Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 11

NOVEMBER 2023

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

* I M A T 2 *

This question paper consists of 12 pages, including 1 page information sheet, and an answer book of 20 pages.

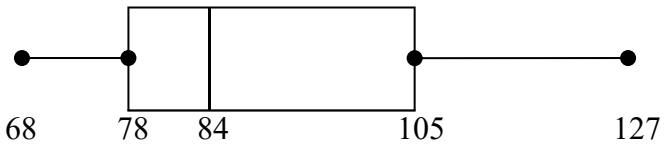
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9 Write neatly and legibly.

QUESTION 1

The box-and-whisker diagram below shows the distribution of the September examination marks of a group of 24 boys in Grade 11. The median is 84 and the mean is 87.



1.1 How many boys scored more than 105? (1)

1.2 In which direction are the marks skewed? (1)

1.3 Calculate the range of the data (2)

1.4 On checking the answer book of the candidate who scored 127, an adding error is discovered and his mark is changed to 147. Determine the resulting value of the following measures:

1.4.1 The median (1)

1.4.2 The mean (2)

[7]

QUESTION 2

55 learners were sampled at the school tuckshop to measure their waiting time in minutes before being served. The results were published in the table below:

Time waiting (minutes)	Number of learners	Cumulative frequency
$0 < x \leq 4$	6	
$4 < x \leq 8$	10	
$8 < x \leq 12$	19	
$12 < x \leq 16$	15	
$16 < x \leq 20$	5	

2.1 Complete the cumulative frequency table in the ANSWER BOOK. (2)

2.2 Hence, draw the cumulative frequency graph on the grid in the ANSWER BOOK. (4)

2.3 Use your graph to estimate the number of learners who waited for more than 14 minutes. (3)

2.4 Write down the modal class of the data. (1)

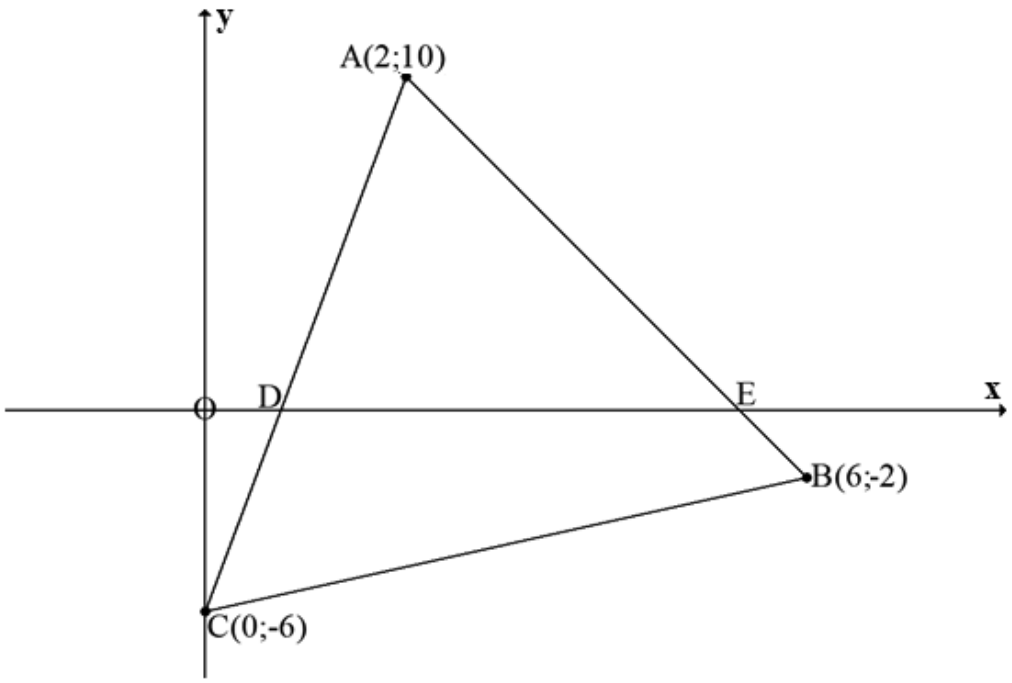
2.5 Use your graph to estimate the interquartile range of the data. (3)

[13]

QUESTION 3

3.1 Collinear points are points that lie on the same straight line. If A$(-8; 0)$, B$(x-5; -8)$ and C$(x; -14)$ are collinear, then calculate the value of x. (5)

3.2 A$(2; 10)$; B$(6; -2)$ and C$(0; -6)$ are vertices of a triangle. Points D and E are x-intercepts of straight-line AC and AB respectively.
Study the diagram below and answer the questions that follow.



