Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

SEPTEMBER 2019

TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages.
Hierdie nasienriglyn bestaan uit 17 bladsye.

NOTE:

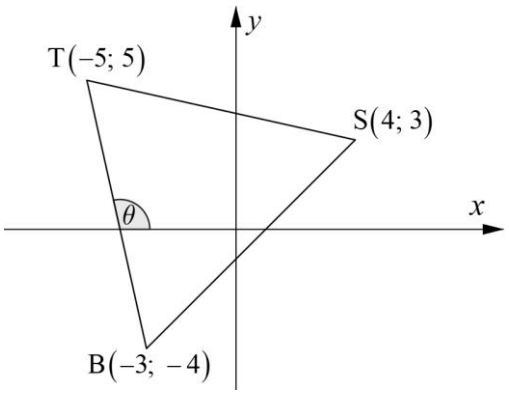
- Continuous accuracy (CA) applies in ALL aspects of the marking guideline.
- After two mistakes, do not apply CA marking.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is deurgaans in ALLE aspekte van die nasienriglyn van toepassing.*
- *Na twee foute word CA nie toegepas nie.*
- *Aanvaarding van waardes/antwoorde om ŉ problem op te los, is onaanvaarbaar.*

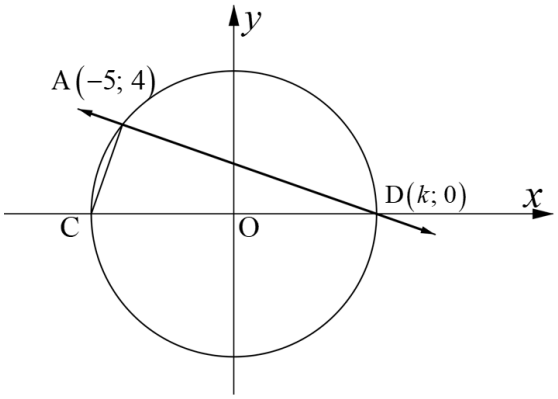
MARKING CODES / *NASIENKODES*	
M	**Method/*Metode***
MA	**Method with accuracy/*Metode met akkuraatheid***
A	**Accuracy/*Akkuraatheid***
CA	**Consistent accuracy/*Deurlopende akkuraatheid***
S	**Simplification or Statement / *Vereenvoudiging of bewering***
R	**Reason/*Rede***
SR	**Statement and correct reason/*Bewering en korrekte rede***
SF	**Substitution correctly in correct formula/ *Korrekte vervanging in die korrekte formule***

QUESTION/*VRAAG* 1



1.1	$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(4+5)^2 + (3-5)^2}$ $= \sqrt{81+4}$ $= \sqrt{85}$ $\approx 9{,}22$	✓**SF** **A** ✓**S** ✓Length as a decimal / *lengte as 'n desimaal*	(3)
1.2	$m_{BT} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5+4}{-5+3}$ $= -\frac{9}{2}$	✓**SF** **A** ✓**S**	(2)
1.3	$\tan\theta = m_{BT}$ $= -\frac{9}{2}$ $\theta = 180° - \tan^{-1}\left(\frac{9}{2}\right)$ $= 180° - 77{,}47°$ $= 102{,}5°$	✓**SF** **CA** ✓**S** 180° - **A** ✓**S** 77,47° **CA** ✓Value of / waarde van θ **CA**	(4)
1.4	$y - y_1 = m(x - x_1)$ $y - 3 = -\frac{9}{2}(x-4)$ $y = -\frac{9}{2}x + 21$	✓gradient / *gradiënt* **A** ✓**SF** **CA** ✓ equation / *vergelyking* **CA**	(3)
			[12]

