regoorst. ∠e*) $\hat{O}ED = 71,57°$ (complementary ∠s) (*komplementêre ∠e*) $\therefore\ \hat{P}RB = 71,57°$ (corr. ∠s:DE ||BC) (*ooreenk. ∠e* : DE ∥ BC)	✓ $\tan^{-1}\dfrac{1}{3}$ ✓ $18,43°$ ✓ answer / *antwoord* ✓ reason / *rede*	(4)
3.4	$DE = \sqrt{(4-0)^2+\left(0+\frac{4}{3}\right)^2}$ $= \dfrac{4\sqrt{10}}{3}$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.5	$\dfrac{AD}{AB} = \dfrac{3}{9} = \dfrac{1}{3}$	✓ ratios / *verhoudings* ✓ simplified answer / *vereenvoudigde antwoord*	(2)

3.6	$\Delta ABC \text{ ||| } \Delta ADE$ $\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{1}{3}$ $BC = 3DE$ $= 3\left(\frac{4\sqrt{10}}{3}\right)$ $\therefore BC = 4\sqrt{10}$	✓ ratio of corresponding sides *verhouding van ooreenstemmende sye* ✓isolating / isoleer BC ✓ substitution / *vervanging*	
	OR/*OF*	**OR/*OF***	
	equation of BC / *vergelyking van BC* $y - 6 = \frac{1}{3}(x - 4)$ $\therefore y = \frac{1}{3}x + \frac{14}{3}$ $m_{AC} = \frac{\frac{5}{3}}{-4} = -\frac{5}{12}$ equation of AC / *vergelyking van AC* $y = -\frac{5}{12}x - \frac{4}{3}$ $\therefore \frac{1}{3}x + \frac{14}{3} = -\frac{5}{12}x - \frac{4}{3}$ $4x + 56 = -5x - 16$ $9x = -72$ $\therefore x = -8$ $y = \frac{1}{3}(-8) + \frac{14}{3}$ $\therefore y = 2$ $C(-8;2)$ and/*en* $B(4;6)$ $\therefore BC = \sqrt{(-8-4)^2 + (2-6)^2} = 4\sqrt{10}$	✓ equation of BC / *vergelyking van* BC ✓ equation of AC / *vergelyking van* AC ✓ answer / *antwoord*	(3)

3.7.1	Midpoint of DE / *Middelpunt van* DE $\left(\frac{4+0}{2};\frac{0-\frac{4}{3}}{2}\right)$ $\left(2;-\frac{2}{3}\right)$	✓✓ coordinates / *koördinate*	(2)
3.7.2	$m_{bisector}/m_{halveerlyn} = -3$ Equation of bisector / *Vergelyking van halveerlyn* $y+\frac{2}{3}=-3(x-2)$ $\therefore\ y=-3x+\frac{16}{3}$	✓ gradient of bisector / *gradiënt van halveerlyn* ✓substitution / *vervanging* ✓ equation / *vergelyking*	(3)
3.8	LHS/*LK* = -3 RHS/*RK* $= -3(4)+\frac{16}{3}$ $=-\frac{20}{3}=-6\frac{2}{3}\approx -6{,}66$ LHS/*LK* ≠ RHS/*RK* No, bisector does not pass through A. / *Nee, die halveerlyn gaan nie deur A nie*	✓ substitution / *vervanging* ✓ conclusion / *gevolgtrekking*	(2)
			[21]