3.2.1 Calculate the coordinates of M, the midpoint of BC. (2)

3.2.2 Calculate the length of AC and leave your answer in simplified surd form. (2)

3.2.3 Write down the coordinates of F if ABCF is a parallelogram. (2)

3.2.4 Show that ABCF is not a rectangle. (3)

3.2.5 Determine the equation of straight lines AB and AC. (5)

3.2.6 Hence or otherwise, calculate the size of $\hat{A}$. (6)

3.2.7 Calculate the area of ΔADE. (5)
[30]

QUESTION 4

4.1 If $7\tan\theta = 3$ and $\cos\theta < 0$, use a sketch in the correct quadrant to determine the value of: $\frac{\sin\theta + \cos\theta}{2\sin\theta}$ without using a calculator. (6)

4.2 If $\sin 32° = p$, express each of the following in terms of p:

4.2.1 $\tan(-32°)$ (3)

4.2.2 $\sin 418°$ (2)

4.3 Simplify the following fully and without using a calculator:

$$\frac{\sin 120°.\tan 300°}{\cos(-60°).\tan 225°}$$ (7)

4.4 Prove the following identity:

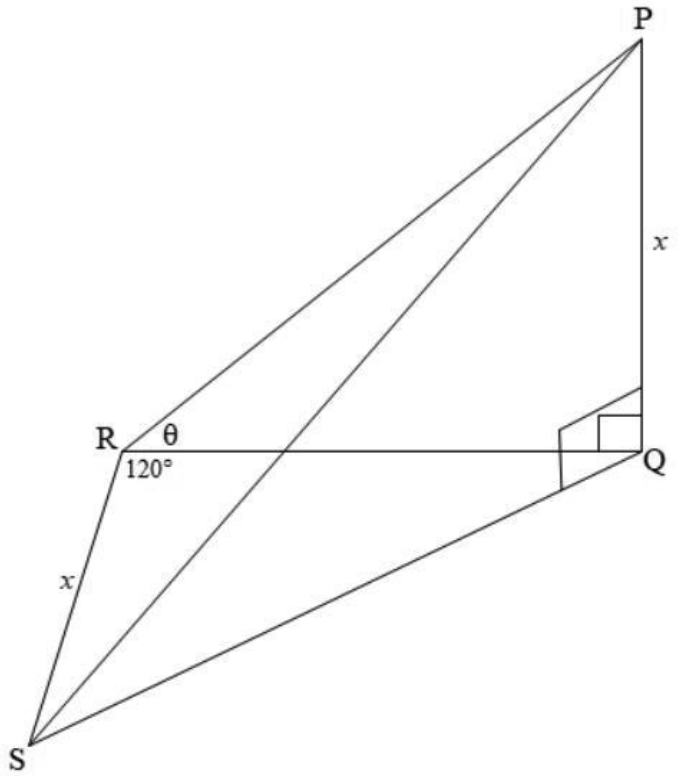
$$\frac{\cos^2 x - \cos x - \sin^2 x}{2\sin x\cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x}$$ (5)

4.5 Determine the general solution of: $\sin(5\theta) = \cos(\theta - 40°)$ (6)

[29]

QUESTION 5

In the diagram below, PQ is a vertical tower. Q, R and S are points on the same horizontal plane. The angle of elevation from R to P is θ. $Q\widehat{R}S = 120°$ and PQ = RS = x.