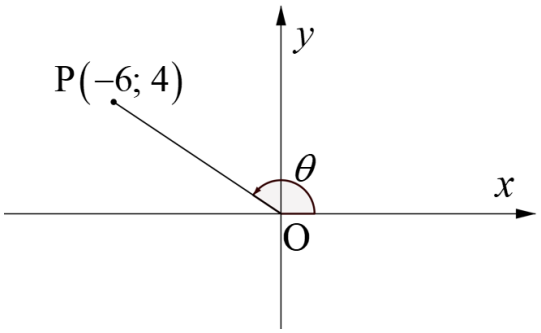
QUESTION/*VRAAG* 2



2.1	$x^2 + y^2 = (-5)^2 + 4^2$ $= 25 + 16$ $= 41$	✓**SF** **A** ✓equation / *vergelyking*	(2)
2.2	$x^2 + 0^2 = 41$ $x = \pm\sqrt{41}$ $\therefore k = \sqrt{41}$	✓**S** **A** ✓**S** **CA** ✓value of / *waarde van* ***k***	(3)
2.3	$m_{OA} = \frac{4}{-5}$	✓**S** **A**	(1)
2.4	$m_{\text{tangent/}raaklyn} = \frac{5}{4}$ $\therefore$ equation of tangent:/*vergelyking van raaklyn* $y - y_1 = m(x - x_1)$ $y - 4 = \frac{5}{4}(x + 5)$ $y = \frac{5}{4}x + \frac{41}{4}$	✓gradient / *gradiënte* C**A** ✓**SF** **CA** ✓ equation / *vergelyking* **CA**	(3)

2.5	$m_{AD} \times m_{CA} = \frac{4-0}{-5-\sqrt{41}} \times \frac{4-0}{-5+\sqrt{41}}$ $= \frac{16}{25-41}$ $= \frac{16}{-16}$ $= -1$ $\therefore$ AD $\perp$ CA $\left(\begin{matrix}\text{prod of grad} = -1 \\ \textit{prod van grad} = -1\end{matrix}\right)$ **OR/*OF*** $AD^2 = \left(-5-\sqrt{41}\right)^2 + (4-0)^2$ $= 25+10\sqrt{41}+41+16$ $= 82+10\sqrt{41}$ $C\left(-\sqrt{41}; 0\right)$ $AC^2 = \left(-5+\sqrt{41}\right)^2 + (4-0)^2$ $= 25-10\sqrt{41}+41+16$ $= 82-10\sqrt{41}$ $AD^2 + AC^2 = 82+10\sqrt{41}+82-10\sqrt{41}$ $= 164$ $CD = CO + OD$ $= \sqrt{41}+\sqrt{41}$ $= 2\sqrt{41}$ $CD^2 = 4 \times 41$ $= 164$ $\therefore AD^2 + AC^2 = CD^2$ $\therefore D\hat{A}C = 90°$ (Converse of Pythagoras/*Omgekeerde van Pythagoras*) **NO MARKS FOR ANGLE IN SEMICIRCLE/** ***GEEN PUNTE VIR HOEK IN HALFSIRKEL***	✓gradients / *gradiënte* C**A** ✓**SF** **CA** ✓ **S** **CA** ✓**R** ✓AD^2 & AC^2 **CA** ✓ AD^2 + AC^2 **CA** ✓ CD^2 **CA** ✓**R**	(4)
			[13]