QUESTION 4/*VRAAG 4*			
4.1	$x^2 - 6x + y^2 = 16$ $(x - 3)^2 + (y - 0)^2 = 16 + 9$ $(x - 3)^2 + (y - 0)^2 = 25$ Hence centre / *Gevolglik middelpunt* (3 ; 0) Radius/*Radius* = $\sqrt{25} = 5$	✓ completing the square *voltooiing van vierkant* ✓ final form / *finale vorm* ✓ taking square root / *vierkantswortel*	(3)
4.2.1	$y = 0$: hence/*gevolglik* $(x - 3)^2 = 25$ $x - 3 = \pm 5$ $x = 3 \pm 5$ B (8 ; 0) **OR / *OF***  	✓ equating / *stel* $y = 0$ ✓ solving for x / *Los op vir* x ✓ coordinates of B *koördinate van* B **OR / *OF*** ✓✓✓ use of horizontal distance / *gebruik van horisontale afstand*	(3)
4.2.2	$x = 0$: then/*dan* $(-3)^2 + y^2 = 25$ $9 + y^2 = 25$ $y^2 = 16$ $y = \pm 4$ C(0 ; − 4) **OR/*OF*** using Pythagoras / *gebruik van Pythagoras* : $MD^2 = OM^2 + OD^2$ $5^2 = 3^2 + OD^2$ $4 = OD = OC$ C(0 ; − 4)	✓ equating x to 0 *stel x gelyk aan* 0 ✓ coordinates of C *koördinate van* C **OR/*OF*** ✓ use of Pythagoras *gebruik van Pythagoras* ✓ coordinates of C *koördinate van* C	(2)
4.3	$M\hat{C}P = 90^o$ $PM^2 = MC^2 + PC^2$ $(8\frac{1}{3})^2 = 5^2 + PC^2$ $PC = \sqrt{\frac{400}{9}} = \frac{20}{3} = 6{,}67$ **OR / *OF*** $P\hat{O}C = 90^o$ $PC^2 = PO^2 + OC^2$ $PO = 8\frac{1}{3} - 5 + 2 = \frac{16}{3}$ and/*en* OC = 4 $PC^2 = (\frac{16}{3})^2 + 4^2$ $PC = \sqrt{\frac{400}{9}} = \frac{20}{3} = 6{,}67$	✓ radius ⊥ tangent *radius ⊥ raaklyn* ✓substitution into Pythag. Thm *vervanging in Stelling van Pyth* ✓ answer / *antwoord* **OR/*OF*** ✓ calculation of PO *berekening van* PO ✓substitution into Pythag. Thm *vervanging in Stelling van Pyth* ✓ answer / *antwoord*	(3)

