



Province of the
EASTERN CAPE
EDUCATION

Iphondo leMpuma Kapa: Isebe leMfundo
Provinsie van die Oos Kaap: Departement van Onderwys
Porafensie Ya Kapa Boijahabela: Lefapha la Thuto

NATIONAL SENIOR CERTIFICATE

KEREITI YA 12

LOETSE 2024

MMETSE P2

MATSWAO: 150

NAKO: Dihora tse 3

Pampiri ena e na maqephe a 14 ho kenyelletsa le leqephe le 1 la
tlhahisoleseding mmoho le buka ya dikarabo e nang le maqephe a 25.

DITAELO LE TLHAHISOLESSEDING

Bala ditaelo tse latelang ka hloko pele o ka araba dipotso.

1. Pampiri ena e na le dipotso tse 10.
2. Arabela dipotso KAOFELA ho BUKA E KGETHEHILENG YA HO ARABELA o e filweng.
3. Bontsha ka ho hlakileng KAOFELA dikhaltjhuleishene, didayakeramo, dikerafo j.j. tseo o di sebedisitseng ho fumana dikarabo.
4. Dikarabo fela di ke ke tsa abelwa matshwao a felletseng.
5. O ka sebedisa khaltjhuleitha ya saentifiki (e sa phorekeremuwang, le e se nang dikerafo), ntle le ha ho boletswe.
6. Ebang ho hlokeha, atametsa dikarabo tsa hao ho didesimale tse PEDI, ntle le ha ho boletswe.
7. Didayakeramo HA DI latele tekanyetso e nepahetseng.
8. Pampiri ya tlhahisoleseding e nang le difomula e kenyelleditswe qetellong ya pampiri ena.
9. Ngola ka mongolo o hlakileng mme o makgethe.

POTSO YA 1

- 1.1 Lenane la dilitara tsa diesel tse rekuweng ke bakganni ba dilori ba 15 seteisheneng sa peterole le rekhophilwe ka mokgwa ona o latelang.

82	64	55	50	41
71	78	88	98	96
63	66	80	84	88

1.1.1 Ngola mode. (1)

1.1.2 Ngola range. (1)

1.1.3 Khaltjhuleitha mean. (2)

1.1.4 Khaltjhuleitha standard deviation ya mean. (1)

1.1.5 Fumana lenane la bakganni ba dilori ba rekileng dilitara tsa diesel tse ka tlase ho one standard deviation ya mean. (3)

- 1.2 Mean ya boima ba batho ba 8 ba kenang ka hara lifti ke 75 kg. Lifti ha e nke boima ba fetang 1 000 kg.

Ke batho ba bakae ka lenane ba ka nnang ba kena ka lifting ebang re nahanela hore mean e dula e le 75 kg? (4)

[12]

POTSO YA 2

Diphetho tsa kereiti ya 8 ya diteko tse pedi moo e le nngwe e ngolletsweng hodima matshwao a 50 di ngotswe ka tlase.

TEKO YA A (x)	39	33	35	44	37	40	24	31	30	5
TEKO YA B (y)	41	45	48	40	47	42	37	44	43	24

2.1 Fumana outlier ho tswa tafoleng e ka hodimo. (1)

2.2 Fumana ekhweishene ya least squares regression line. (3)

2.3 Sebedisa ikhweishene ya least squares regression line ho nahanela matshwao a TEKO YA B ebang moithuti o fumane matshwao a 14 ho TEKO YA A. Atametsa karabo ya hao ho whole namba e haufi. (2)