5.1 Determine QR in terms of θ and x. (2)

5.2 Show that QS = $x\sqrt{\frac{1}{tan^2\theta} + \frac{1}{\tan\theta} + 1}$ (4)

5.3 If $x = 15$ cm and $\theta = 22°$, calculate

5.3.1 QS (2)

5.3.2 $Q\widehat{P}S$ (2)

[10]

QUESTION 6

Given: $f(x) = \sin 2x$ and $g(x) = \tan x - 1$

6.1 Draw the sketch graphs of f and g on the same set of axes provided in the answer book for $x \in [-45°;180°]$. Indicate all endpoints, intercepts with the axes, turning points as well as asymptotes. (6)

6.2 Use your graphs to determine value(s) of x for which:

6.2.1 $\sin 2x > 0$ (1)

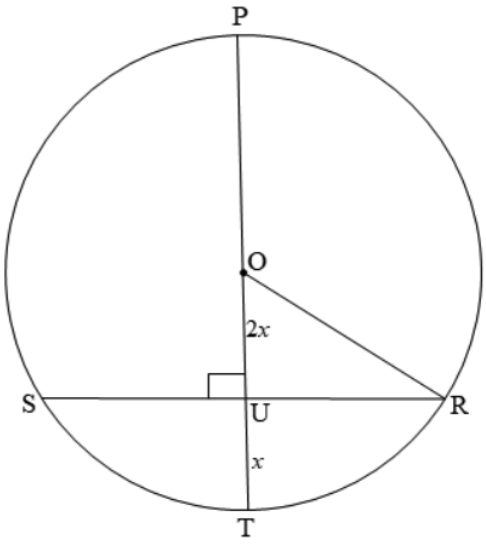
6.2.2 $f(x).g(x) \geq 0$ (3)

6.3 If $h(x) = f(x) + 2$, write down the range of h. (2)

[12]

QUESTION 7

7.1 In the diagram below, O is the centre of the circle. SR = 40 cm and OT ⊥ SR. It is also given that UT $= x$ and OU $= 2x$.

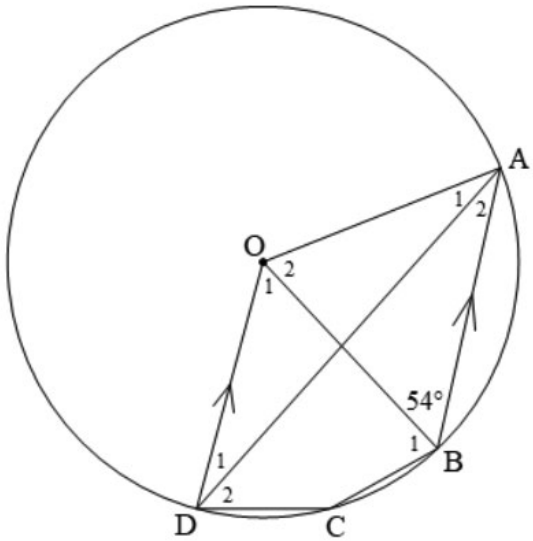


Determine:

7.1.1 The length of OR in terms of x (1)

7.1.2 The value of x, leaving your answer in simplest surd form (5)

7.2 In the diagram below, O is the centre of the circle. ABCD are points on the circumference of the circle. OD ∥ AB and $O\widehat{B}A = 54°$.



Determine:

7.2.1 The size of $\widehat{A}_2$ (4)

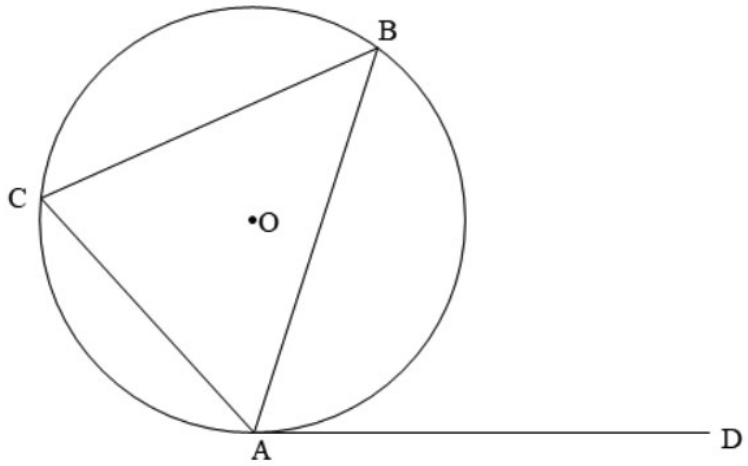
7.2.2 The size of $\widehat{C}$ (2)