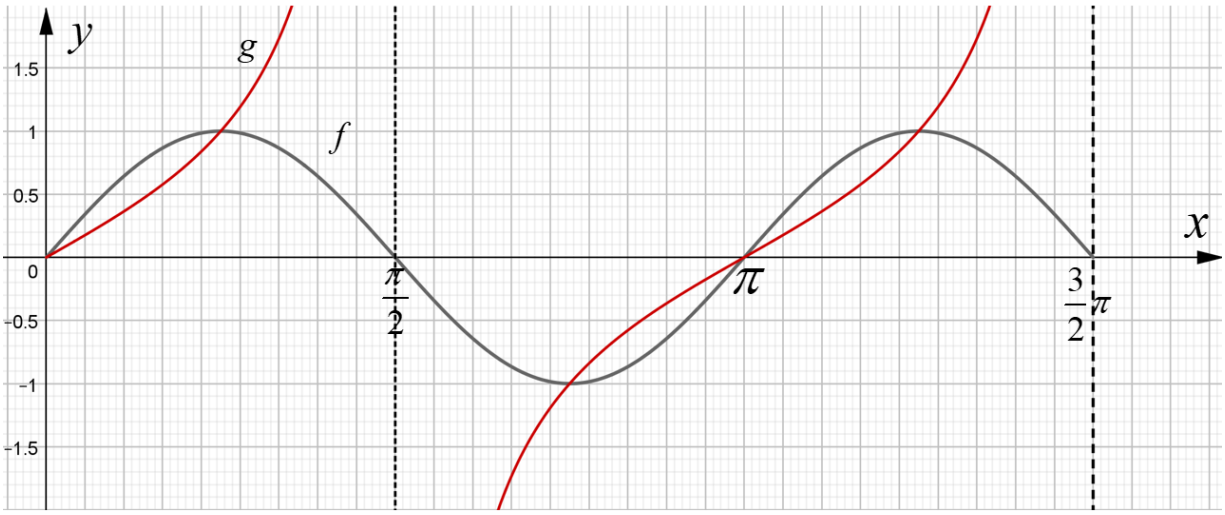
QUESTION/*VRAAG* 3

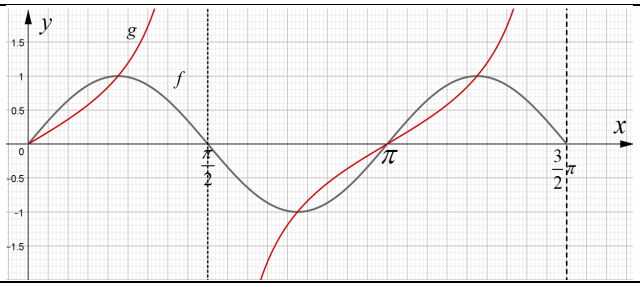


3.1.1	$OP = \sqrt{(-6)^2 + 4^2}$ $= \sqrt{36+16}$ $= \sqrt{52}$ $= 2\sqrt{13}$	✓applying Pythagoras/ *Pas Pythagoras toe* **M** ✓value of / *waarde van* OP	(2)
3.1.2	$\cos^2\theta - \sin^2\theta$ $= \left(\frac{-6}{\sqrt{52}}\right)^2 - \left(\frac{4}{\sqrt{52}}\right)^2$ $= \frac{36}{52} - \frac{16}{52}$ $= \frac{20}{52}$	✓value of / *waarde van* cos θ **CA** ✓value of / *waarde van* sin θ **CA** ✓**SF** **A** ✓**S** **CA**	(4)
3.1.3	$\cot\theta - 2$ $= \frac{4}{-6} - 2$ $= -\frac{8}{3}$	✓value of / *waarde van* cot θ **CA** ✓**S** **CA**	(2)
3.2	$\cot^2 2\beta - \text{cosec}^2\theta = \cot^2 2(73{,}2°) - \text{cosec}^2 64{,}5°$ $= \frac{1}{(\tan 146{,}4°)^2} - \frac{1}{(\sin 64{,}5°)^2}$ $\approx 1{,}04$	✓**SF** **A** ✓**S** 146,4° **A** ✓ $\frac{1}{\tan 146{,}4°}$ **M** ✓ $\frac{1}{\sin 64{,}5°}$ **M** ✓✓ value / *waarde* **CA**	(6)

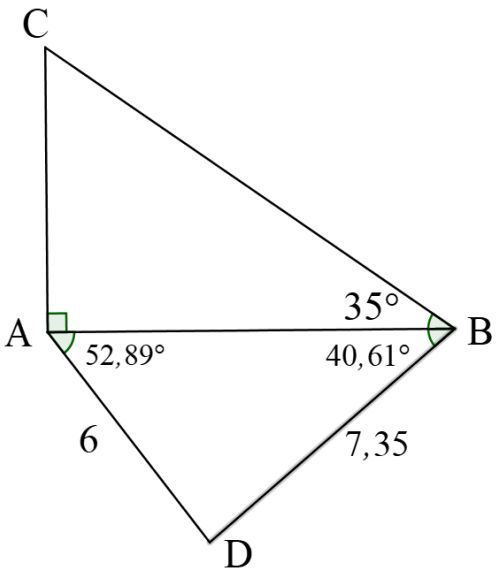
3.3	$\sin(180°-x).\cos(180°+x).\sec(360°+x).\sin^2\frac{\pi}{3}$ $=\sin x(-\cos x)\sec x.\left(\frac{\sqrt{3}}{2}\right)^2$ $=-\sin x.\cos x.\frac{1}{\cos x}.\left(\frac{3}{4}\right)$ $=-\frac{3}{4}\sin x$	✓ **sin *x*** **A** ✓ **– cos *x*** **A** ✓ **sec *x*** **A** ✓ $\frac{\sqrt{3}}{2}$ **A** ✓ **I** $\frac{1}{\cos x}$ **A** ✓ $\frac{3}{4}$ **CA** ✓ $-\frac{3}{4}$ **sin *x*** **CA**	**(7)**
3.4.1	$4\cos(2\theta+20°)=2,178$ $\cos(2\theta+20°)=0,5445$ $2\theta+20°=57°$ $\theta=18,5°$	✓ **S** **A** ✓ **S** only 1st quadrant / *slegs 1ste kwadrant* **CA** ✓ value of / *waarde van* θ **CA**	(3)
3.4.2	$\text{cosec}\ (\theta-30°)=1,57$ Ref/*verw* $\angle$ $=\sin^{-1}\left(\frac{1}{1,57}\right)=39,56°$ $\theta-30°=39,56°$ OR/*OF* $\theta-30°=180°-39,56°$ $\theta=69,56°$ OR/*OF* 170,44°	✓✓ Ref / *verw* $\angle$ **A** ✓ 1st quad / *1ste kwadrant* **A** ✓ 2nd quad / *2de kwadrant* **A** ✓ value quad 1 / *waarde kwad* ***1*** ✓value quad 2 / *waarde kwad* ***2***	(6)
			[30]