2.4 Tshwaela ka strength sa correlation dipakeng tsa TEKO YA A le TEKO YA B. (2)

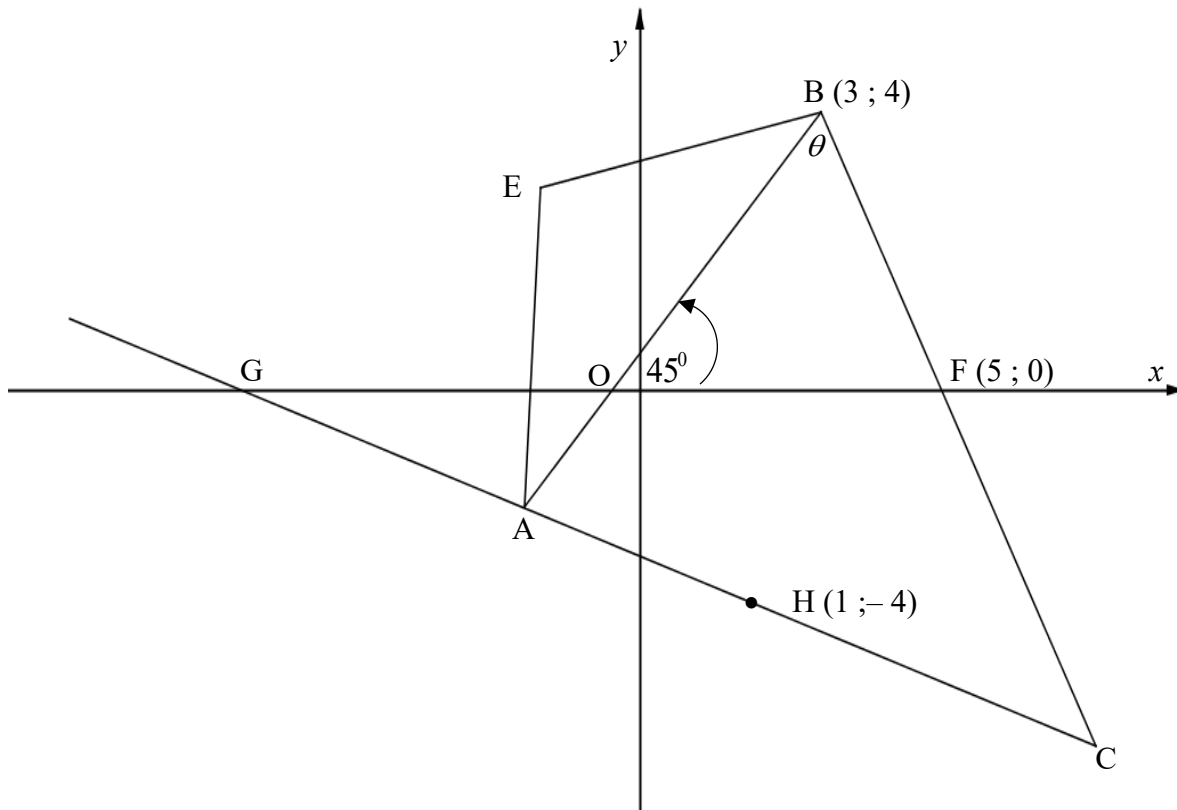
[8]

POTSO YA 3

Khwadrilatherale AEBC e teroilwe. Dikhoodineithi tsa B ke (3; 4). G, O le F(5; 0) ke x -inthesepthe tsa dilaene AC, AB le BC ka ho latellana. H (1; -4) ke poente e ho laene AC.

$\hat{ABC} = \theta$. Area of $\triangle OBF = 12$ square units le inclination ya laene AB ke 45° .

