Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 11

NOVEMBER 2023

TECHNICAL MATHEMATICS P2

MARKS: 150

TIME: 3 hours

ITMATE2

This question paper consists of 17 pages, including a 2-page information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.

6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.

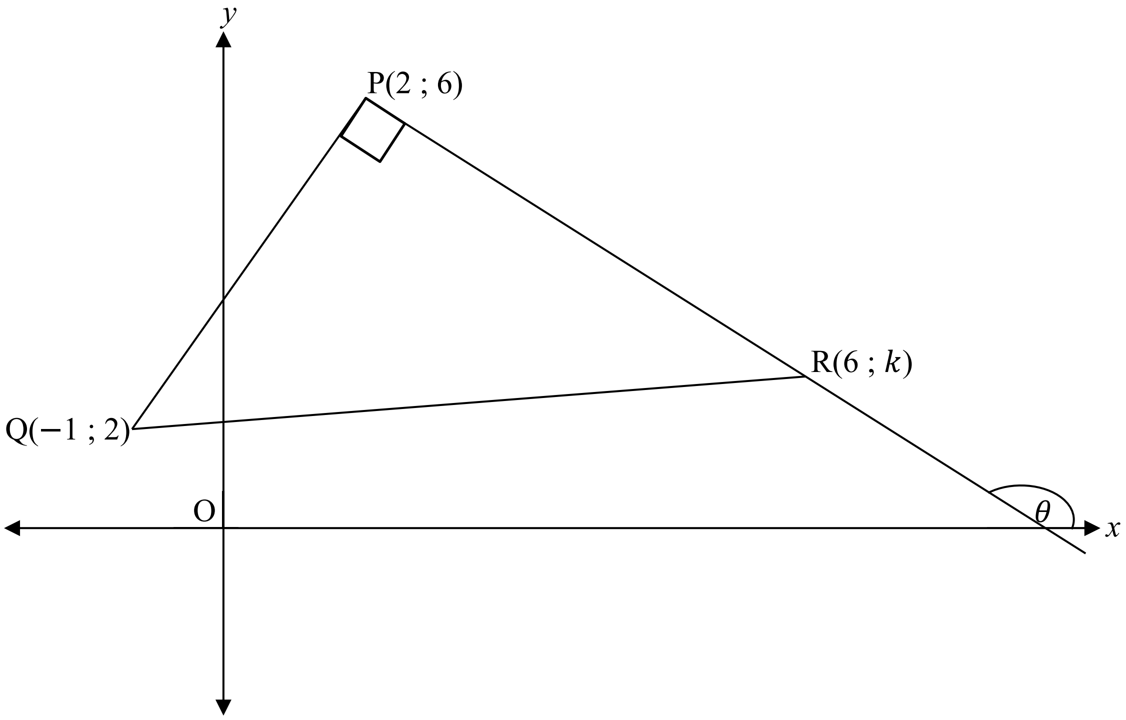
7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

In the diagram below, ΔPQR has a right angle at P. The coordinates of the vertices are $P(2\,;6)$; $Q(-1\,;2)$ and $R(6\,;k)$. θ is the angle of inclination of line PR.



1.1 Complete the statement:

"When two lines are perpendicular the … of the gradients must be equal to -1." (1)

Determine:

1.2 The gradient of PQ (3)

1.3 Show that the value of $k = 3$ (3)

1.4 The coordinates of the midpoint of QR (3)

1.5 The coordinates of S, so that QPRS is a rectangle (4)

1.6 The equation of PR (4)

1.7 θ, the inclination angle of line PR (3)

1.8 If the length of PQ = 5 units, calculate the size of $\widehat{Q}$ (5)

[26]

QUESTION 2

2.1 Given: $x = 30{,}5^\circ$ and $y = 130{,}5^\circ$

Determine the following:

2.1.1 $\tan(x + y)$ (2)

2.1.2 $\text{cosec}(y - x)$ (3)

2.2 If $\sin 36° = k$, express the following in terms of k .

2.2.1 $\cos 36°$ (4)

2.2.2 $\sin\ 216°$ (2)