QUESTION/*VRAAG* 4



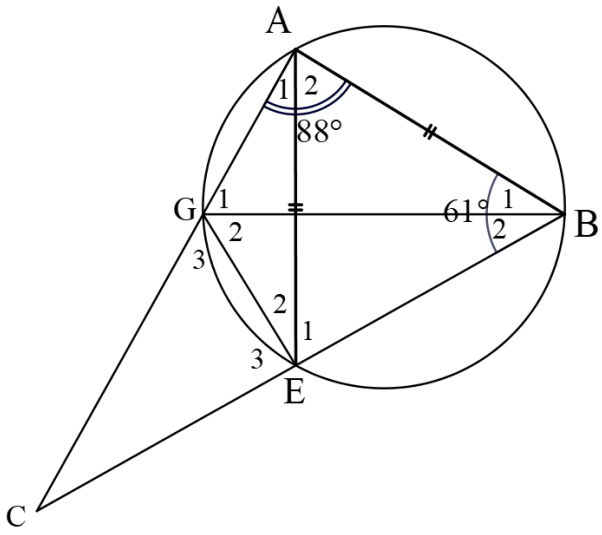
4.1.1	A: $x=\frac{\pi}{2}=90°$	✓**S** **A**	(1)
4.1.2	$p=2$	✓**S** **A**	(1)
4.1.3	$y=-1$	✓**S** **A**	(1)
4.2		✓shape / *vorm* **A** (intercept at the turning points of *f* / *sny by die draaipunte van f*) ✓*x*-intercepts / *x-afsnitte* **A** ✓ asymptotes / *asimptote* **CA**	(3)
4.3.1	$x\in\{180°;\ 225°\}$ OR/*OF* $x\in\left\{\pi;\ \frac{5\pi}{4}\right\}$	✓ $x=180°=\pi$ **CA** ✓ $x=225°=\frac{5}{4}\pi$ **CA**	(2)
4.3.2	$x\in[0°;\ 45°]=\left[0;\ \frac{\pi}{4}\right]$ **AND/*EN*** $x\in(90°;\ 135°]=\left(\frac{\pi}{2};\ \frac{3}{4}\pi\right]$	✓critical values / *kritiese waardes* **CA** ✓ notation / *notasie* **CA** ✓critical values / *kritiese waardes* **CA** ✓ notation / *notasie* **CA**	(4)
			[12]

QUESTION/*VRAAG* 5



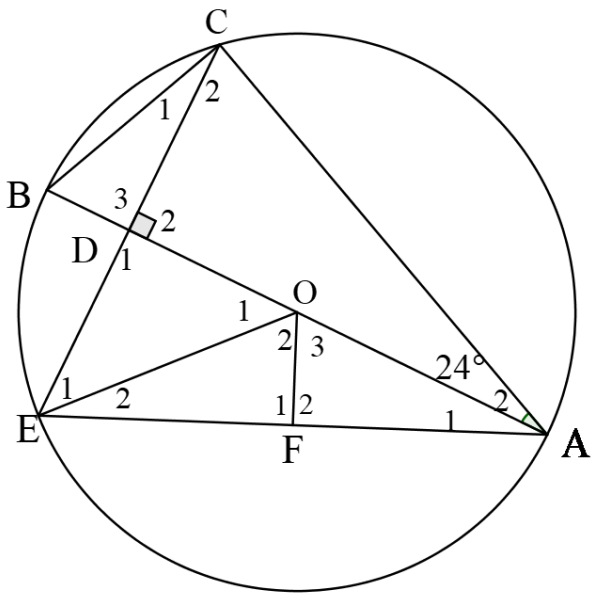
	$\frac{CA}{AB} = tan\,35°$ $CA = AB\,tan\,35°$	✓**S** ✓**S**	
	In ΔABD: $\hat{D} = 86,5°$ $\left(\text{Int } \angle\text{s of } \Delta \text{ / } \textit{Binne } \angle \textit{e van } \Delta\right)$	✓**SR**	
	$\frac{AB}{sin\,86,5} = \frac{AD}{sin\,40,61}$ $AB = \frac{6\,sin\,86,5}{sin\,40,61}$ $\approx 9,2$ m **OR/*OF***	✓**S** ✓**SF** ✓**S** **CA**	
	$AB^2 = AD^2 + BD^2 - 2AD.BD\,cos\,86,5$ $= 6^2 + 7,35^2 - 2(6)(7,35)\,cos\,86,5$ $= 84,638...$ $AB \approx 9,2$ m	**OR/*OF*** ✓**S** ✓**SF** ✓**S** **CA** ✓	
	$\therefore CA = 9,2\,tan\,35°$ $\approx 6,4$ m	✓**S** ✓**S** **CA**	(6)
			[6]

