Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2022

TECHNICAL MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTIONS 4.2.7 and 7.4 on the ANSWER SHEETS provided. Write your name and school's name in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+7)+10=0$ (3)

1.1.2 $2x-1=\frac{4}{x}$ (Correct to TWO decimal places) (3)

1.1.3 $x^2+\frac{7x}{2}+3\leq 0$ (3)

1.2 Solve for x and y if:

$x-y-1=0$ and $xy+y^2=x$ (5)

1.3 The following formula for Estimation of Blood Alcohol Content (EBAC) is used for programming the breathalyser, an instrument used to estimate the amount of alcohol in someone's blood:

$$\text{EBAC} = \frac{(\text{BWb} \times \text{SD}) \times C}{\text{GBW} \times \text{BWt}} - \text{GMR} \times \text{DP}$$

Where,

- BWb is a constant for Body Water in the Blood stream.
- SD is a number of Standard Alcoholic Drinks taken.
- GBW is a Gender Body Water Constant.
- BWt is Body Mass of the person that drank SD (in kilograms, kg).
- C is a conversion factor.
- GMR is the Gender Metabolism Rate.
- DP is the Drinking Period (in hours, h)

1.3.1 Make SD the subject of the formula. (2)

1.3.2 A person, after a night shift work, consumed some alcoholic drinks at 2 am.

If the person's:

EBAC = 0,07
BWb = 1,806
GBW = 0,58
BWt = 140 kg
C = 3,2
GMR = 0,18

(a) Express the person's EBAC in Scientific Notation. (1)

(b) Determine the number of standard alcoholic drinks (SD) the person consumed on the night, correct to a whole number. (2)

(c) If it is punishable by law to consume more than 4 alcoholic drinks in one night in the person's country, determine the number of drinks the person exceeded according to the set limit. (1)

1.4 Simplify, WITHOUT using a calculator:

$1110_2 + 11_2$ (1)

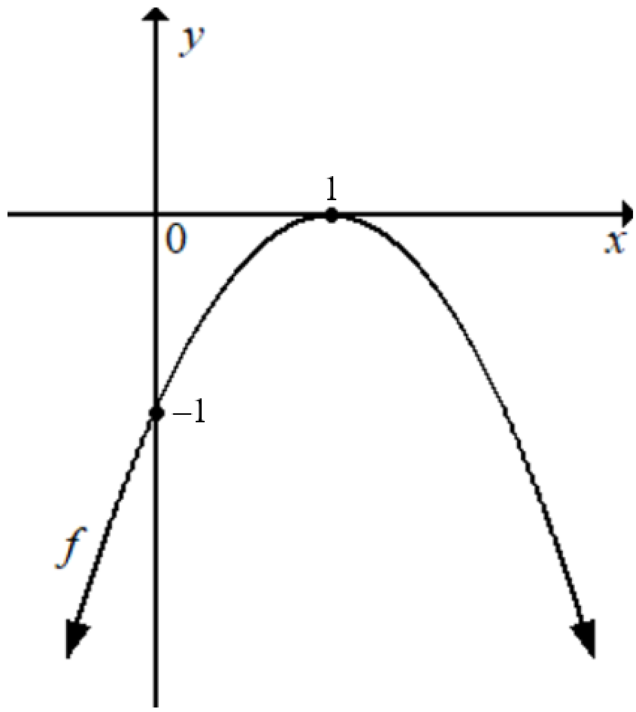
[21]

QUESTION 2

2.1 Given: $g\left(x\right)=\dfrac{3-\sqrt{x}}{25-x^2}$

Determine the values of x for which g is undefined. (2)

2.2 Consider the following sketch of a function defined by $f(x) = -(x+1)^2$:



2.2.1 Write down the value of the discriminant of f. (1)

2.2.2 Hence or otherwise, describe the nature of roots of the above function. (2)

2.2.3 If $f(x) + k = 0$, determine the value(s) of k for which f will have two distinct real roots. (1)

[6]

QUESTION 3

3.1 Simplify the following WITHOUT using a calculator:

3.1.1 $\dfrac{5^{x+1}.2^{2x-3}}{20^x}$ (Leave the solution with a POSITIVE exponent) (3)