2.3 Solve for θ, $\theta \in [0°; 360°]$ rounded off to ONE decimal digit:

$\tan\theta = 2\sin\ 38{,}1°$ (4)

[15]

QUESTION 3

3.1 Simplify:

$$\frac{\cos(360° - \theta).\frac{1}{\cot(180° + \theta)}.\tan(360° + \theta)}{\cos(180° + \theta).\tan(180° - \theta)}$$ (6)

3.2 Prove that:

$$\left(tanx + \frac{1}{cosx}\right)^2 = \frac{1 + sinx}{1 - sinx}$$ (4)

[10]

QUESTION 4

Given $f(x) = \cos x$ and $g(x) = \sin x + 1$; $x \in (0°; 360°)$

4.1 On the same axes, given in your SPECIAL ANSWER BOOK, draw the graphs of $f(x) = \cos x$ and $g(x) = \sin x + 1$. Clearly show the intercepts with the axes, turning points and endpoints. (7)

4.2 Write down the range of g. (2)

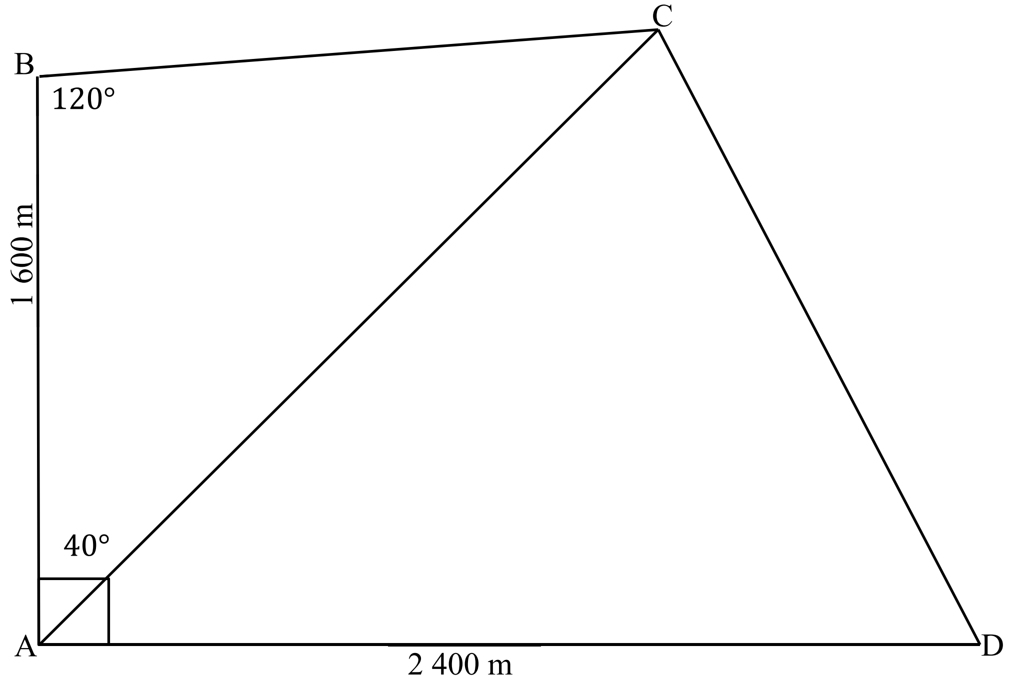
4.3 Write down the period of f. (1)

4.4 Use your graphs to determine for which values of x, is $f(x).g(x) \leq 0$. (2)

[12]

QUESTION 5

A farm has the shape shown in the figure below with a fence dividing it into two triangles. The lengths of two adjacent sides of the farm are 1 600 m and 2 400 m and makes an angle of 90°, with one another at A.
$B\hat{A}C = 40°$ and $C\hat{B}A = 120°$.