QUESTION/*VRAAG* 6



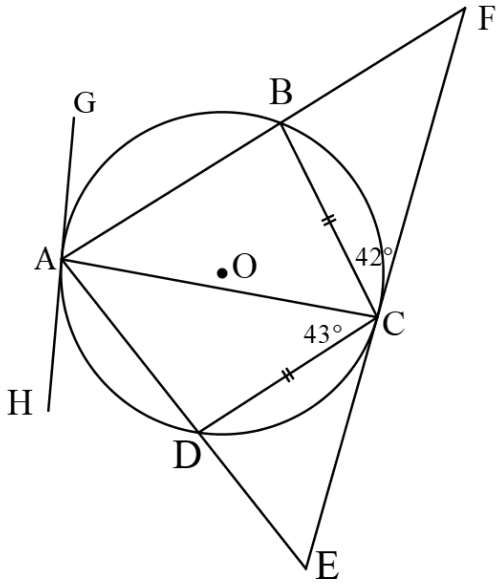
6.1	$\hat{E}_1 = 61°$ $(\angle s \text{ opp} = \text{sides}/\angle e \text{ } teenoor = sye)$ $\hat{G}_1 = 61°$ $(\angle s \text{ in same seg}/\angle e \text{ } in \text{ } dies \text{ } segm)$ $\hat{G}_3 = 61°$ $(\text{ext } \angle \text{ of cyclic quad}/buite \angle van \text{ } kdvh)$	✓ **SR** ✓ **S** ✓ **R** ✓ **S** ✓ **R**	(5)
6.2.1	$\hat{E}_2 = 180° - 88° - 61°$ $\begin{pmatrix}\text{opp } \angle s \text{ of cyclic quad}/\\ teenoorst \angle e \text{ } van \text{ } kdvh\end{pmatrix}$ $= 31°$	✓ **S** ✓ **R**	(2)
6.2.2	$\hat{B}_1 = 31°$ $(\angle s \text{ in same segm}/\angle e \text{ } in \text{ } dies \text{ } segm)$ $\therefore \hat{B}_2 = 30°$	✓ **S** ✓ **R** ✓ **S**	(3)
			[10]