$HC = 2AH$

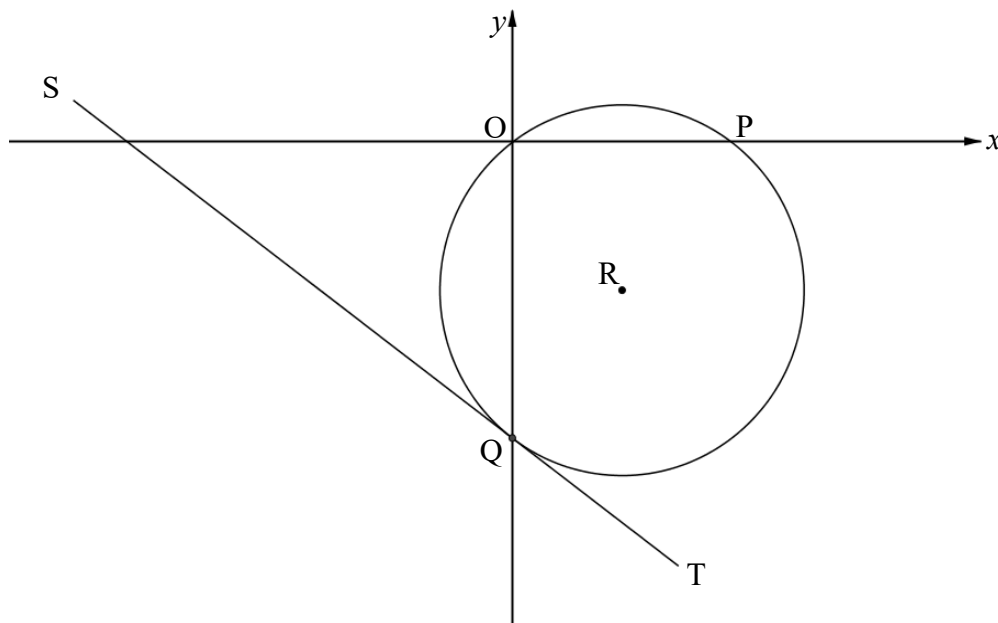


- 3.1 Khaltjhuleitha bolelele ba BF. O siye karabo ya hao e le ho surd form e bebofaditsweng. (2)
- 3.2 Khaltjhuleitha gradient ya BF. (2)
- 3.3 Khaltjhuleitha saeze ya θ . (3)
- 3.4 Pruva hore $HF \parallel AB$. (4)
- 3.5 Ho boetse ho fuwe hore, EC o baesektha AB perpendicularly. AEBC ke mofuta ofe wa khwadrilaterale? (1)
- 3.6 O nto, kapa ka tsela e nngwe o khaltjhuleithe bolelele ba AC. (4)
- 3.7 Khaltjhuleitha eriya ya khwadrilaterale AOFC. (3)

[19]

POTSO YA 4

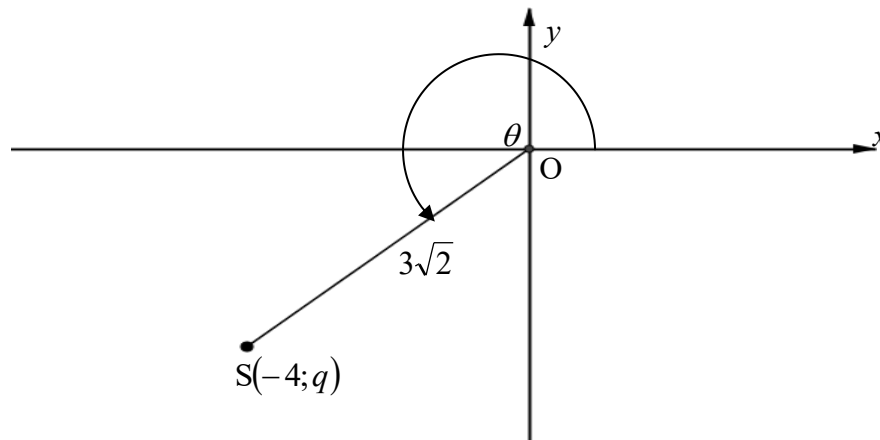
- 4.1 Ho dayakeramo e ka tlase, R ke mokgubu wa sekele OPQ. Poente Q ke y -inthasepthe ya sekele. SQT ke thanjente ya sekele ho Q. Ikhweishene ya SQT ke $y = -\frac{3}{4}x - 8$.



- 4.1.1 Khaltjhuleitha dikhoodineithi tsa Q. (2)
- 4.1.2 Fumana ikhweishene ya QR ka mokgwa ona $y = mx + c$. (3)
- 4.1.3 Khaltjhuleitha dikhoodineithi tsa P, x -inthasepthe ya laene QR. (2)
- 4.1.4 Khaltjhuleitha dikhoodineithi tsa R mokgubu wa sekele. (3)
- 4.1.5 Ngola ikhweishene ya sekele e nang le mogubu ho R ka mokgwa:
 $(x-a)^2 + (y-b)^2 = r^2$. (3)
- 4.1.6 Ebang $y = k$ ke thanjente ho sekele, fumana di/velu tsa/ya k . (3)
- 4.2 Khaltjhuleitha bolelele bo hodimodimo ba radius ya sekele e nang le ikhweishene
 $x^2 + y^2 - 2x \sin \theta - 4y \sin \theta = -2$. (5)
- [21]**

POTSO YA 5

- 5.1 Dayakeramong e katlsase, poente $S(-4; q)$ le reflex angle θ di bontshitswe. O ke poent e ho origin. $OS = 3\sqrt{2}$.