3.1.2 $\dfrac{\sqrt{405}-\sqrt{80}}{\sqrt{5}}$ (3)

3.2 Given: $\log_a 3x = \log_a \left(2x^2 - 9\right)$

3.2.1 Show, by means of calculations, that $x > \dfrac{3}{\sqrt{2}}$ (3)

3.2.2 Hence or otherwise, solve for the exact value(s) of x. (5)

3.3 Given the complex numbers $z_1 = 2 - 5i$ and $z_2 = 1 + i$

Determine:

3.3.1 The complex number P, if $\mathrm{P} = \dfrac{z_1}{z_2}$ (4)

3.3.2 The modulus of P (2)

3.3.3 The size of θ, the angle of inclination of P (3)

3.3.4 Hence, express P in polar form (where θ is in degrees) (1)

3.4 Solve for x and y if $x + 2yi = -3$ (2)

[26]

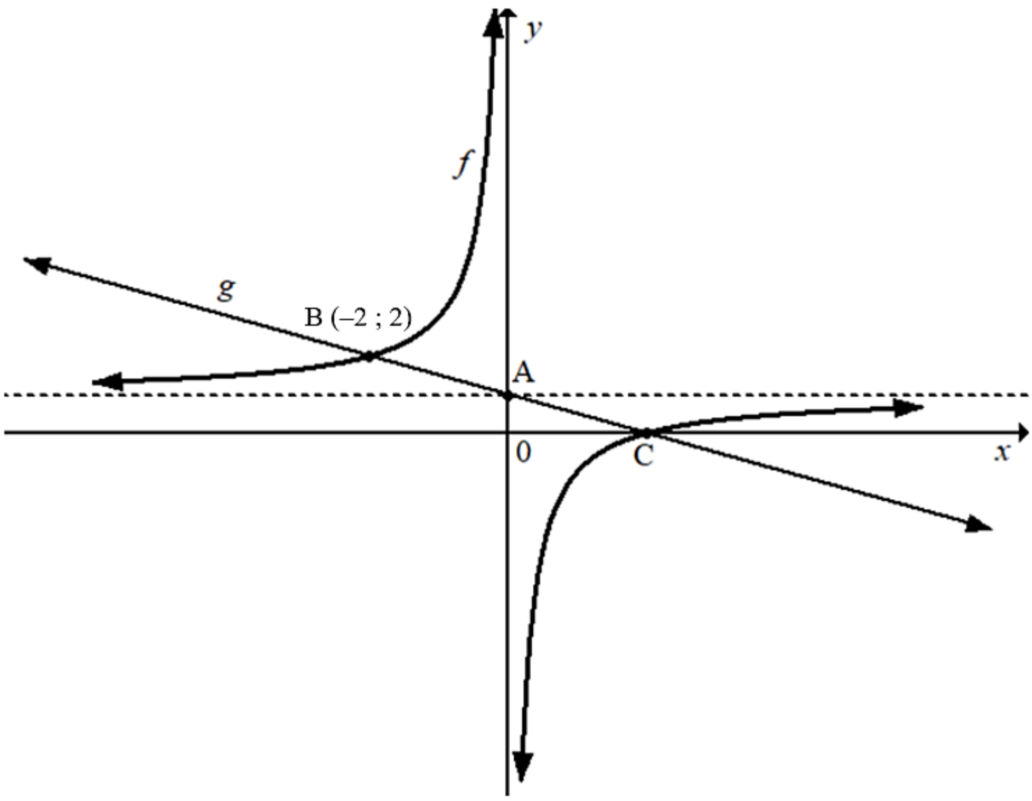
QUESTION 4

4.1 The diagram below shows sketch graphs of functions defined by

$f(x)=-\frac{2}{x}+1$ and $g(x)=-\frac{x}{2}+1$

The two graphs cut each other at point B(–2 ; 2) and at C.

The horizontal asymptote cuts the y-axis at A.



