



**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

NOVEMBER 2023

**TECHNICAL MATHEMATICS P2
(DEAF LEARNERS)**

MARKS: 150

TIME: 3 hours

This question has 17 pages, with a 2-page information sheet.

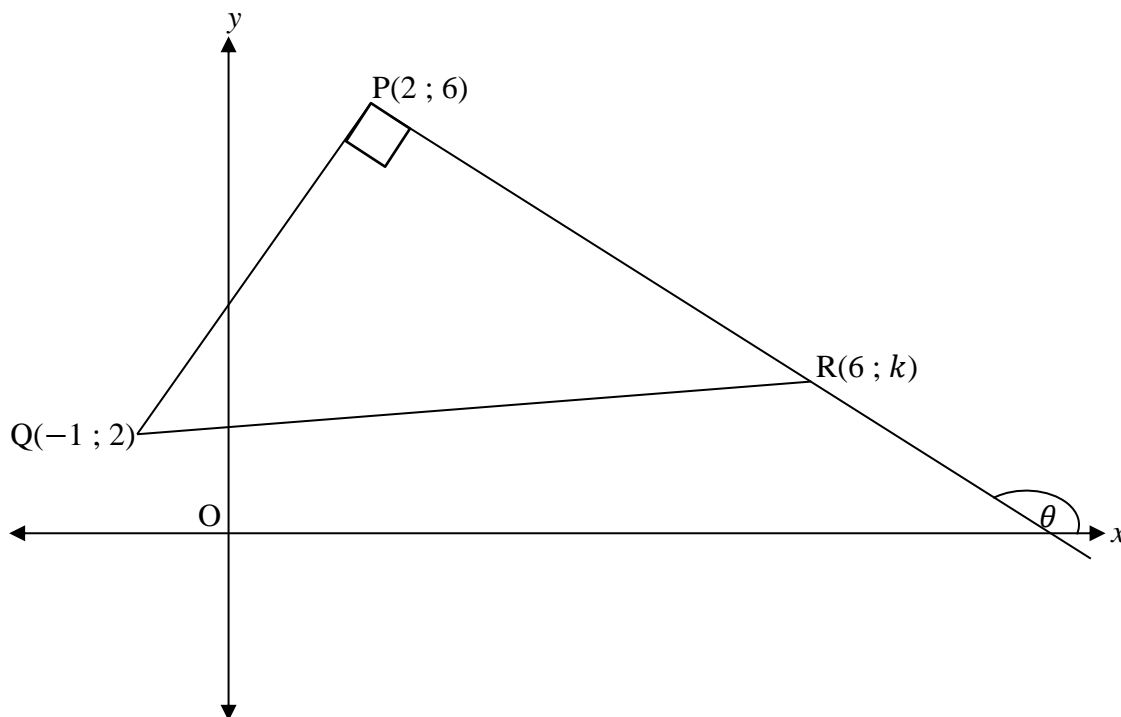
INSTRUCTIONS AND INFORMATION

Read the instructions.

1. This **question paper** has **TEN (10)** questions.
2. **Answer ALL** the **questions**.
Use the SPECIAL ANSWER BOOK.
3. **Show ALL calculations, diagrams, graphs, et cetera** that you have **used** in **working out** your **answers**.
4. **Answers only** will **NOT** always be awarded **full marks**.
5. You **may use** a prescribed **calculator**.
Some questions will **tell** you **NOT** to use a **calculator**.
6. **Round off** answers to **TWO decimal places**.
Some questions will **tell** you **how to round off**.
7. **Diagrams** are **NOT** always drawn to **scale**.
Some questions will **tell** you to **use the scale**.
8. An **information sheet** with **formulae** is included at the **end** of the question paper.
9. Write **neatly**.
Your **work** must be **easy to read**.

QUESTION 1

Diagram:

 ΔPQR has a **right angle** at **P**.The **coordinates** of the **vertices** are **P(2 ; 6)**; **Q(-1 ; 2)** and **R(6 ; k)**. θ is the **angle of inclination** of line **PR**.