Ntle le ho sebedisa khaltjhuleitha, fumana velu ya:

5.1.1 q (2)

5.1.2 $\sin(\theta + 45^\circ)$ (4)

5.1.3 $\cos(2\theta - 360^\circ)$ (4)

- 5.2 Simplifaya tse latelang ntle le ho sebedisa khaltjhuleitha:

$$\frac{\sin(90^\circ - \theta) \cdot \cos 480^\circ + \cos(180^\circ - \theta)}{\cos \theta \cdot \sin 150^\circ - \tan 180^\circ} \quad (5)$$

5.3 Pruva hore $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2 \sin x} = \sin x$ (5)

5.4 O fuwe: $\frac{\cos 60^\circ}{\sin x} - \frac{\sin 60^\circ}{\cos x} = 2$

5.4.1 Bontsha hore ikhweishene $\frac{\cos 60^\circ}{\sin x} - \frac{\sin 60^\circ}{\cos x} = 2$ e ka ngolwa e le $\cos(x + 60^\circ) = \cos(90^\circ - 2x)$ (3)

5.4.2 O nto, kapa ka moko gwa o mong, o fumana general solution ya $\frac{\cos 60^\circ}{\sin x} - \frac{\sin 60^\circ}{\cos x} = 2$ (4)

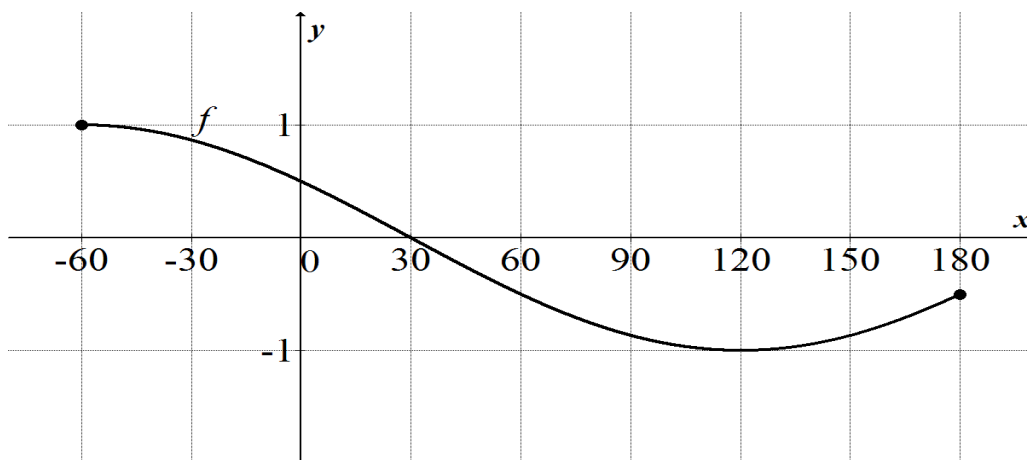
5.5 O fuwe hore $\cos 22,5^\circ = \frac{a}{c}$ le $a^2 + b^2 = c^2$.

Ka thuso ya dayakeramo, kapa ka moko gwa o mong, bontsha hore $\frac{2ab}{c^2} = \frac{\sqrt{2}}{2}$. (5)

[32]

POTSO YA 6

Kerafo ya $f(x) = -\sin(x - 30^\circ)$ e teroilwe ho inthavale ya $x \in [-60^\circ; 180^\circ]$.



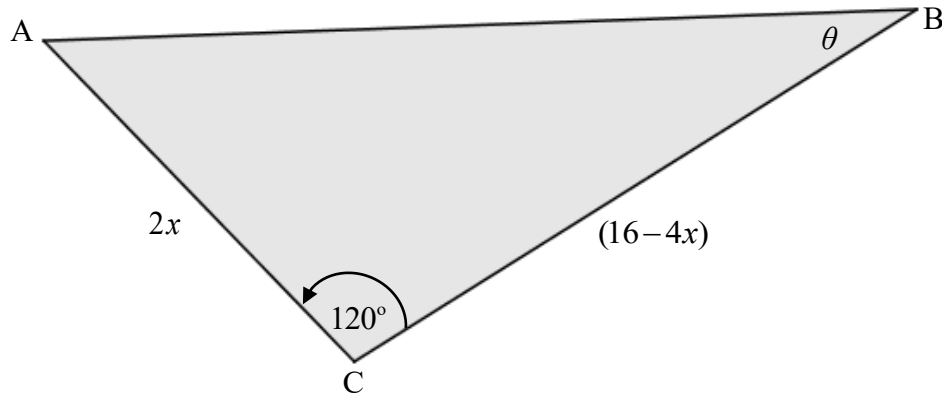
Sebedisa kerafo ho araba dipotso tse latelang.