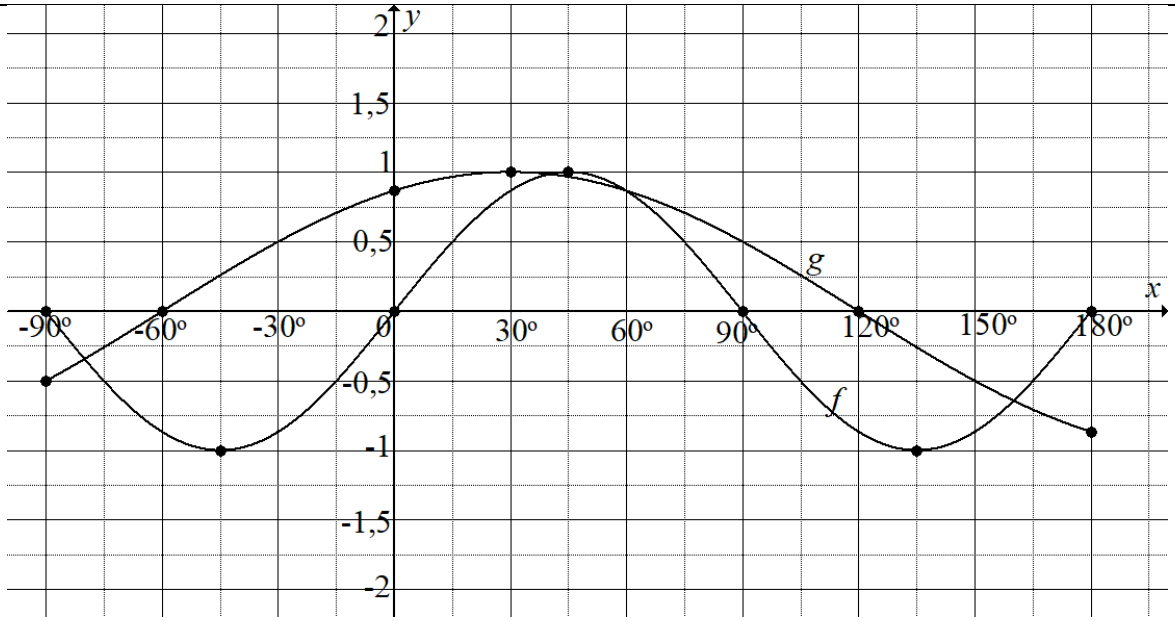
4.4	 $m_{BD} = -\frac{1}{2}$ $\tan^{-1}(-\frac{1}{2}) = -26{,}57^o$ $\therefore \alpha = 180 - 26{,}57^o$ $\therefore \theta = 180^o - 153{,}43^o = 26{,}57^o$ $D\hat{B}C = 2 \times 26{,}57^o$ $= 53{,}14^o$ **OR / *OF*** $\sin\theta = \frac{4}{\sqrt{(8)^2+(-4)^2}}$ $\sin^{-1}(0{,}4472\ldots) = 26{,}57^o$ $D\hat{B}C = 2 \times 26{,}57^o = 53{,}14^o$ **OR / *OF*** $\tan D\hat{B}O = \frac{4}{8} = \frac{1}{2}$ $D\hat{B}O = 26{,}57^o$ $D\hat{B}C = 2 \times 26{,}57^o$ $= 53{,}14^o$	✓ gradient of BD / *gradiënt van BD* ✓ calculation of α / *berekening van* α ✓ angle θ / *hoek* θ ✓ doubling the angle / *verdubbeling van hoek* **OR / *OF*** ✓ sin definition / sin-*definisie* ✓ use of arcsin / gebruik van $\sin^{-1}$ ✓ angle θ / *hoek* θ ✓ doubling the angle / *verdubbeling van hoek* **OR / *OF*** ✓ tan ratio / tan *verhouding* ✓use of arctan to find angle / *gebruik van* $\tan^{-1}$ *om hoek te bepaal* ✓ angle / *hoek* ✓ doubling the angle / *verdubbeling van hoek*	(4)
4.5	New circle centre/*Nuwe sirkel middelpunt* - M/(5 ; 1) C/(2 ; – 3) $m_{M'C'} = \frac{4}{3}$ $m_{tan} = -\frac{3}{4}$ $-3 = -\frac{3}{4}(2) + c$ $\therefore c = -\frac{3}{2}$ $\therefore y = -\frac{3}{4}x - \frac{3}{2}$	✓ coordinates of M/ and C/ *koördinate van* M/ *en* C/ ✓ gradient of MC / *gradiënt van* MC ✓ gradient of tangent / *gradiënt van raaklyn* ✓ equation / *vergelyking*	

	OR / *OF*	**OR / *OF***	
	Gradient of MC / *Gradiënt van* MC = $\frac{4}{3}$ Hence gradient of M'C' / *Gevolglik gradiënt van* M'C' = $\frac{4}{3}$ (MC ∥ M'C') Gradient of tangent at C' / *Gradiënt van raaklyn by* C' = $-\frac{3}{4}$ (M'C' ⊥ tang at C') (M'C' ⊥ *raaklyn by* C') C' = $(2 ; -3)$ $\therefore y + 3 = -\frac{3}{4}(x - 2)$ $\therefore y = -\frac{3}{4}x - \frac{3}{2}$	✓ gradient of MC / *gradiënt van* MC ✓ gradient of M'C' / gradiënt van M'C' ✓ coordinates of C' / koördinate van C' ✓ equation / *vergelyking*	(4)
			[19]
QUESTION 5/*VRAAG 5*			
5.1.1	1	✓ answer / *antwoord*	(1)
5.1.2	cos 2A	✓ answer / *antwoord*	(1)
5.2.1	$3^2 = 2^2 + k^2$ $k = -\sqrt{5}$	✓ distance formula / *afstand formule* Pythagoras Thm / *Stelling* ✓ answer with correct sign *antwoord met korrekte teken*	(2)
5.2.2(a)	$\tan(\theta - 180^\circ) = \tan\theta$ $= -\frac{\sqrt{5}}{2}$	✓ reduction / *reduksie* ✓ answer / *antwoord*	(2)
5.2.2(b)	$\frac{1-\sin^2 2\theta}{1-2\sin^2\theta} = \frac{\cos^2 2\theta}{\cos 2\theta}$ $= \cos 2\theta$ $= 1 - 2\sin^2\theta$ $= 1 - 2\left(-\frac{\sqrt{5}}{3}\right)^2$ $= 1 - \frac{10}{9}$ $= -\frac{1}{9}$ **OR / *OF*** $\frac{1-\sin^2 2\theta}{1-2\sin^2\theta} = \frac{\cos^2 2\theta}{\cos 2\theta}$ $= \cos 2\theta$ $= 2\cos^2\theta - 1$ $= 2\left(\frac{2}{3}\right)^2 - 1$ $= \frac{8}{9} - 1$ $= -\frac{1}{9}$	✓ $1 - \sin^2 2\theta = \cos^2 2\theta$ ✓ $1 - 2\sin^2\theta = \cos 2\theta$ ✓ substitution for sin *vervanging vir* sin ✓ answer / *antwoord* **OR / *OF*** ✓ $1 - \sin^2 2\theta = \cos^2 2\theta$ ✓ $1 - 2\sin^2\theta = \cos 2\theta$ ✓ substitution for cos *vervanging vir* cos ✓ answer / *antwoord*	