1.1 Complete the statement:

“When **two lines** are **perpendicular** the ... of the **gradients** must be **equal** to -1.” (1)

Determine:

1.2 The **gradient** of **PQ** (3)1.3 **Show** that the **value** of $k = 3$ (3)1.4 The **coordinates** of the **midpoint** of **QR** (3)1.5 The **coordinates** of **S**, so that **QPRS** is a **rectangle** (4)1.6 The **equation** of **PR** (4)1.7 θ , the **inclination angle** of **line PR** (3)1.8 If the **length** of **PQ** = 5 units, **calculate** the **size** of \hat{Q} (5)

[26]

QUESTION 2

2.1 **Given:** $x = 30,5^\circ$ and $y = 130,5^\circ$

Determine:

2.1.1 $\tan(x + y)$ (2)

2.1.2 $\operatorname{cosec}(y - x)$ (3)

2.2 If $\sin 36^\circ = k$, **express the following in terms of k .**

2.2.1 $\cos 36^\circ$ (4)

2.2.2 $\sin 216^\circ$ (2)

2.3 **Solve for θ , $\theta \in [0^\circ; 360^\circ]$ rounded off to ONE decimal digit:**

$\tan \theta = 2 \sin 38,1^\circ$ (4)

[15]

QUESTION 33.1 **Simplify:**

$$\frac{\cos(360^\circ - \theta) \cdot \frac{1}{\cot(180^\circ + \theta)} \cdot \tan(360^\circ + \theta)}{\cos(180^\circ + \theta) \cdot \tan(180^\circ - \theta)} \quad (6)$$

3.2 **Prove that:**

$$\left(\tan x + \frac{1}{\cos x} \right)^2 = \frac{1 + \sin x}{1 - \sin x} \quad (4)$$

[10]

QUESTION 4

Given $f(x) = \cos x$ **and** $g(x) = \sin x + 1$; $x \in (0^\circ; 360^\circ)$

4.1 Use the **grid** in the SPECIAL ANSWER BOOK.

On the same axes, draw the graphs of $f(x) = \cos x$ and $g(x) = \sin x + 1$.

Show ALL:

- The intercepts with the axes
- Turning points
- Endpoints

(7)

4.2 Write down the **range** of g .

(2)

4.3 Write down the **period** of f .

(1)

4.4 Use your **graphs**.

Determine for which values of x , is $f(x) \cdot g(x) \leq 0$.

(2)

[12]

QUESTION 5

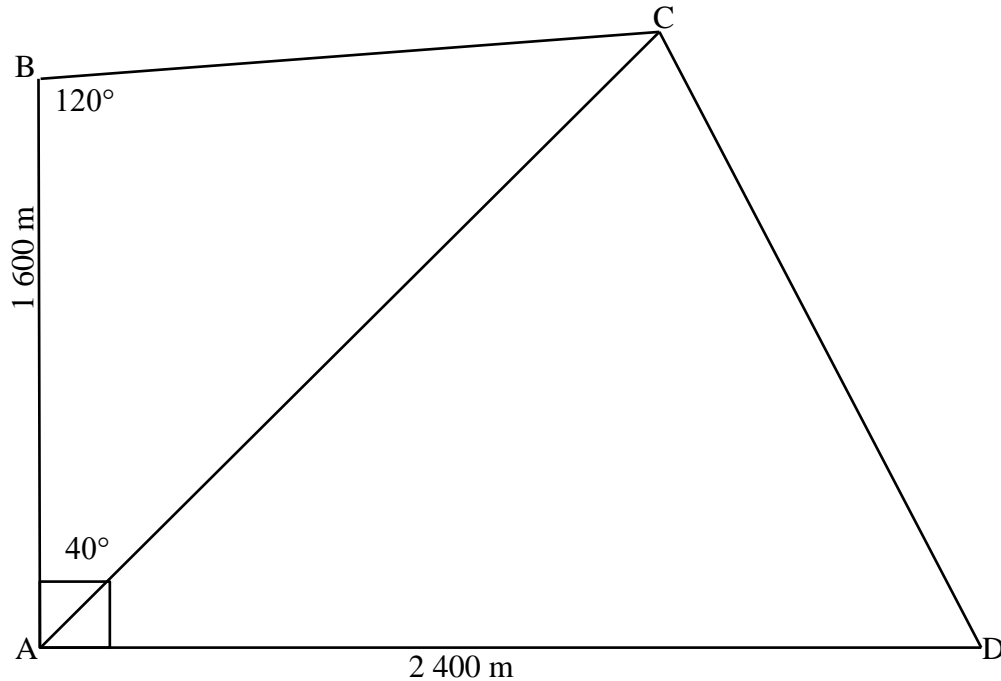
A farm has the shape shown in the figure below.