- 6.1 Ngola period ya f . (1)
- 6.2 Ngola minimum velu ya f . (1)
- 6.3 Fumana range ya $f(x) + 1$. (2)
- 6.4 Ke divelu dife tsa x moo kerafo ya f e inkhrisang, moo $x \in [-60^\circ; 180^\circ]$? (2)
- 6.5 Kerafo ya f e shiftile 60° ho ya ho le letona mme ya reflekhtwa ka ho x -axis ho etsa kerafo e ntjha ya g . Fumana ekhweishene ya g ka mokgwa o bobebe. (3)
- 6.6 Teroya kerafo ya g ho sete e le nngwe ya di-axes. Bontsha ka ho hlakileng diinthasepthe ho di-axis le turning point ya $x \in [-60^\circ; 180^\circ]$. (3)

[12]

POTSO YA 7

Ho $\triangle ABC$ e ka tlase, $AC = 2x$, $BC = (16 - 4x)$, $\widehat{C} = 120^\circ$, $\widehat{B} = \theta$.



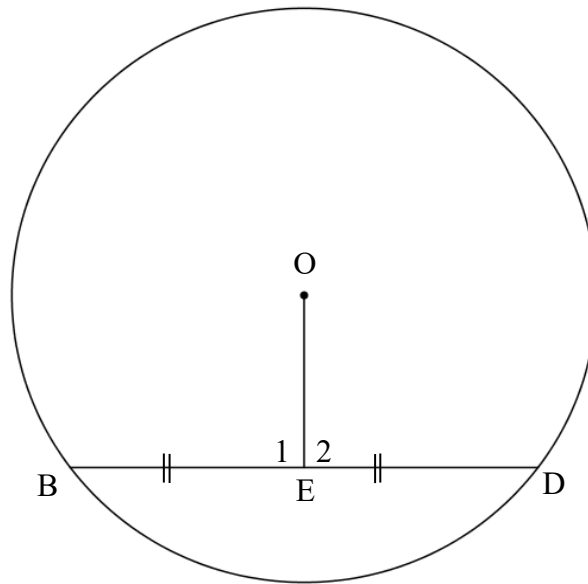
7.1 Fumana eriya ya $\triangle ABC$ ka mokgwa wa x , ntle le ho sebedisa khaltjhuleitha. (3)

7.2 Ke di/velu dife/eke tsa/ya x moo eriya ya $\triangle ABC$ e tla ba maximum? (3)

[6]

POTSO YA 8

- 8.1 Dayakeramong e ka tlase, O ke mokgubu wa sekele. BD ke chord ya sekele. E ke midpoente ya chord BD.

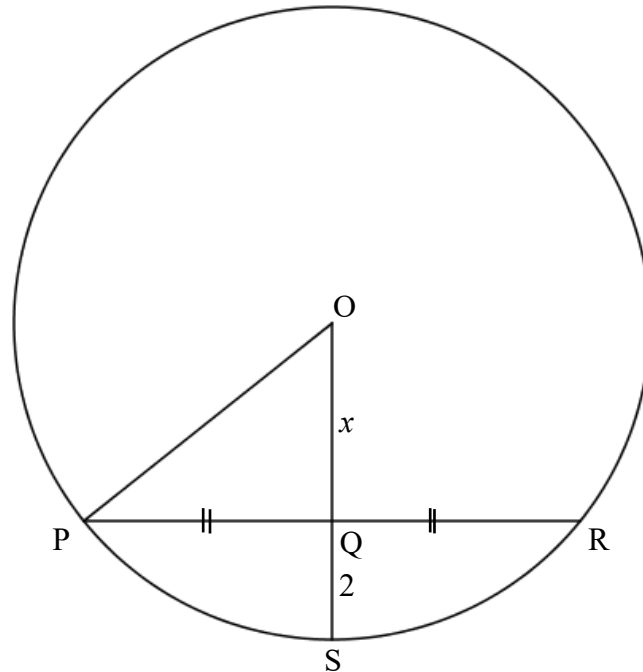


Sebedisa tayakeramo o e filweng ho HO BUKA YA DIKARABO ho prua theorem e reng: Laene e teroilweng ho tloha mokgubung wa sekele e baesekthang chord e perpendicular ho chord.

Ka mantswe a mang, prua hore: $OE \perp BD$.