	OR / *OF* $\frac{1-\sin^2 2\theta}{1-2\sin^2\theta} = \frac{1-(2\sin\theta\cos\theta)^2}{1-2\sin^2\theta}$ $= \frac{1-\left[2(-\frac{\sqrt{5}}{3})(\frac{2}{3})\right]^2}{1-2\left(-\frac{\sqrt{5}}{3}\right)^2}$ $= -\frac{1}{9}$	**OR / *OF*** ✓$\sin^2 2\theta = (2\sin\theta\cos\theta)^2$ ✓ substitution for sin θ *vervanging vir* sin θ ✓ substitution for cos θ *vervanging vir* cos θ ✓ answer / *antwoord*	(4)
5.3	sin (-200^o).cos 310^o + tan (-135^o).cos 380^o .sin 230^o = sin (20^o)(cos 50^o) + (tan45^o)(cos 20^o)$(-\sin 50^o)$ = sin 20^o cos 50^o – (1)cos 20^o sin 50^o = sin $(20^o - 50^o)$ = sin (-30^o) = – ½	✓ ✓ ✓ ✓ reductions to acute angles / *reduksie tot skerphoeke* ✓ sin expansion / sin *uitbreiding* ✓ answer / *antwoord*	(6)
5.4	LHS/*LK* = sin 2θ + cos $(2\theta - 90^o)$ = sin 2θ + sin 2θ = 2 sin 2θ = 2(2sin θ cos θ) = 4 sin θ cos θ = RHS/*RK*	✓ cos $(2\theta - 90^o)$ = sin 2θ ✓ simplification *vereenvoudiging* ✓ expansion / *uitbreiding*	(3)
5.5	$10^{\sin x} + 10^{\sin x}.10^1 = 110$ $10^{\sin x}(1+10) = 110$ $10^{\sin x} = 10^1$ ∴ sin x = 1 ∴ $x = -270^o$ or/*of* $x = 90^o$	✓ split into product of 2 bases *skei in product van 2 basisse* ✓simplification / factorisation *vereenvoudiging/faktorisering* ✓division by 11 / *deel deur* 11 ✓ equating the exponents *gelykstel van eksponente* ✓ both solutions / *beide oplossings*	(5)
			[24]

QUESTION 6/*VRAAG 6*

6.1



✓ completing / *voltooiing van* $f(x) = \sin 2x$ graph/*grafiek* (1)

6.2 See graph in 6.1 / *Sien grafiek in* 6.1

$g(x) = \cos(x - 30^\circ)$
✓ *y* intercept / *y-afsnit*
✓ end points / *eindpunte*
✓ turning point / *draaipunte*
✓ *x* intercepts / *x-afsnitte* (4)

6.3
$\sin 2x = \sin(90^\circ - (x - 30^\circ))$
$\sin 2x = \sin(120^\circ - x)$
$2x = 120^\circ - x + k.360^\circ$ or/*of* $2x = 180^\circ - (120^\circ - x) + k.360^\circ$