4.1.1 Write down the y-coordinate of A. (1)

4.1.2 Determine the equation of the vertical asymptote of f. (1)

4.1.3 Write down the x-intercept of f. (1)

4.1.4 Write down the y-intercept of g. (1)

4.1.5 Determine the range of f. (2)

4.1.6 Determine the value(s) of x for which $f(x)<g(x)$ (3)

4.2 Given: $k(x) = 2x^2 - 7x + 3$ and $w(x) = \sqrt{9 - x^2}$

4.2.1 Explain why the graph of w is a function. (1)

4.2.2 Write down the y-intercept of k. (1)

4.2.3 Calculate the y-intercept of w. (1)

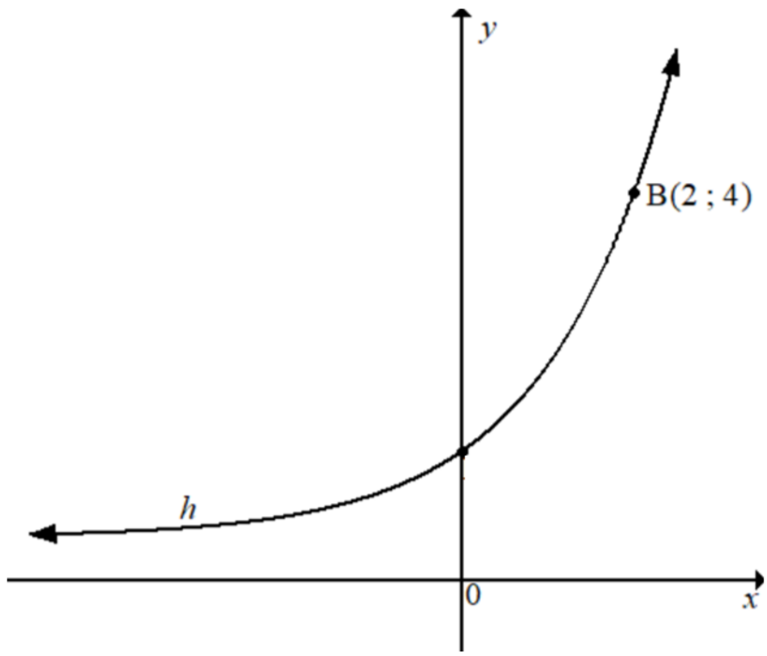
4.2.4 Determine the coordinates of the x-intercepts of k. (2)

4.2.5 Determine the x-intercepts of w. (1)

4.2.6 Determine the turning point of k. (4)

4.2.7 Sketch the graph of k and w on the same set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes and all asymptote(s). (5)

4.3 Consider the sketch graph of a function defined by $h(x) = a^x$. The graph passes through point B(2;4).



4.3.1 Write down the y-intercept of h. (1)

4.3.2 Determine the value of a. (3)

[28]

QUESTION 5

5.1 Melody buys her car radio which costs R2 960,00 on hire purchase and agrees to pay a deposit of R350 and a R145 monthly instalment for a period of 24 months.

Calculate:

5.1.1 Melody's deposit, as a percentage (1)

5.1.2 The hire purchase value after paying the deposit (1)

5.1.3 The interest rate charged on the hire purchase agreement (3)

5.2 Cype invests R20 000 for 7 years into an investment account that grows at 6 % interest rate per annum, compounded monthly for the first 3 years. The interest rates increased to 7,5% per annum on simple interest calculated quarterly p.a. for the remaining years.

5.2.1 Calculate the amount Cype got at the end of the 7th year investment period. (4)

5.2.2 Determine the amount Cype would have withdrawn at the end of the 3rd year in order to get R30 000 paid to her at the end of the investment period. (4)

[13]

QUESTION 6

6.1 Determine $f'(x)$ by using FIRST PRINCIPLES if $f(x) = -1 - 2x$. (5)

6.2 Determine:

6.2.1 $D_x\left(x^2 + x - 2\right)$ (2)

6.2.2 $\dfrac{dy}{dx}$ if $xy = x\sqrt{x} - 9x^2 - 1$ (5)

6.3 Determine the average gradient of a function between the points $(-3\,;\,0)$ and $(2\,;\,5)$. (2)

[14]