QUESTION/*VRAAG* 7



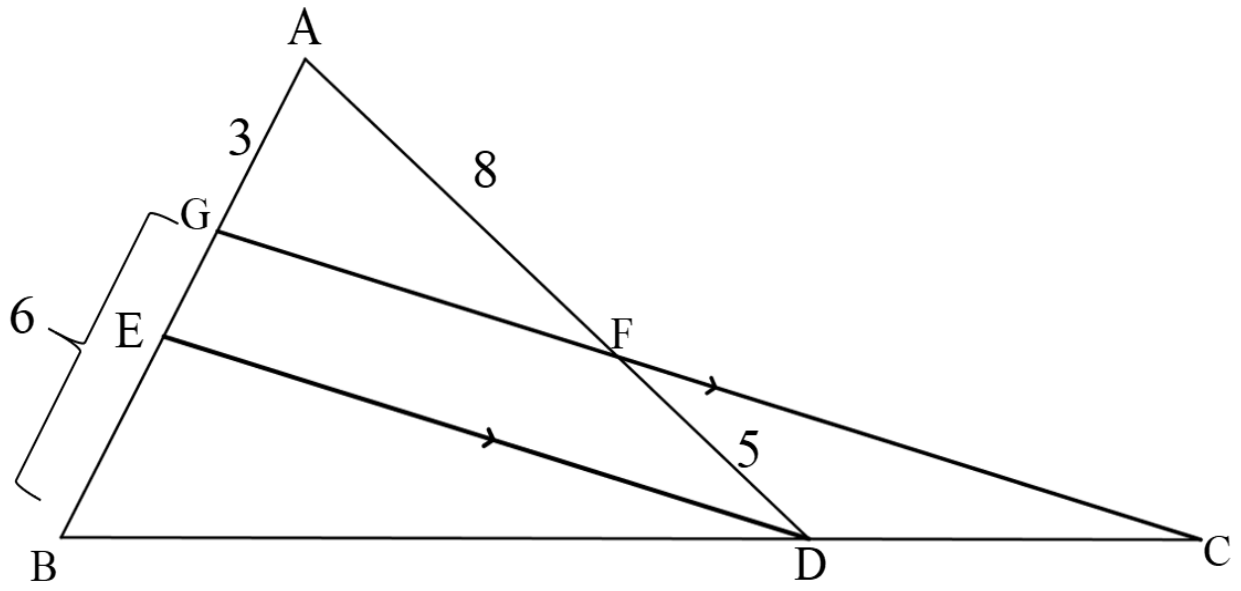
7.1	$\hat{C} = 90°$ $(\angle$ in semi circle/$\angle$ *in semi - sirkel*$)$ $\hat{B} = 66°$ $(\text{int } \angle\text{s of } \Delta$ / *binne* $\angle e$ *van* $\Delta)$	✓**S** ✓**R** ✓**SR**	(3)
7.2.1	In ΔADC and/en ΔADE AD is common/*gemeen* CD = DE (line from centre $\perp$ to chord/ *lyn vanaf middelpt* $\perp$ *na koord*) $\hat{D}_1 = 90° = \hat{D}_2$ $\therefore \Delta ADC \equiv \Delta ADE$ (SAS)	✓**S** ✓**SR** ✓**S** ✓**R**	(4)
7.2.2	$\hat{A}_1 = \hat{A}_2$ $(\equiv \Delta s)$ $\therefore$ *DA* bisect/*halveer* $\hat{A}$	✓**S**	(1)
7.2.3	$\hat{O}_1 = 48°$ $(\angle$ at centre = $2 \times \angle$ at circumf/*middelpts* $\angle = 2 \times$ *omtreks*$\angle)$	✓**S** ✓**R**	(2)
7.3	$\hat{F}_2 = 90°$ (line from centre to midpt of chord/ *lyn vanaf middelpt van sirkel na middelpt koord*) $\therefore$ DOFE is cyclic (converse ext $\angle$ of cyclic quad) $\therefore$ *DOFE is 'n kdvh* $(omgekeerde\ buite\angle\ van\ kdvh)$	✓**S** ✓**R** ✓**R**	(3)
			[13]

QUESTION/*VRAAG* 8



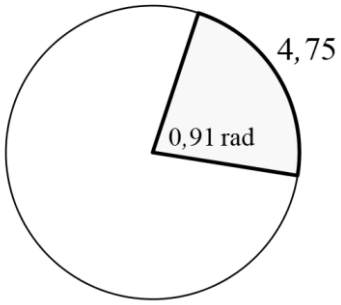
8.1.1	$B\hat{A}C = 42°$ (tan-chord/*raaklyn – koord*) $D\hat{A}C = 42°$ (equal chords; equal ∠s / *gelyke koorde; gelyke∠e*)	✓**S** ✓**R** ✓**S** ✓**R**	(4)
8.1.2	$A\hat{D}C = 95°$ (Int ∠s of Δ / *binne ∠e van Δ*) $A\hat{B}C = 85°$ (opp ∠s of cyclic quad / *teenoorst ∠e van kdvh*) $\hat{F} = 43°$ (ext ∠ of Δ / *buite∠ van Δ*)	✓**SR** ✓**S** ✓**R** ✓**SR**	(4)
8.2	$H\hat{A}D = 43°$ (tan-chord/*raaklyn - koord*) $\hat{F} = 43°$ (proved in 8.1.2/ *bewys in* 8.1.2) GAH a tangent to AFE (converse tan-chord th) *GAH 'n raaklyn aan AFE (omgekeerde raaklyn - koord st)*	✓**S** ✓**R** ✓**R**	(3)
			[11]