$3x = 120^\circ + k.360^\circ$ or/*of* $x = 60^\circ + k.360^\circ$
$\therefore x = 40^\circ + k.120^\circ$ or/*of* $x = 60^\circ + k.360^\circ$; $k \in Z$

$k = 0$: $\theta = 40^\circ$; 60°
$k = -1$: $\theta = -80^\circ$

✓ co-ratio on RHS *ko-verhouding RK*
✓ simplification *vereenvoudiging*

✓ $x = 40^\circ + k.120^\circ$
✓ $x = 60^\circ + k.360^\circ$

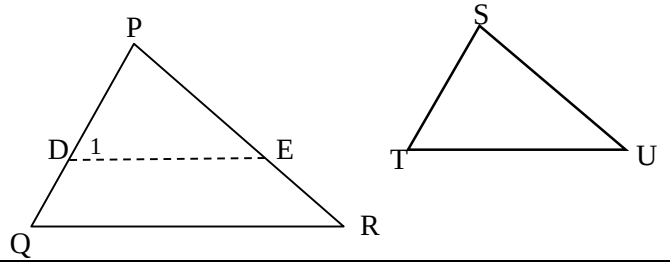
✓✓ solutions *oplossings* (6)

[11]

QUESTION 7/*VRAAG 7*			
7.1	$\tan y = \frac{PB}{k}$ $\therefore PB = k.\tan y$ $\tan x = \frac{PB}{BC}$ $\therefore BC = \frac{PB}{\tan x}$ Hence/*Gevolglik* $BC = \frac{k.\tan y}{\tan x}$	✓ use of tan ratio *gebruik van tan verhouding* ✓ $PB = k.\tan y$ ✓ $BC = \frac{PB}{\tan x}$	(3)
7.2	$(AC)^2 = (4{,}73)^2 + (3)^2 - 2(4{,}73)(3)(\cos 100^o)$ AC = 6,03 m	✓ use of cosine rule *gebruik van cosinusreël* ✓correct substitution in cosine rule *korrekte vervanging in cosinusreël* ✓ answer / *antwoord*	(3)
			[6]

QUESTION 8 / *VRAAG8*			
8.1.1	$B\hat{A}E = 90^o$ ($\angle$ in a semi-circle /$\angle$ *in halwe sirkel*)	✓ statement/*stelling* (S) ✓ reason / *rede* (R)	(2)
8.1.2	$\hat{E}_1 = 40^o$ (sum of $\angle$ s of Δ / *som van* $\angle$*e van* Δ)	✓ statement / S ✓ reason / R	(2)
8.1.3	$\hat{C}_1 = 40^o$ ($\angle$s subtended by AB OR $\angle$s in the same segment) ($\angle$*e onderspan deur* AB *OF* $\angle$*e in dieselfde segment*)	✓ statement / S ✓ reason / R	(2)
8.1.4	$\hat{C}_2 = 62^o$ (ext. $\angle$ of cyclic quad. / *buite* $\angle$ *van koordev.*)	✓ statement / S ✓ reason / R	(2)
8.1.5	$A\hat{B}D = 62^o$ ($\angle$s subtended by AD OR $\angle$s in the same segment OR ext. $\angle$ of cyclic quad.) ($\angle$*e onderspan deur* AD *OF* $\angle$*e in dieselfde segment OF buite* $\angle$ *van koordevierhoek*)	✓ statement / S ✓ reason / R	(2)
8.2	equal to the angle in the alternate segment/ *gelyk aan die hoek in die oorstaande segment*	✓ answer / *antwoord*	(1)
8.3	R.T.P / Te Bewys: STUR cyclic / *koordevierhoek* Proof : $\hat{U}_2 = x$ (tan chord thm)/(*raaklyn koord stelling*) $\hat{R}_3 = y$ (tan chord thm)/(*raaklyn koord stelling*) $\hat{R}_2 = 180^o - (x + y)$ ($\angle$ s on str line)/($\angle$e op reguitlyn) **OR/*OF*** $\hat{R}_2 = 180^o - (x + y)$ (3 $\angle$ s Δ)/(3 $\angle$e Δ) T + $\hat{R}_2 = 180^o$ STUR cyclic (opposite angles are suppl.) STUR *is 'n koordevierhoek* (*oorstaande hoeke is suppl.*)	✓ statement / S ✓ reason / R ✓ S & R ✓ S & R ✓ opp. $\angle$ s of quad. supplementary *teenoorst.* $\angle$*e van vierhoek supplementêr*	(5)
8.4	PN = NQ = 8 units/*eenhede* (line from the centre $\perp$ chord) (*lynstuk vanaf middelpunt* $\perp$ *koord*) $MP^2 = 6^2 + 8^2$ (Pyth. Theorem)/(*Pyth. Stelling*) $\therefore$ MP = 10 units/*eenhede* MP = RM = MT = 10 (radii)/(*radiusse*) $\therefore$ RT = 20 units/*eenhede*	✓ S & R ✓ S & R ✓ S & R ✓ answer / *antwoord*	(4)
			[20]