5.1 Determine the size of $B\hat{C}A$, stating a reason. (2)

5.2 Determine the length of AC, to the nearest whole number. (3)

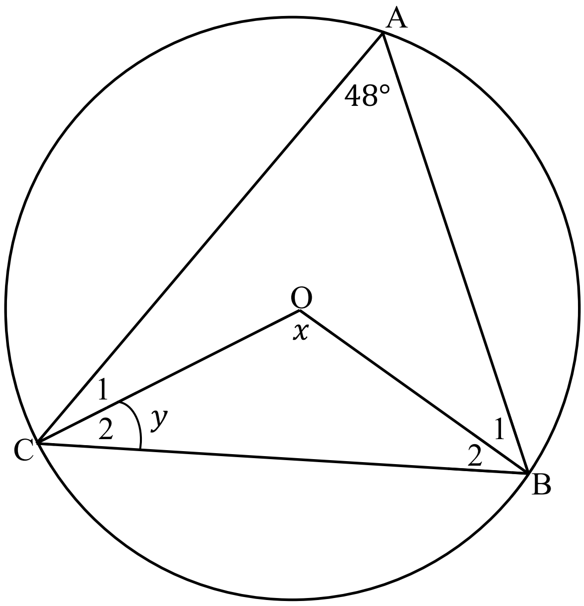
5.3 Determine the total area of the farm, ABCD. (6)
[11]

QUESTION 6

6.1 Complete the following theorem:

"The angle subtended by an arc at the centre of a circle is … the size of the angle subtended by the same arc at the circumference of the circle." (1)

6.2 In the diagram, O is the centre of the circle passing through A, B and C.
$C\hat{A}B = 48°, C\hat{O}B = x$, and $\hat{C}_2 = y$



Determine, with reasons, the sizes of the following:

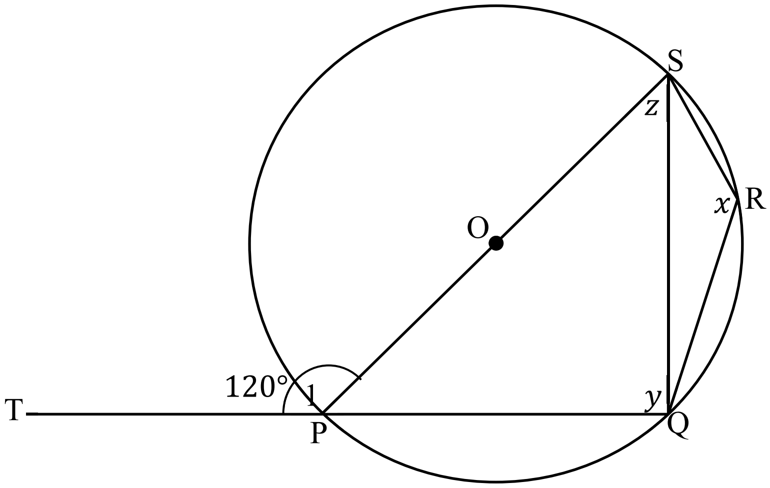
6.2.1 x (2)

6.2.2 y (2)

6.3 Complete the following statement:

"The exterior angle of a cyclic quadrilateral is … to the interior opposite angle." (1)

6.4 O is the centre of the circle, with PS as the diameter. $T\hat{P}S = 120°$.



Determine, with reasons, the sizes of the following:

6.4.1 x (2)

6.4.2 y (2)

6.4.3 z (2)