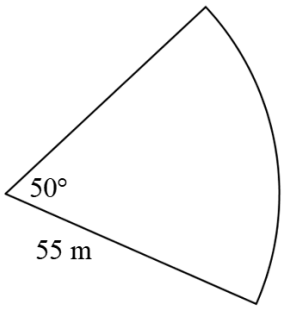
QUESTION/*VRAAG* 9

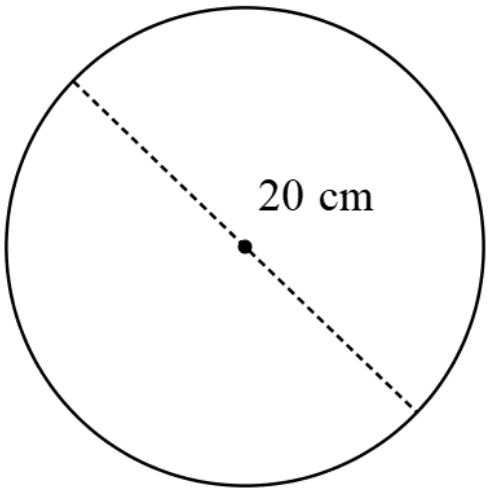


9.1	Let/*Laat* GE = x $\frac{GE}{AG} = \frac{FD}{AF}$ (prop th; ED ∥ GF / *ewer st; ED ∥ GF*) $\frac{x}{3} = \frac{5}{8}$ $x = \frac{15}{8}$ $x = 1{,}875$ $x \approx 2$	✓**S** ✓**R** ✓**S** (ratio/ *verhouding)* ✓**S** value of *x / waarde van x* ✓**S** rounding / *afronding*	(5)
9.2	In ΔGBC $\frac{BC}{BD} = \frac{BG}{BE}$ (prop th; GC ∥ ED / *ewer st; GC ∥ ED*) $= \frac{6}{4}$ $= \frac{3}{2}$	✓**SR** ✓**S** **CA** value of/ *waarde van* BE ✓**S** value / *waarde*	(3)
			[8]