QUESTION 9/*VRAAG9*			
9.1.1	$\frac{AD}{DE}=\frac{AC}{CF}$ (prop. theorem; DC‖EF) (*Eweredigh. stelling* ; DC‖EF) $\frac{12}{3}=\frac{AC}{5}$ $20=AC$	✓ S & R ✓ substitution / *vervanging* ✓ answer / *antwoord*	(3)
9.1.2	AO = 10 (diagonals of a parallelogram bisect) (*hoeklyne van 'n parallelogram halveer*)	✓ answer / *antwoord*	(1)
9.1.3	$\Delta ADC \text{ ||| } \Delta AEF$ ($\angle,\angle,\angle$) $\frac{AD}{AE}=\frac{DC}{EF}$ (||| Δs: sides in prop.) (||| Δe : *sye is eweredig*) $\frac{12}{15}=\frac{14}{EF}$ $EF=\frac{14\times15}{12}$ $EF=17,5$	✓ S & R ✓ statement / S ✓ answer / *antwoord*	(3)
9.2	$\frac{\text{Area }\Delta ADC}{\text{Area }\Delta AEF}=\left(\frac{12}{15}\right)^2$ $=\left(\frac{4}{5}\right)^2=\frac{16}{25}$ $\frac{\text{Area }\Delta ADO}{\text{Area }\Delta ADC}=\frac{1}{2}$ $\frac{\text{Area }\Delta ADC}{\text{Area }\Delta AEF}\times\frac{\text{Area }\Delta ADO}{\text{Area }\Delta ADC}=\frac{\text{Area }\Delta ADO}{\text{Area }\Delta AEF}$ But/*Maar*: $\frac{\text{Area }\Delta ADC}{\text{Area }\Delta AEF}\times\frac{\text{Area }\Delta ADO}{\text{Area }\Delta ADC}=\frac{16}{25}\times\frac{1}{2}$ $=\frac{8}{25}$ $\therefore \frac{\text{Area }\Delta ADO}{\text{Area }\Delta AEF}=\frac{8}{25}$ **OR/*OF*** $\frac{\text{Area }\Delta ADO}{\text{Area }\Delta AEF}=\frac{\frac{1}{2}AD.AO\sin D\hat{A}O}{\frac{1}{2}AE.AF\sin D\hat{A}O}$ $=\frac{12\times10}{15\times25}$ $=\frac{8}{25}$	**(Area => Oppervlakte)** ✓ $\frac{\text{Area }\Delta ADC}{\text{Area }\Delta AEF}=\frac{16}{25}$ ✓ $\frac{\text{Area }\Delta ADO}{\text{Area }\Delta ADC}=\frac{1}{2}$ ✓simplification / *vereenvoudiging* **OR/*OF*** ✓ use of area rule *gebruik van oppervlakte reël* ✓ area of ΔADO ✓ area of ΔAEF	(3)
			[10]