It has a fence dividing it into two triangles.

The lengths of two adjacent (adjoining) sides of the farm are 1 600 m and 2 400 m.

They make an angle of 90° , with one another at A.

$\widehat{BAC} = 40^\circ$ and $\widehat{CBA} = 120^\circ$.



- 5.1 Determine the size of \widehat{BCA} .
State a reason. (2)
- 5.2 Determine the length of AC, to the nearest whole number. (3)
- 5.3 Determine the total area of the farm, ABCD. (6)
- [11]

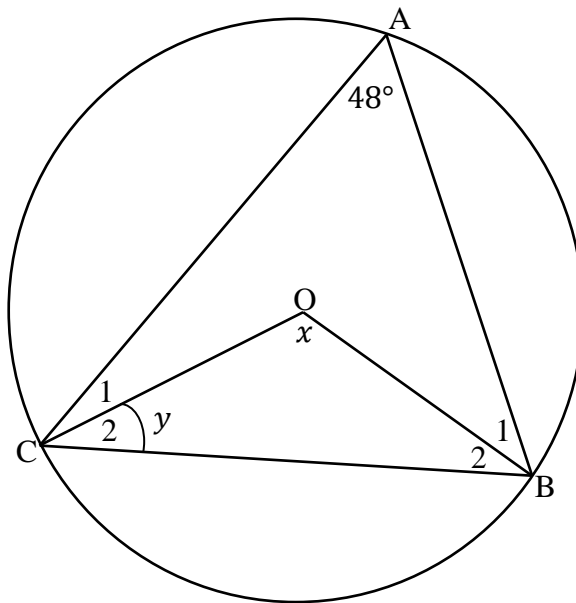
QUESTION 6

6.1 Fill in the **missing word(s)** in the **theorem** statement:

“The **angle subtended** by an **arc** at the **centre** of a **circle** is ... the **size** of the **angle subtended** by the **same arc** at the **circumference** of the **circle**.” (1)

6.2 **Diagram:**

O is the **centre** of the **circle** passing through A, B and C.
 $\hat{CAB} = 48^\circ$, $\hat{COB} = x$, and $\hat{C}_2 = y$



Determine the sizes of the following.
Give reasons.

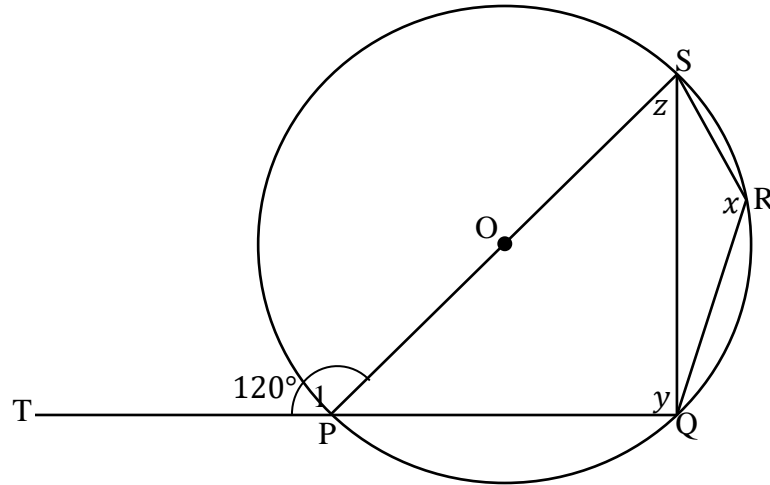
6.2.1 x (2)

6.2.2 y (2)

6.3 Fill in the **missing word(s)** in the **theorem** statement:

“The **exterior angle** of a **cyclic quadrilateral** is ... to the **interior opposite angle**.” (1)

6.4 **O** is the **centre** of the **circle**, with **PS** as the **diameter**.
 $\widehat{TPS} = 120^\circ$.



Determine the sizes of the following.
Give reasons.