7.2.3 Prove that DA bisects $O\widehat{A}B$ (3)

[15]

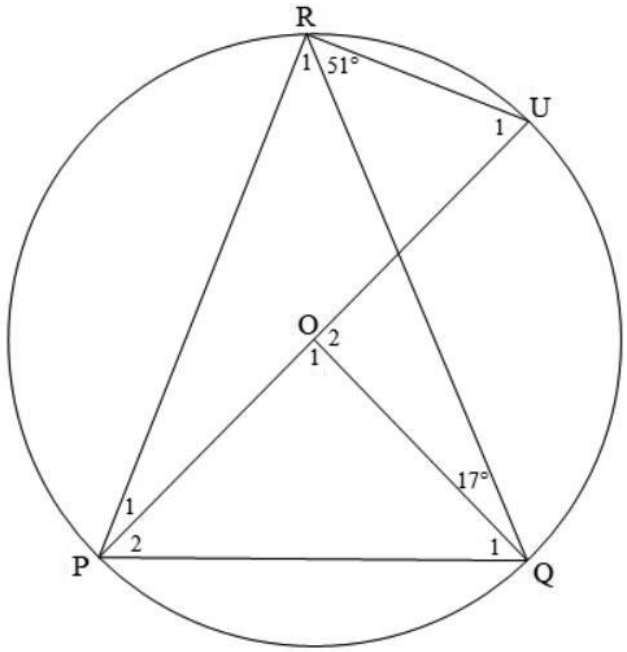
QUESTION 8

8.1 In the diagram below, DA is a tangent to the circle with centre O.



Use the diagram above to prove the theorem that states that:
$D\hat{A}B = B\hat{C}A$. (5)

8.2 In the diagram below, O is the centre of circle QPRU. $O\hat{Q}R = 17°$ and $Q\hat{R}U = 51°$.



Calculate, with reasons, the size of:

8.2.1 $\hat{R}_1$ (2)

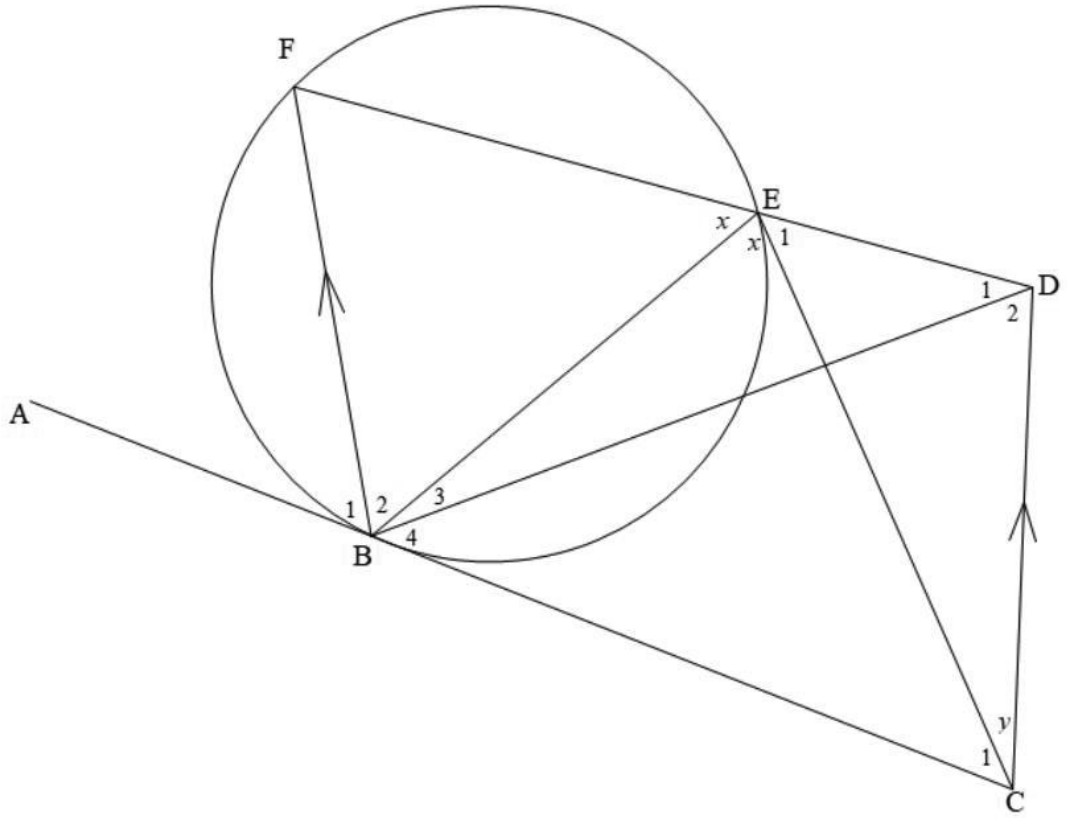
8.2.2 $\hat{O}_1$ (2)