QUESTION 10/*VRAAG 10*			
10.1			
	Construction: Mark D on PQ and E on PR such that PD = ST and PE = SU. Join DE. *Konstruksie: Merk D op PQ en E op PR sodat PD = ST en PE = SU. Verbind DE.* Proof/*Bewys*: In ΔPDE and/*en* ΔSTU $\hat{P} = \hat{S}$ (given) / (*gegee*) PD = ST (construction) / (*konstruksie*) PE = SU (construction) / (*konstruksie*) $\Delta PDE \equiv \Delta STU$ (SAS) $\therefore \hat{D}_1 = \hat{T}$ ($\equiv \Delta's$) But/*Maar*: $\hat{Q} = \hat{T}$ (given) / (*gegee*) $\therefore \hat{D}_1 = \hat{Q}$ $\therefore$ DE || QR (corr. $\angle$ s =) / (*ooreenk.* $\angle e$ =) $\therefore \frac{PD}{PQ} = \frac{PE}{PR}$ (prop. theorem; DE || QR) $\therefore \frac{ST}{PQ} = \frac{SU}{PR}$	✓construction / *konstruksie* ✓ S & R ✓ $\hat{D}_1 = \hat{T}$ ✓ $\hat{D}_1 = \hat{Q}$ ✓ reason for lines || *rede vir || lyne* ✓ prop. theorem; DE || QR *eweredigh. stelling*; DE||QR	(6)
10.2.1	$\hat{A} = x$ (tan chord thm)/(raakl koord stelling) $\hat{C}_3 = x$ (angles opp. = sides)/(hoeke teenoor = sye)	✓ statement(S) ✓ reason(R) ✓ S & R	(3)
10.2.2	R.T.P/*Te Bewys*: ΔTOC ⫼ Δ BPC $\hat{ACB} = 90°$ ($\angle$ in the semi-circle)/($\angle$ *in halwe sirkel*) Proof/*Bewys* : $\hat{C}_3 = \hat{C}_1 = x$ (proved)/(*reeds bewys*) $\hat{T}_2 = 90^o + x$ (ext. $\angle$ of Δ) / (*buite* $\angle$ *van* Δ) $\hat{B}_1 = 90^o + x$ (ext. $\angle$ of Δ) / (*buite* $\angle$ *van* Δ) $\therefore \hat{T}_2 = \hat{B}_1$ $\therefore \hat{O}_2 = \hat{P}$ (sum of $\angle$ s in Δ) / (*som van* $\angle e$ *in* Δ) $\therefore$ ΔTOC ⫼ Δ BPC ($\angle, \angle, \angle$) **OR/*OF*** $\hat{ACB} = 90°$ ($\angle$ in the semi-circle) / (*hoek in halwe sirkel*) $\hat{B}_2 = 90° - x$ (sum of $\angle$s in Δ)/(*som van* $\angle e$ *in* Δ) $\therefore \hat{P} = 90° - 2x$ (ext. $\angle$of Δ) / (*buite* $\angle$ *van* Δ) $\hat{O}_1 = 2x$ ($\angle$at centre = 2 × $\angle$ at circum.)/(Middelpunts $\angle$ = 2×Omtrekshoek) $\therefore \hat{O}_2 = 90° - 2x$ (SO ⊥ AB) $\therefore \hat{O}_2 = \hat{P}$ $\hat{C}_3 = \hat{C}_1 = x$ (proved above) / (*alreeds bo bewys*) $\therefore$ ΔTOC|||ΔBPC($\angle, \angle, \angle$)	✓ S & R ✓ S ✓ S & R ✓ S ✓ R **OR/*OF*** ✓ S & R ✓ S ✓ S & R ✓ S ✓ R	(5)

10.2.3	R.T.P/*Te Bewys*: TO. PC = OB . BP Proof/*Bewys* : $\frac{TO}{BP} = \frac{OC}{PC}$ But/*Maar* OC = OB (radii)/(radiusse) $\therefore \frac{TO}{BP} = \frac{OB}{PC}$ $\therefore$ TO. PC = OB . BP	✓sides in prop. *sye is eweredig* ✓ S	(2)
10.2.4	In ΔOPC: $OP^2 = OC^2 + PC^2$ (Pyth. theorem/*stelling*) But/*Maar*: OB = OC = BP (radii)/(*radiusse*) $\therefore (2OC)^2 = OC^2 + PC^2$ $4OC^2 = OC^2 + PC^2$ $\therefore PC^2 = 3OC^2$	✓ S & R ✓ OB = OC = BP ✓ $(2OC)^2 = OC^2 + PC^2$	(3)
			[19]
		TOTAL/*TOTAAL*:	**150**