QUESTION 7

Given: $f(x) = x^3 - 7x + 6$

7.1 Write down the y-intercept of f. (1)

7.2 Determine the x-intercepts of f. (5)

7.3 Determine the coordinates of the turning point of f. (5)

7.4 Sketch the graph of f on the ANSWER SHEET provided. Clearly show all the coordinates of the turning points and intercepts with the axis. (4)

7.5 Determine the equation of a tangent to the curve of f at $x = -2$ (4)

[19]

QUESTION 8

A certain state is applying strict tax laws to control the private selling of indigenous products by increasing the tax for more items sold. The following is an Objective Profit function that is used by the state to control the sale of this indigenous item:

$P(x) = -x^2 + 5x$, in thousand dollars, where x is the number of the indigenous items sold.

8.1 Write 1,5 thousand as a whole number. (1)

8.2 Determine the number of items that generate no profit when sold. (4)

8.3 Calculate, how much profit, in thousand dollars, does a single indigenous item sold, generate. (2)

8.4 Determine the maximum number of indigenous items that should be sold to generate the maximum profit. (2)

8.5 Calculate the maximum profit, in thousand dollars, that can be made in sales. (2)
[11]

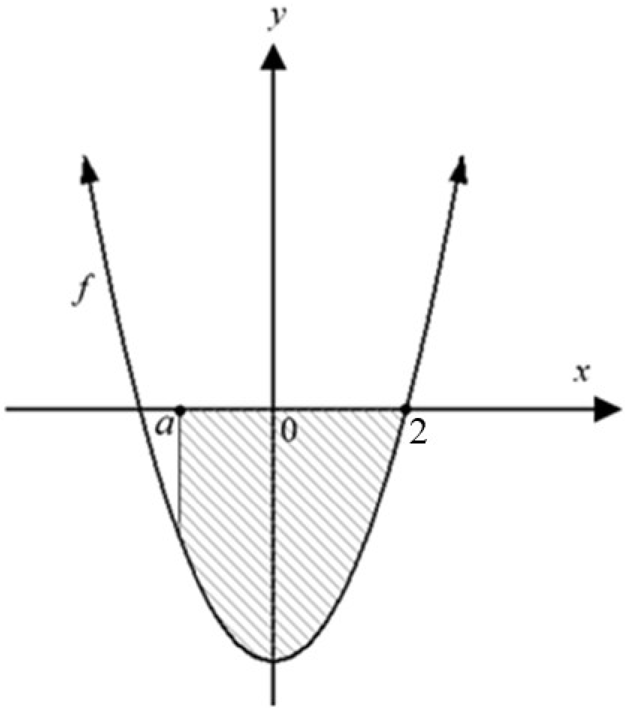
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int x^{\frac{1}{2}}\,dx$ (2)

9.1.2 $\int\left(x^{-2}-\frac{\pi}{x}\right)dx$ (2)

9.2 The sketch below represents the shaded area bounded by the function defined by $f(x)=5x^2-20x$ and the x-axis from $x = a$ to $x = 2$



Determine the value of a if the shaded area is 45 square units. (8)

[12]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\dfrac{b}{2a}$ $\qquad y = \dfrac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b\,,\quad a > 0,\ a \neq 1 \text{ and } b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$ $\qquad A = P(1 + i)^n$ $\qquad A = P(1 - i)^n$

$i_{eff} = \left(1 + \dfrac{i}{m}\right)^m - 1$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + C\,,\ n \neq -1$ $\qquad \int k x^n\, dx = k.\dfrac{x^{n+1}}{n+1} + C\,,\ n \neq -1$

$\int \dfrac{1}{x} dx = \ln x + C\,,\ x > 0$ $\qquad \int \dfrac{k}{x} dx = k.\ln x + C\,,\ x > 0$

$\int a^x\, dx = \dfrac{a^x}{\ln a} + C\,,\ a > 0$ $\qquad \int k a^{nx}\, dx = k.\dfrac{a^{nx}}{n \ln a} + C\,,\ a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad M\left(\dfrac{x_2 + x_1}{2}\,;\,\dfrac{y_2 + y_1}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad \tan\theta = m$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area of } \Delta\, ABC = \dfrac{1}{2} ab.\sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta$