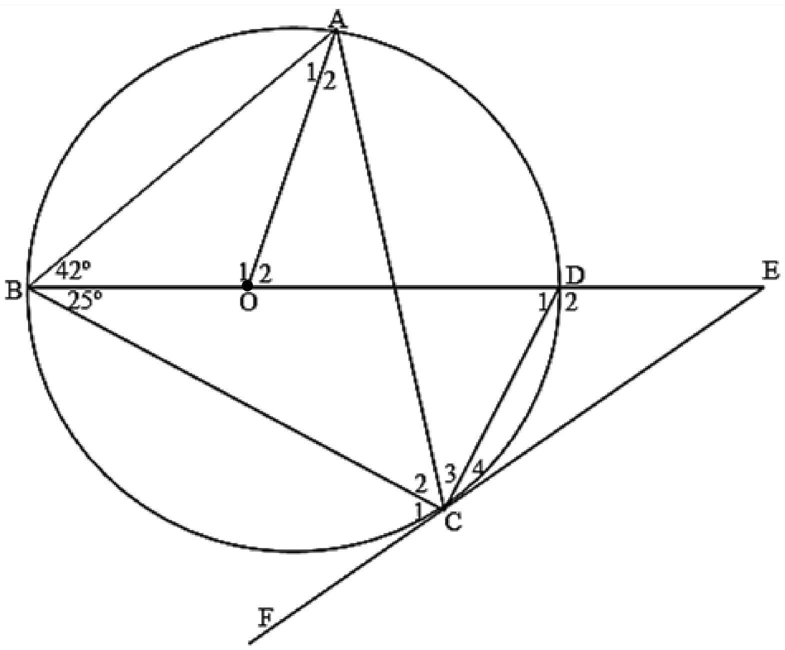
[12]

QUESTION 7

7.1 Complete the following theorem:

"The angle between the tangent to a circle and the chord drawn from the point of contact is … to the angle in the alternate segment." (1)

7.2 In the given diagram, O is the centre of the circle and tangent FC touches the circle at C.
Diameter BD extended meets tangent FC in E.
$A\hat{B}O = 42°$ and $O\hat{B}C = 25°$



Determine, with reasons, the sizes of the following:

7.2.1 $\hat{C}_4$ (2)

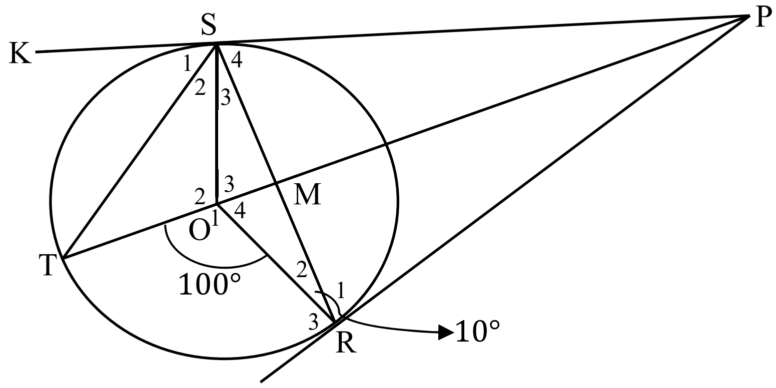
7.2.2 $\hat{C}_3$ (2)

7.2.3 $\hat{D}_1$ (4)

7.3 Complete the following theorem:

"Two tangents drawn to a circle from the same point … the circle are equal in length." (1)

7.4 PS and PR are tangents. O is the centre of the cirle. POT is a straight line.
$\hat{O}_1 = 100°$ and $\hat{R}_2 = 10°$.



Determine, with reasons, the sizes of the following:

7.4.1 $\hat{S}_2$ (4)

7.4.2 $\hat{S}_4$ (2)

7.4.3 $\hat{P}$ (4)

[20]

QUESTION 8

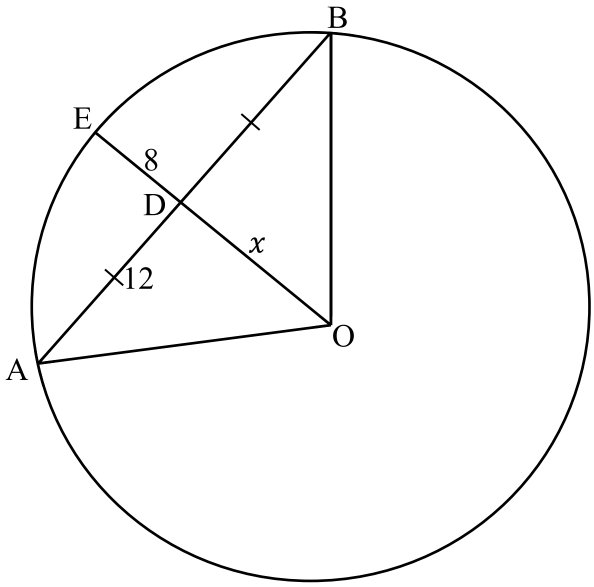
8.1 Complete the following theorem:

"The line drawn from the centre of a circle to the midpoint of a chord is … to the chord." (1)

8.2 In the given diagram, AB is a chord of a circle with centre O.

OE bisects AB.