(5)

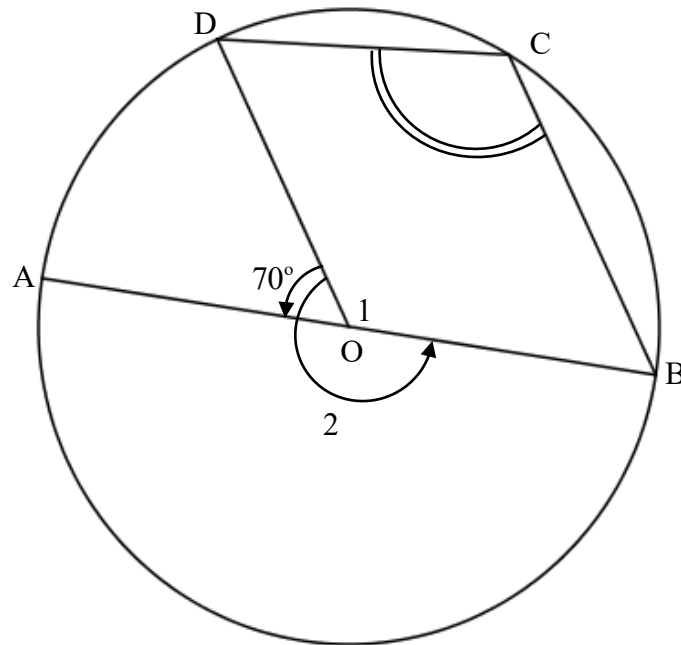
- 8.2 Dayakeramong e ka tlase, O ke mokgubu wa sekele. Q ke midpoente ya chord PR. OQS ke rediyase ya sekele. $PR = 8$ di-unithi, $OQ = x$ di-unithi and $QS = 2$ di-unithi.



- 8.2.1 Fumana, o fana ka mabaka, saeze ya \hat{OQP} . (2)
- 8.2.2 Khaltjhuleitha bolelele ba PO. (5)
- [12]

POTSO YA 9

- 9.1 A, B, C le D ke dipointe tse ho sekhamfarensa ya sekele e nang le mokgubu ho O. AOB ke dayametha ya sekele. $\hat{AOD} = 70^\circ$.



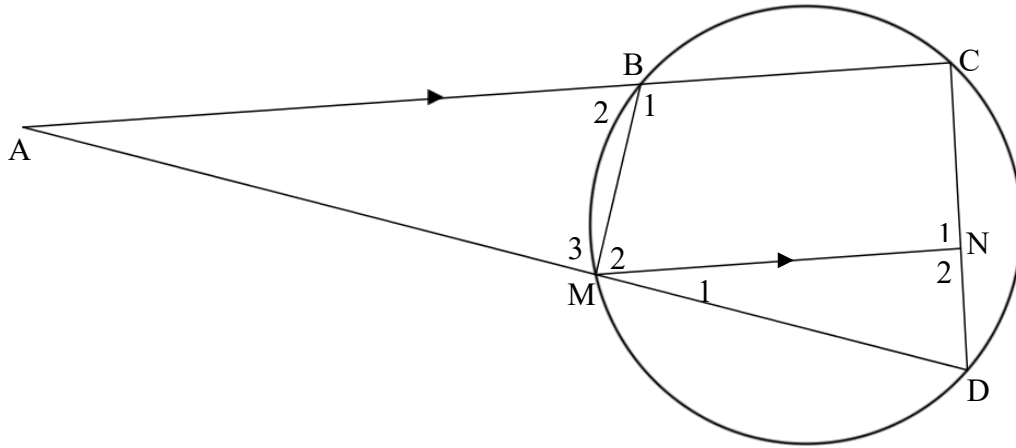
Khaltjhuleitha saeze ya \hat{C} , o fana ka mabaka.

(5)

POTSO YA 10

BCDM ke cyclic quadrilateral. Chords MD le BC di ekeditswe mme tsa kopana ho poente A.
N ke poente ho CD. $AC \parallel MN$ mme $AM = CD$.

AC = 36 di-unithi, AD = 48 di-unithi le BM = 24 di-unithi.



- 10.1 Pruva hore $\triangle ABM \parallel \triangle ADC$. (4)
- 10.2 Pruva hore $CD^2 = BM \times AC$. (3)
- 10.3 Khaltjhuleitha bolelele ba CN. (6)
- [13]**

MATSHWAO KAOFELA: 150

PAMPIRI YA TLHAHISOLESSEDING: MMETSE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n} \quad \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$