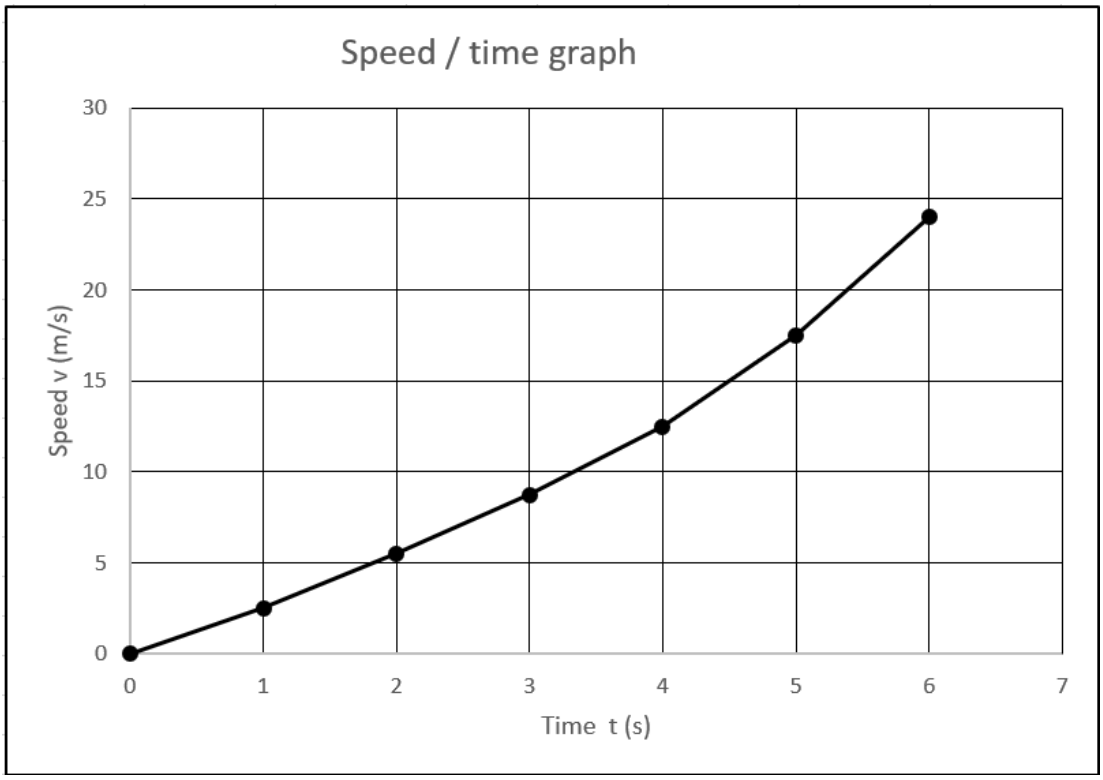
QUESTION/*VRAAG* 10



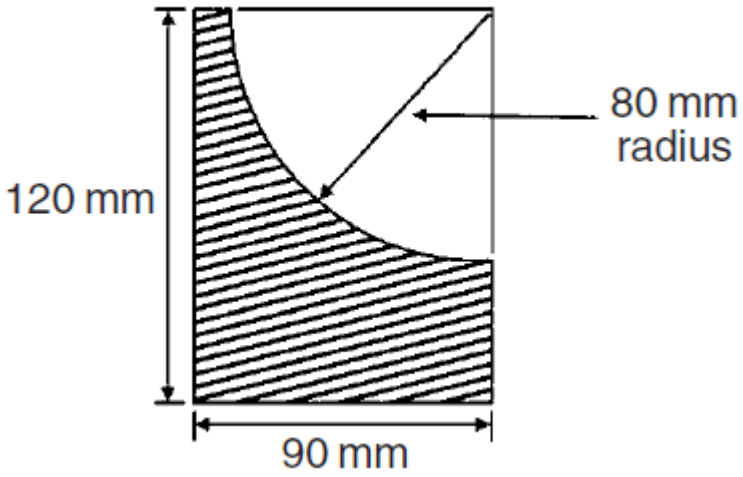
10.1.1	$s = r\theta$ $r = \frac{s}{\theta}$ $= \frac{4{,}75}{0{,}91}$ $= 5{,}22$ cm	✓ formula / *formule* **A** ✓ *r* subject / *onderwerp* **A** ✓**SF** **A** ✓**S** **CA**	(4)
10.1.2	Circumference/Omtrek $= 2\pi r$ $= 2\pi(5{,}22)$ $\approx 32{,}80$ cm	✓ formula / *formule* **A** ✓**SF** **A** ✓**S** **CA**	(3)
10.2			
10.2.1	$50° = 50° \times \frac{\pi}{180°} \approx 0{,}873$ rad	✓**M** ✓**A** ✓ rounding / *afronding*	(3)
10.2.2	Floodlit area/*Sprei beligte area* = area of sector/*area van sektor* $= \frac{1}{2}r^2\theta$ $= \frac{1}{2}(55)^2(0{,}873)$ ≈ 1320 m^2	✓ formula / *formule* **A** ✓**SF** ✓**S** **CA**	(3)

10.3			
10.3.1	$r = \frac{20}{2} = 10 \text{ cm}$	✓**A**	(1)
10.3.2	$n = 215 \text{ rev/min}$ $\omega = 2\pi n$ $= 2\pi(215)$ $= 430\pi \text{ rad/min}$	✓ formule / *formule* **A** ✓**SF A** ✓ value of / *waarde van* ω **CA**	(3)
10.3.3	$\upsilon = \omega r$ $= 430\pi \times 10$ $= 4300\pi \text{ cm/min}$	✓ formule / *formule* **A** ✓**SF A** ✓ value of / *waarde van* υ **CA**	(3)
10.3.4	$4300\pi \text{ cm/min} = \frac{4300\pi \text{ cm}}{1 \text{ min}} \times \frac{60 \text{min}}{1 \text{ hr}} \times \frac{1 \text{ km}}{10\,000 \text{ cm}}$ $= 25{,}8\pi$ $\approx 81 \text{ km/h}$	✓**M × 60** ✓**M** $\times \frac{1}{10\,000}$ ✓**S CA** ✓ rounding/*afronding* **CA**	(4)
			[24]

QUESTION/*VRAAG* 11								
11.1	Time *t* (s)	0	1	2	3	4	5	6
	Speed v (m/s)	0	2,5	5,5	8,75	12,5	17,5	24



	$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ $= 1\left(\frac{0+24}{2} + 2,5 + 5,5 + 8,75 + 12,5 + 17,5\right)$ $= 12 + 46,75$ $= 58,75 \text{ m}^2$	✓ formula / *formule* **A** ✓ value of / *waarde* van *a* **A** ✓**SF** **A** ✓value of / *waarde van* $\boldsymbol{A_T}$ **CA**	(4)



11.2	Area of full rectangle/*Area van volle reghoek* $= l \times b$ $= 120 \times 90$ $= 10800 \text{ mm}^2$ Area of quarter circle/*Area van kwart sirkel* $= \frac{1}{4}\pi r^2$ $= \frac{1}{4}\pi (80)^2$ $= 1600\pi$ Area of figure/*Area van figuur* $= 10800 - 1600\pi$ $= 5773{,}45 \text{ mm}^2$	✓formula / *formule* ✓**SF A** ✓ value of rectangle / *waarde van reghoek* ✓formula / *formule* ✓**SF A** ✓ value of circle / *waarde van sirkel* ✓ value of figure / *waarde van figuur*	(7)
			[11]

TOTAL/*TOTAAL*: 150