AD = 12 cm, ED = 8 cm and OD = x.



8.2.1 Determine the length of OE, in terms of x . (1)

8.2.2 Complete, with a reason, the statement:

$OA^2 = 12^2 + \ldots$ and determine the length of OA, in terms of x . (3)

8.2.3 Determine the length of x . (4)

8.2.4 Hence, determine the length of the radius. (1)

[10]

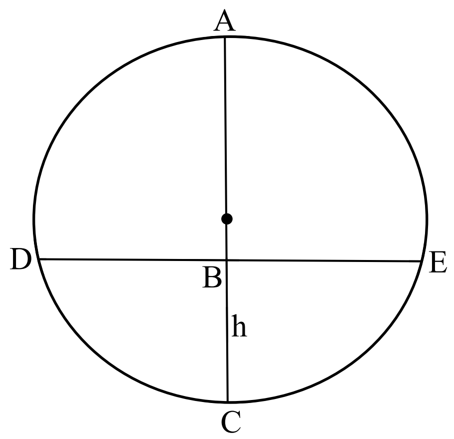
QUESTION 9

9.1 A wheel rotates at 12 revolutions per second. Determine the angular velocity of the wheel. (3)

9.2 The radius of a circular spinning toy is 40 mm. It rotates at 20 revolutions per minute. Determine the circumferential velocity of the toy. (4)

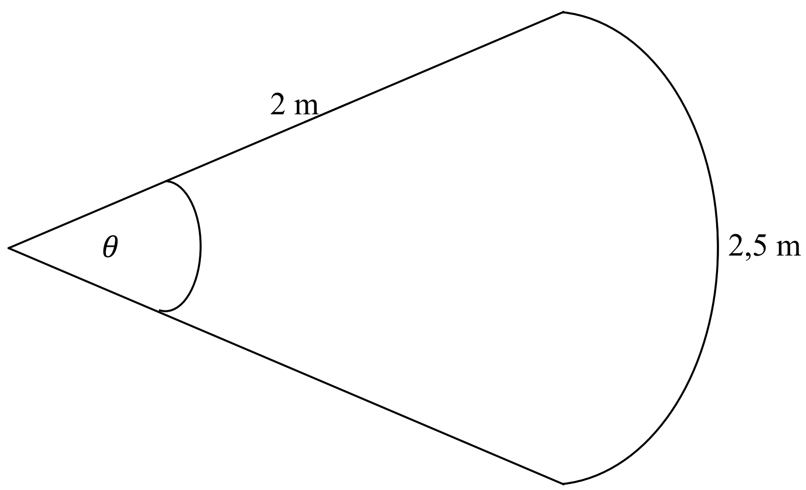
9.3 In the diagram below, the circle with centre O, has a chord, DE, length of 500 mm and the diameter, AC, is 56,6 cm.

The chord, DE, divides the circle into two segments.



Calculate the height of the minor segment, h (BC), in cm. (6)

9.4 A metal plate is cut into the shape of a sector of a circle. The radius of the circle is 2 m and the arc length is 2,5 m.



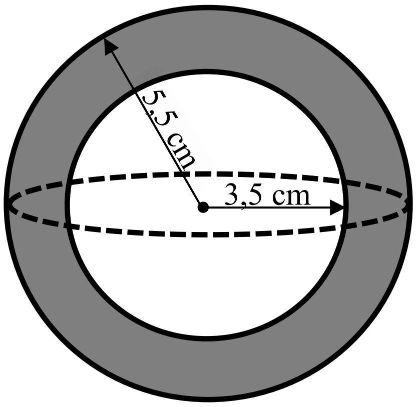
9.4.1 Determine the central angle of the sector in radians. (3)

9.4.2 Determine the area of the sector. (3)

9.4.3 The sector is bent into a cone. Determine the perpendicular height of the cone. (5)

9.5 A toymaker wants to make a toy in the shape of a hollow sphere. The hollow sphere has an internal radius of 3,5 cm and an external radius of 5,5 cm. One cubic centimetre of metal weighs 30 grams.