6.4.1 x (2)

6.4.2 y (2)

6.4.3 z (2)

[12]

QUESTION 7

7.1 Fill in the **missing word(s)** in the **theorem** statement:

“The **angle** between the **tangent** to a **circle** and the **chord** drawn from the **point of contact** is ... to the **angle** in the **alternate segment**.” (1)

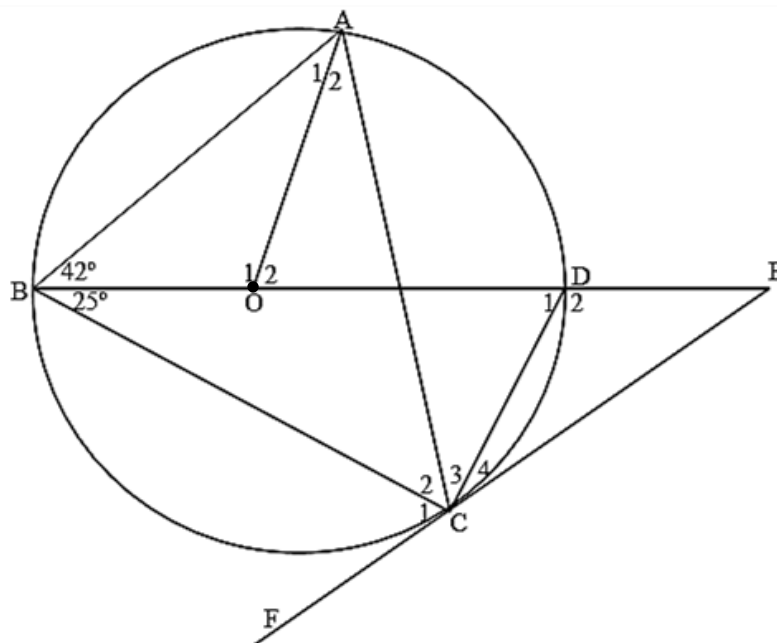
7.2 **Diagram:**

O is the **centre** of the **circle**.

Tangent FC touches the circle at **C**.

Diameter BD extended meets tangent **FC** in **E**.

$\widehat{ABO} = 42^\circ$ and $\widehat{OBC} = 25^\circ$



Determine the sizes of the following.
Give reasons.

7.2.1 \widehat{C}_4 (2)

7.2.2 \widehat{C}_3 (2)

7.2.3 \widehat{D}_1 (4)

7.3 Fill in the **missing word(s)** in the **theorem** statement:

“Two tangents drawn to a circle from the same point ... the circle are equal in length.” (1)

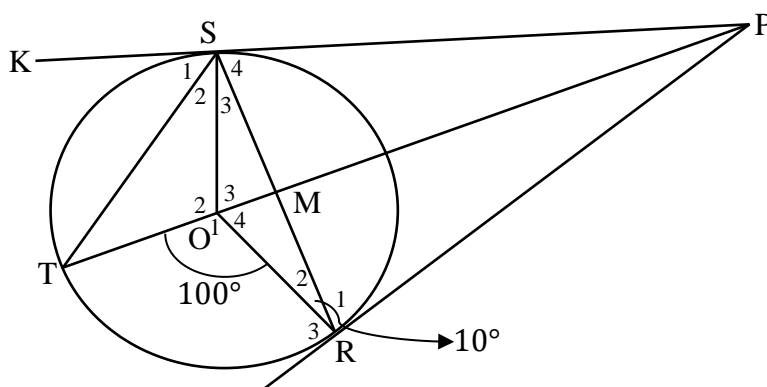
7.4 **Diagram:**

PS and **PR** are **tangents**.

O is the **centre** of the **circle**.

POT is a straight line.

$\hat{O}_1 = 100^\circ$ and $\hat{R}_2 = 10^\circ$.



Determine the sizes of the following.

Give reasons.

7.4.1 \hat{S}_2 (4)

7.4.2 \hat{S}_4 (2)

7.4.3 \hat{P} (4)

[20]

QUESTION 8

8.1 Fill in the **missing word(s)** in the **theorem** statement:

“The **line drawn** from the **centre** of a **circle** to the **midpoint** of a **chord** is ... to the **chord**.”