$\pi\, rad = 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity = $v = 2\pi r n$ where r = radius and = rotation frequency

Arc length = $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length and θ = central angle in radians

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

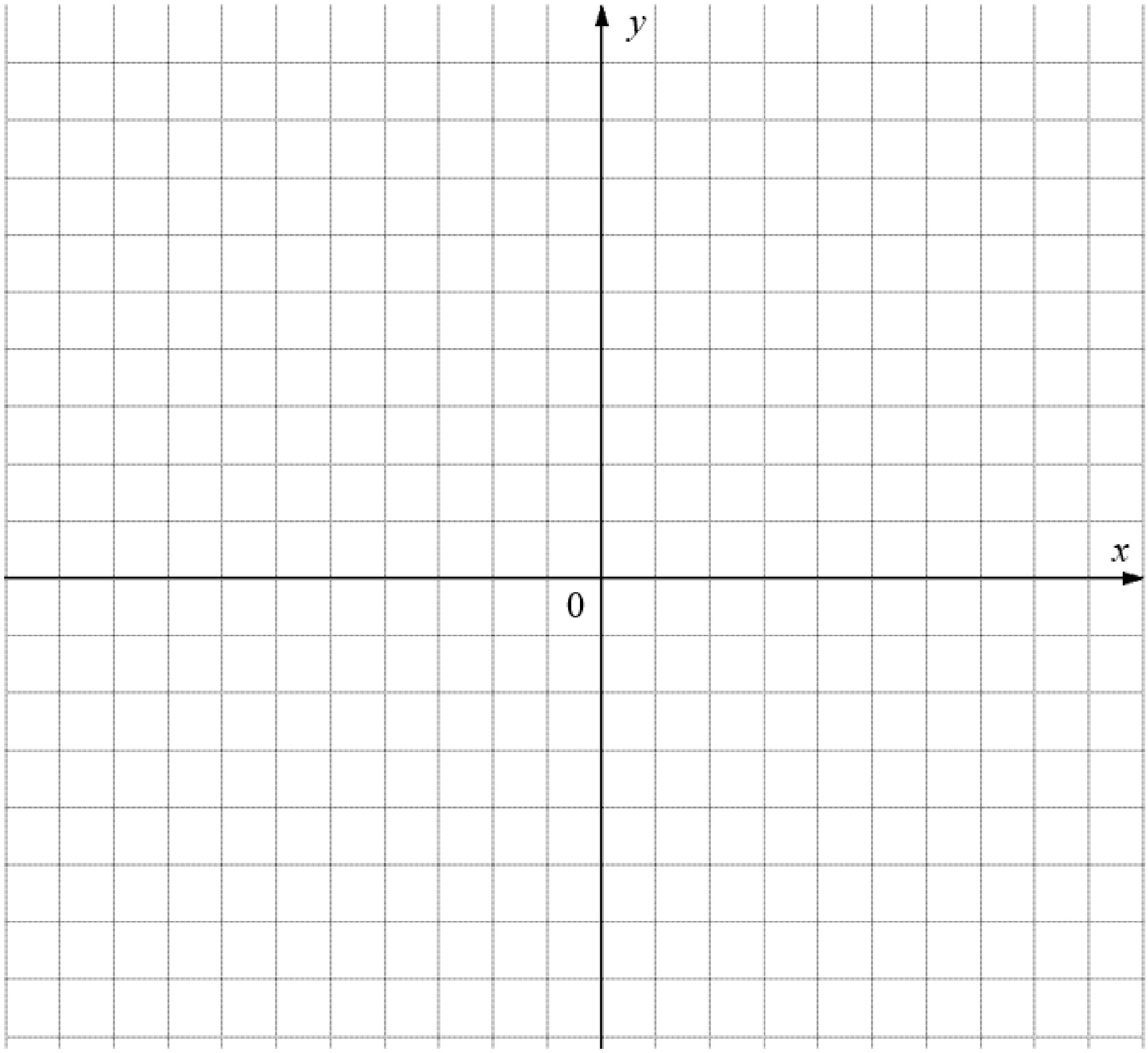
$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates

QUESTION 4.2.7



QUESTION 7.4