8.2.3 $\hat{Q}_1$ (2)

8.2.4 $\hat{U}_1$ (2)
[13]

QUESTION 9

ABC is a tangent to circle BFE at B. From C, a straight-line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $F\hat{E}B = B\hat{E}C = x$ and $E\hat{C}D = y$.



9.1 Calculate, in terms of x and y the sizes of the following angles. Give reasons for your answers.

9.1.1 $A\hat{B}F$ (2)

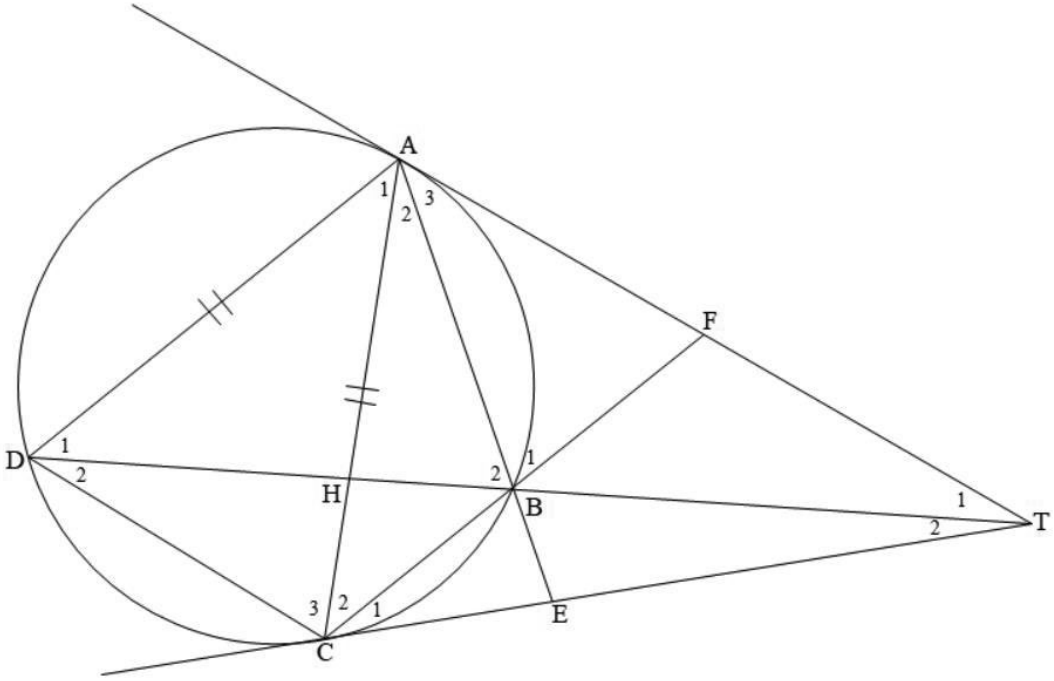
9.1.2 $B\hat{C}D$ (2)

9.2 Why is BEDC a cyclic quadrilateral? (1)

9.3 Which other two angles are equal to x? Give reasons for your answers. (4)
[9]

QUESTION 10

In the diagram below, ABCD is a cyclic quadrilateral with AC = AD. Tangents AC and CT touch the circle at A and C respectively.



Prove that:

10.1 $\widehat{B}_1 = \widehat{B}_2$ (4)

10.2 BECH is a cyclic quadrilateral. (3)

10.3 CA is a tangent to the circle passing through points A, B and T. (5)
[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$