(1)

8.2 **Diagram:**

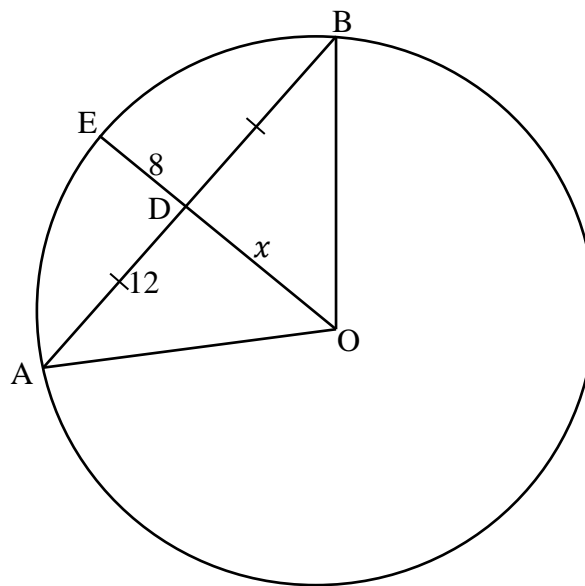
AB is a **chord** of a **circle** with **centre O**.

OE **bisects** **AB**.

AD = **12 cm**.

ED = **8 cm**

OD = x .



8.2.1 **Determine the length of OE, in terms of x .**

(1)

8.2.2 **Complete the statement.**

Give a reason.

$OA^2 = 12^2 + \dots$ and **determine the length of OA, in terms of x .**

(3)

8.2.3 **Determine the length of x .**

(4)

8.2.4 **Determine the length of the radius.**

(1)

[10]

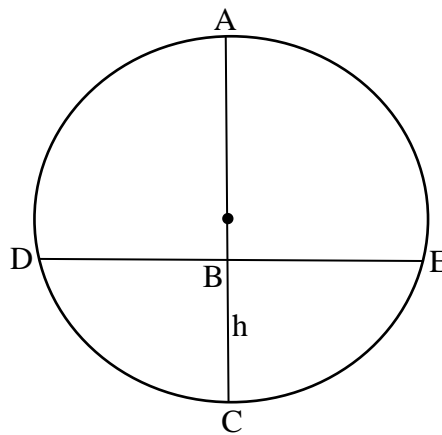
QUESTION 9

- 9.1 A wheel rotates at 12 revolutions per second.
Determine the angular velocity of the wheel. (3)

- 9.2 The radius of a circular spinning toy is 40 mm.
It rotates at 20 revolutions per minute.
Determine the circumferential velocity of the toy. (4)

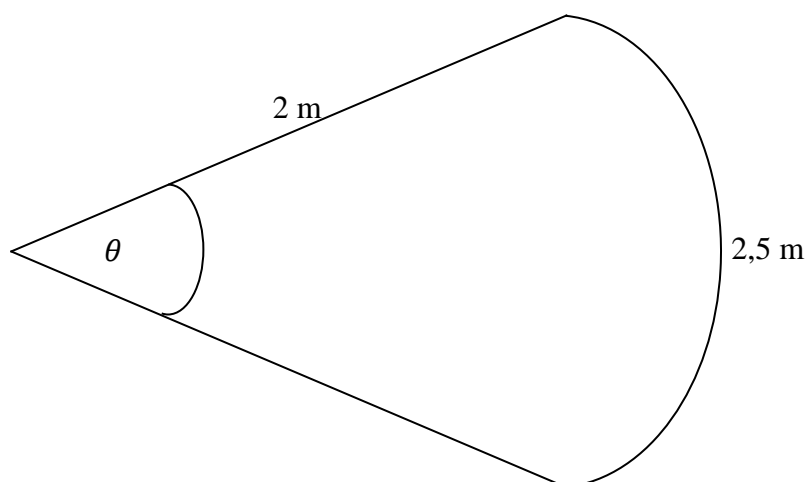
- 9.3 **Diagram:**

The circle with centre O, has a chord, DE, with a length of 500 mm.
The diameter, AC, is 56,6 cm.
Chord, DE, divides the circle into two segments.



- Calculate the height of the minor segment, h (BC), in cm. (6)

- 9.4 A metal plate is cut into the shape of a sector of a circle.
The radius of the circle is 2 m and the arc length is 2,5 m.