$V = \frac{4}{3}\pi r^3$

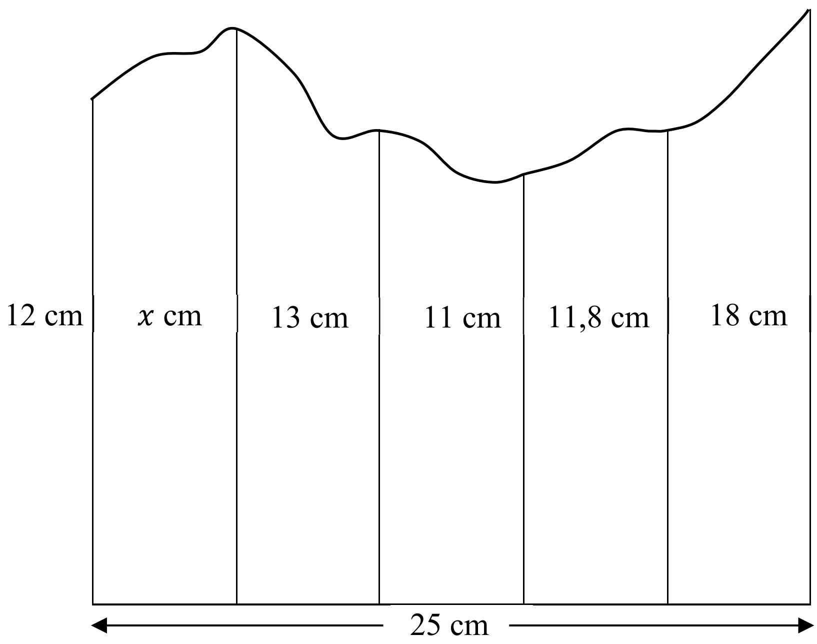


Determine the mass of the toy. (5)

[29]

QUESTION 10

The irregular shape below has an area of 329 cm^2.
The horizontal side is 25 cm in length and divided into five equal parts.
The ordinates are 12 cm, x cm, 13 cm, 11 cm, 11,8 cm and 18 cm.



Determine, using the mid-ordinate rule, the value of x . (5)

[5]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \quad \Leftrightarrow x = \log_a b \;\; a > 0, a \neq 1$ and $b > 0$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n \qquad i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = tan\theta \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$\int kx^n dx = \frac{kx^{n+1}}{n + 1} + C, n, k \in \mathbb{R}$ with $n \neq -1$ and $k \neq 0$

$\int \frac{k}{x} dx = k \ln(x) + C, x > 0 \; and \; k \in \mathbb{R}; k \neq 0$

$\int ka^{nx} dx = \frac{ka^{nx}}{nlna} + C, a > 0; a \neq 1 \; and \; k, a \in \mathbb{R}; k \neq 0$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ where $n =$ rotation frequency

Circumferential velocity $= v = \pi Dn$ where $D =$ diameter and $n =$ rotation frequency

Circumferential velocity $= v = \omega r$ where $\omega =$ angular velocity and $r =$ radius

Arc length = $s = r\theta$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.cosA \qquad Area = \frac{1}{2}ab.sinC$$

$$sin^2\theta + cos^2\theta = 1 \qquad 1 + tan^2\theta = sec^2\theta \qquad cot^2\theta + 1 = cosec^2\theta$$

$A_T = a\left(\frac{O_1 + O_n}{2} + O_2 + O_3 + O_4 + \ldots + O_{n-1}\right)$ where a = width of equal parts, $O_i = i^{th}$ ordinate and n = number of ordinates

OR

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ where a = width of equal parts, $m_i = \frac{O_i + O_{i+1}}{2}$ and n = number of ordinates; i = 1;2;3; …;n-1