- 9.4.1 Determine the central angle of the sector in radians. (3)

- 9.4.2 Determine the area of the sector. (3)

- 9.4.3 The sector is bent into a cone.
Determine the perpendicular height of the cone. (5)

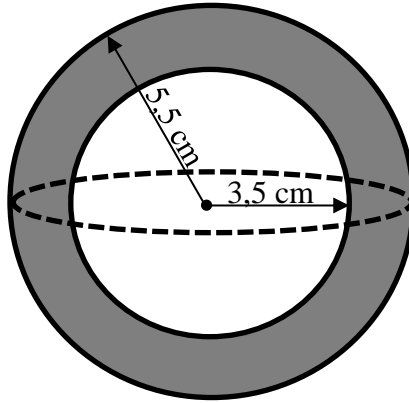
9.5 **Diagram:**

A toymaker wants to **make a toy** in the **shape** of a **hollow sphere**.

The **hollow sphere** has an **internal radius** of **3,5 cm** and an **external radius** of **5,5 cm**.

One cubic centimetre of metal weighs 30 grams.

$$V = \frac{4}{3}\pi r^3$$



Determine the mass of the toy.

(5)
[29]

QUESTION 10

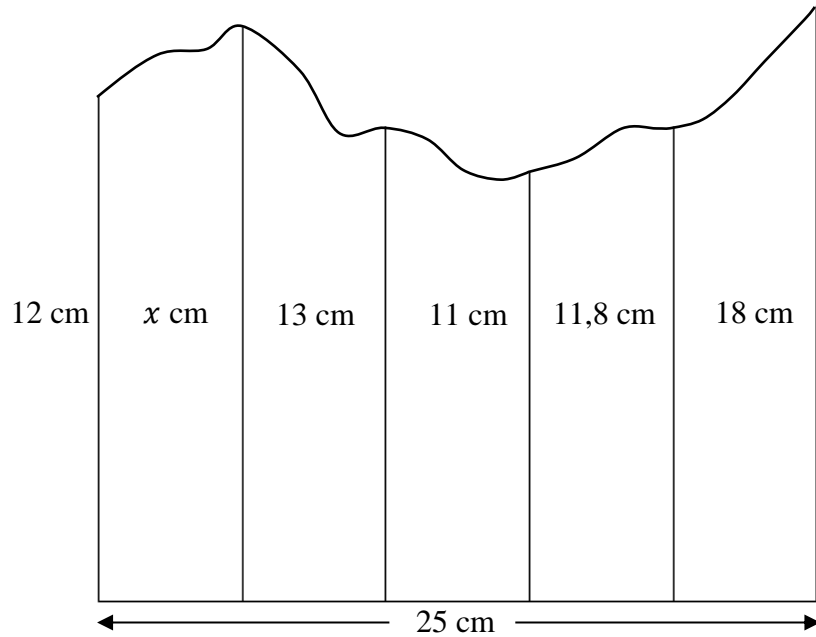
Diagram:

The **irregular shape** below has an **area** of **329 cm²**.

The **horizontal side** is **25 cm** in **length**.

It is **divided** into **five equal parts**.

The **ordinates** are **12 cm**, **x cm**, **13 cm**, **11 cm**, **11,8 cm** and **18 cm**.



Determine, using the **mid-ordinate rule**, the **value** of x .

(5)

[5]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 + i)^n$$

$$A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C, n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln(x) + C, x > 0 \text{ and } k \in \mathbb{R}; k \neq 0$$

$$\int ka^{nx} dx = \frac{ka^{nx}}{n \ln a} + C, a > 0; a \neq 1 \text{ and } k, a \in \mathbb{R}; k \neq 0$$

$$\pi \text{ rad} = 180^\circ$$

Angular velocity = $\omega = 2\pi n = 360^\circ n$ where n = rotation frequency

Circumferential velocity = $v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity = $v = \omega r$ where ω = angular velocity and r = radius

Arc length = $s = r\theta$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

Area of a sector = $\frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{Area} = \frac{1}{2}ab \cdot \sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

$$A_T = a \left(\frac{O_1 + O_n}{2} + O_2 + O_3 + O_4 + \dots + O_{n-1} \right) \quad \text{where } a = \text{width of equal parts, } O_i = i^{\text{th}} \text{ ordinate and } n = \text{number of ordinates}$$

OR

where a = width of equal parts, $m_i = \frac{O_i + O_{i+1}}{2}$ and

$$A_T = a(m_1 + m_2 + m_3 + \dots + m_{n-1}) \quad n = \text{number of ordinates; } i = 1; 2; 3; \dots; n-1$$